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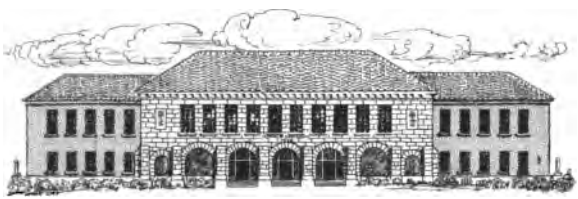

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MATHEMATICS FOR COMMON SCHOOLS

PART II

GRAMMAR-SCHOOL ARITHMETIC

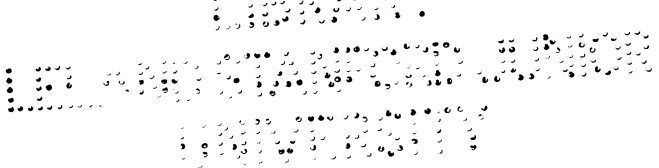
INCLUDING

EASY ALGEBRAIC EQUATIONS AND SIMPLE GEOMETRICAL PROBLEMS

BY

JOHN H. WALSH

ASSOCIATE SUPERINTENDENT OF PUBLIC INSTRUCTION,
BROOKLYN, N.Y.



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PREFACE.

THE PRIMARY ARITHMETIC, Part I., of the two-part edition of MATHEMATICS FOR COMMON SCHOOLS is designed to cover the work of the first four years, and contains those portions of the subject needed by all pupils of the common schools: addition, subtraction, multiplication, and division of whole numbers; simple fractions; and the most commonly used denominations of compound numbers.

Part II., the GRAMMAR SCHOOL ARITHMETIC, completes the ordinary grammar-school course in this subject, and contains, besides, two chapters on algebraic equations and one on elementary constructive geometry, with applications. The first algebra chapter should be taken up with the seventh year's work in percentage and interest. The remaining one, Chapter XV., may be profitably studied where there is time to continue this subject. Although placed at the end of the book, it is intended that suitable portions of the geometry work of Chapter XVI. be taught from time to time during the last two years of the grammar school.

The special features of the work are its division of the arithmetical portion into half-yearly chapters, instead of the ordinary arrangement by topics; the omission, as far as possible, of rules and definitions; the very great number and variety of the examples; the use of the equation in the solution of arithmetical problems, especially in those of percentage and interest; and the introduction of the elements of algebra and geometry.

Believing that there is some foundation for the complaints frequently made by business men and high-school teachers that grammar-school graduates are too often slow and inaccurate in ordinary computations, the author has furnished throughout the entire work systematic drills and reviews in the addition, subtraction, multiplication, and division of ordinary numbers and of fractions.

In this endeavor to enrich the grammar-school course in mathematics, the attempt has not been made to shorten it so much as some may desire. The intelligent teacher can and should do the remainder for himself, by rigorously omitting all such topics as he finds unnecessary.

J. H. W.

BROOKLYN, N.Y., January, 1895.

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CHAPTER VI.

MIXED NUMBERS. — FEDERAL MONEY. — BILLS. — DENOMINATE NUMBERS. — DECIMALS. — MEASUREMENTS.

MIXED NUMBERS.

447. Oral Exercises.

How many halves in 1? How many fourths in 1? Six halves = ? 12 fourths = ? 6 thirds = ? 12 sixths = ?

448. Slate Exercises.

Add:

1.	2.	3.	4.	5.
$7\frac{1}{2}$	$5\frac{3}{4}$	$18\frac{2}{3}$	$3\frac{2}{5}$	$74\frac{1}{9}$
18	$39\frac{1}{4}$	$150\frac{2}{3}$	$\frac{1}{5}$	$37\frac{7}{9}$
$27\frac{1}{2}$	17	$57\frac{2}{3}$	$27\frac{2}{5}$	$\frac{1}{9}$

449. Oral Exercises.

$\frac{2}{2}$ = ? $\frac{3}{2}$ = ? $\frac{5}{2}$ = ? $\frac{9}{2}$ = ? $\frac{11}{2}$ = ? $\frac{13}{2}$ = ?

450. A **mixed number** is a whole number and a fraction.

451. Reduce to a whole number or to a mixed number:

$\frac{12}{3}$ $\frac{15}{4}$ $\frac{17}{2}$ $\frac{18}{6}$ $\frac{23}{4}$ $\frac{21}{5}$ $\frac{11}{3}$

452. Slate Exercises.

Add:

6.	7.	8.	9.	10.
$3\frac{1}{2}$	$8\frac{3}{4}$	$9\frac{2}{3}$	$318\frac{1}{3}$	$87\frac{9}{10}$
95	$29\frac{1}{2}$	$48\frac{1}{3}$	$5\frac{3}{4}$	$17\frac{2}{5}$
$254\frac{1}{2}$	$73\frac{1}{4}$	$35\frac{1}{3}$	$527\frac{7}{8}$	$3\frac{8}{10}$
$7\frac{1}{2}$	$6\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{1}{2}$	$69\frac{1}{5}$

453. Oral Exercises.

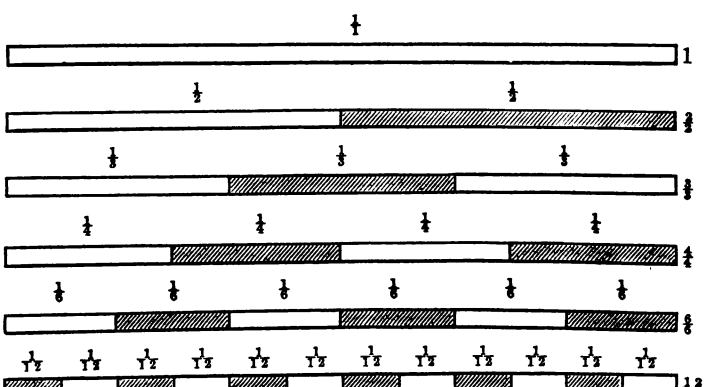
How many quarts in a gallon?

What part of a gallon is a quart?

$\frac{1}{2}$ gallon = how many quarts? $\frac{1}{2}$ = how many fourths?

How many quarts in a peck? What part of a peck is one quart? One-half peck is how many quarts? One-half peck = how many eighths?

$\frac{1}{4}$ peck is how many quarts? $\frac{1}{4}$ = how many eighths? $\frac{3}{4}$ = how many eighths? $\frac{5}{4}$ = how many eighths?



454. Draw a line one foot long. Draw a second line of the same length; divide it into halves. Divide a third line of the same length into three equal parts. Divide three other lines, one into fourths, one into sixths, and one into twelfths.

How many inches in a foot? What part of a foot is one inch? $\frac{1}{2}$ foot = how many inches? $\frac{1}{2}$ = how many twelfths?

$\frac{1}{3}$ = how many twelfths? $\frac{2}{3}$ = how many twelfths? Change $\frac{1}{4}$ to twelfths. Change $\frac{2}{4}$, $\frac{3}{4}$ to twelfths. How many twelfths = $\frac{1}{6}$? $\frac{2}{6}$? $\frac{3}{6}$? $\frac{4}{6}$? $\frac{5}{6}$? $\frac{6}{6}$?

$\frac{2}{12} = \frac{?}{6}$ $\quad \frac{3}{12} = \frac{?}{4}$ $\quad \frac{4}{12} = \frac{?}{6} = \frac{?}{3}$

$\frac{6}{12} = \frac{?}{6} = \frac{?}{4} = \frac{?}{2}$ $\quad \frac{8}{12} = \frac{?}{6} = \frac{?}{3}$ $\quad \frac{9}{12} = \frac{?}{4}$

$\frac{10}{12} = \frac{?}{6}$ $\quad \frac{12}{12} = \frac{?}{6} = \frac{?}{4} = \frac{?}{3} = \frac{?}{2}$

How many inches in $\frac{1}{2}$ ft. + $\frac{1}{3}$ ft. + $\frac{1}{4}$ ft. + $\frac{1}{6}$ ft. + $\frac{1}{12}$ ft.? How many feet and inches?

How many 12ths in $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$? Change to a mixed number. Change the fractional part to a different fraction having the same value.

What fraction of a dime is 1 cent? $\frac{1}{2}$ dime = how many cents? $\frac{1}{2} = \frac{?}{10}$.

$\frac{1}{5}$ dime = how many cents? $\frac{1}{5} = \frac{?}{10}$. Change $\frac{2}{5}$ to tenths. $\frac{3}{5}$. $\frac{4}{5}$. $\frac{5}{5}$.

Add $\frac{1}{2}$ dime, $\frac{1}{5}$ dime, and $\frac{1}{10}$ dime. How many cents? How many tenths $= \frac{1}{2} + \frac{1}{5} + \frac{1}{10}$? Can you change the answer to a different fraction having the same value?

455. Slate Exercises.

Add:

11.	12.	13.	14.	15.
$91\frac{1}{2}$	$62\frac{1}{2}$	$34\frac{1}{2}$	$9\frac{1}{2}$	$31\frac{1}{2}$
$270\frac{1}{4}$	$53\frac{1}{5}$	$6\frac{1}{4}$	$16\frac{1}{3}$	$183\frac{1}{3}$
$3\frac{1}{3}$	$95\frac{1}{10}$	$17\frac{1}{6}$	$25\frac{1}{6}$	$2\frac{1}{6}$
$\frac{1}{16}$	4	$28\frac{1}{3}$	7	$30\frac{1}{4}$

456. Oral Exercises.

Show by a diagram that $\frac{1}{4}$ is the same as $\frac{2}{8}$.

How do we add $\frac{1}{3}$ and $\frac{1}{4}$? Show by a diagram.

How many hours in a day? In $\frac{1}{2}$ day? In $\frac{1}{3}$ day? In $\frac{1}{4}$ day? In $\frac{1}{6}$ day? In $\frac{1}{8}$ day? In $\frac{1}{12}$ day?

Change $\frac{1}{2}$ to twenty-fourths. $\frac{1}{3}$. $\frac{1}{4}$. $\frac{1}{6}$. $\frac{1}{8}$. $\frac{1}{12}$.

Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{12}$, $\frac{5}{8}$, $\frac{11}{12}$ to 24ths.

457. Slate Exercises.

Add:

16. $9\frac{1}{2}$, $8\frac{1}{8}$, $26\frac{3}{8}$

17. $27\frac{3}{4}$, $9\frac{1}{8}$, $100\frac{1}{2}$

18. $33\frac{5}{24}$, $6\frac{1}{12}$, $92\frac{1}{8}$

19. $63\frac{1}{2}$, $9\frac{3}{8}$, $\frac{1}{24}$

20. $87\frac{1}{4}$, $53\frac{5}{8}$, $9\frac{1}{12}$

21. $46\frac{5}{16}$, $5\frac{1}{2}$, $\frac{1}{8}$, $207\frac{1}{4}$

22. 36, $74\frac{3}{8}$, $9\frac{1}{25}$, $81\frac{3}{8}$

23. $275\frac{1}{2}$, $54\frac{3}{10}$, $27\frac{1}{4}$, $6\frac{1}{8}$

24. $93\frac{3}{8}$, $6\frac{1}{2}$, $74\frac{1}{8}$, 87

25. $23\frac{3}{8}$, $65\frac{1}{8}$, $23\frac{1}{8}$, $9\frac{1}{2}$

458. Oral Problems.

1. I spent $\frac{1}{2}$ of a dollar for a ball and $\frac{1}{10}$ of a dollar for a bat. What part of a dollar did I spend for both?

2. $\frac{1}{2}\frac{2}{4}$ = how many fourths?

3. What will be the cost of a pen-knife at $\frac{3}{5}$ of a dollar, and a book at $\frac{1}{2}$ of a dollar?

4. Write $\frac{1}{2}\frac{8}{0}$ with smaller numerator and denominator.

5. I need $\frac{1}{3}$ of a yard of ribbon for one hat and $\frac{1}{8}$ of a yard for another. How much ribbon must I buy?

6. Write a fraction equal to $\frac{9}{12}$ with the smallest numbers you can. (This is called reducing a fraction to lowest terms.)

7. Sold $\frac{3}{8}$ of a pound of tea to one customer and $\frac{7}{8}$ to another. How much was sold to both?

8. What quantity of oats must I buy to give $\frac{3}{4}$ of a peck to one horse and $\frac{5}{8}$ to another?

9. If I sell $\frac{1}{8}$ of a dozen of oranges to one person and $\frac{1}{4}$ of a dozen to another person, what part of a dozen do I sell?

10. What part of an hour is 45 minutes?

11. $\frac{3}{8}$ of an hour is how many minutes?

12. I spent $\frac{1}{5}$ of an hour reading and $\frac{3}{10}$ of an hour writing. What part of an hour did I spend at both?

13. A boy is carrying $6\frac{1}{4}$ lb. of flour, and $6\frac{1}{8}$ lb. of ham. What is the weight of his load?

14. Reduce $\frac{45}{60}$ to lowest terms.

15. 18 hours is what part of a day?

16. Reduce $\frac{18}{24}$ to lowest terms.

ADDITION OF MIXED NUMBERS.

459. Add $12\frac{1}{2}$, $6\frac{3}{8}$, $8\frac{1}{4}$, $15\frac{5}{6}$, $\frac{3}{8}$.

In the fractions $\frac{1}{2}$, $\frac{3}{8}$, $\frac{1}{4}$, $\frac{5}{6}$, $\frac{3}{8}$, the numbers above the line, 1, 2, 1, 5, 3, are called *numerators;* the numbers below the line, 2, 3, 4, 6, 8, are called *denominators.*

To add fractions, they must have the same denominator. An inspection of the denominators, 2, 3, 4, 6, 8, shows that 48 or 24 will contain each without remainder. 24, which is the smallest number that will contain all, is called the *least common denominator.*

	24
$12\frac{1}{2}$	12
$6\frac{3}{8}$	16
$8\frac{1}{4}$	6
$15\frac{5}{6}$	20
$\frac{3}{8}$	9
$43\frac{5}{8}$	$\frac{63}{24} = 2\frac{15}{24} = 2\frac{5}{8}$.

Ans. $43\frac{5}{8}$.

Instead of writing the common denominator 24, with each fraction, we may place it above, and write only the new numerators. $\frac{1}{2} = \frac{12}{24}$, $\frac{3}{8} = \frac{9}{24}$, $\frac{1}{4} = \frac{6}{24}$, etc. Write 12, 16, 6, 20, 9. The sum of these fractions is $\frac{63}{24} = 2\frac{15}{24} = 2\frac{5}{8}$. $\frac{5}{8}$ is placed under the fractions to be added, 2 is carried to the whole numbers, making 43.

NOTE. — The fractional parts of answers should be reduced to *lowest terms.*

460. Slate Exercises.

Add:

1.	2.	3.	4.	5.
$23\frac{1}{2}$	$73\frac{1}{4}$	$93\frac{3}{8}$	$11\frac{3}{7}$	$18\frac{1}{3}$
$63\frac{1}{4}$	$8\frac{1}{8}$	$2\frac{1}{6}$	$3\frac{1}{2}$	$7\frac{2}{5}$
$7\frac{2}{5}$	$39\frac{3}{10}$	$74\frac{5}{12}$	$20\frac{3}{14}$	$9\frac{3}{10}$
$3\frac{3}{10}$	$16\frac{1}{2}$	$6\frac{1}{8}$	$5\frac{1}{7}$	$\frac{1}{8}$

6. $12\frac{1}{8} + 3\frac{5}{16} + 27\frac{3}{4} + 8\frac{1}{2}$
7. $19\frac{1}{2} + 7\frac{2}{3} + 34\frac{5}{6} + \frac{1}{3}$
8. $73\frac{3}{4} + 98\frac{1}{6} + \frac{1}{3} + 33\frac{1}{2}$
9. $5\frac{11}{12} + 38\frac{1}{4} + 23\frac{2}{3} + 17\frac{1}{6}$
10. $100\frac{1}{2} + 75\frac{1}{3} + 9\frac{1}{6} + 49\frac{1}{4}$
11. $33\frac{2}{3} + 17\frac{3}{5} + 24\frac{5}{12} + 69\frac{1}{24}$
12. $6\frac{1}{10} + 18\frac{1}{5} + 32\frac{1}{4} + 94\frac{1}{2}$
13. $103\frac{1}{3} + 84\frac{2}{3} + 25\frac{1}{2} + 9\frac{5}{6}$
14. $218\frac{3}{8} + 301\frac{3}{8} + 18\frac{1}{2} + 24\frac{3}{4}$
15. $444\frac{4}{5} + 518\frac{5}{6} + 37\frac{1}{2} + 95\frac{2}{3}$
16. $93\frac{3}{8} + 127\frac{2}{5} + 18\frac{1}{3} + 509\frac{4}{5}$
17. $8\frac{9}{20} + 64\frac{4}{5} + 126\frac{3}{20} + 1{,}057\frac{1}{4}$
18. $24\frac{1}{2} + 9\frac{3}{4} + 48\frac{5}{12} + 17\frac{1}{6}$
19. $99\frac{1}{2} + 88\frac{3}{10} + 77\frac{3}{4} + 66\frac{4}{5}$
20. $105\frac{5}{18} + 32\frac{2}{3} + 47\frac{1}{6} + 8\frac{1}{3}$

MULTIPLES AND FACTORS.

461. A number that contains another number an exact number of times is a *multiple* of that number.

24 is a multiple of 12; 36, 48, etc., are also multiples of 12.

30 is a multiple of 2, 3, 5, 6, 10, 15.

462. Oral Exercises.

1. 95 is a multiple of what two numbers?
2. Give two factors of 51.
3. What number is a multiple of both 8 and 6?
4. Mention another number that is a multiple of both 8 and 6.
5. Find the smallest number that can be exactly divided by 8 and 12.
6. Give two factors of 91.
7. 57 is a multiple of what two numbers?
8. What is the smallest number that can be exactly divided by 4, 6, and 8.

463. The smallest number that is a multiple of two or more numbers is called the **least common multiple** of such numbers.

9. Find the least common multiple of 5, 10, 15.
10. Find the least common multiple of 2, 7, 14.
11. What part of a dollar is 70 cents?
12. $\frac{3}{5}$ = how many 40ths?
13. Reduce $17\frac{2}{3}$ to thirds.
14. Change $\frac{74}{5}$ to a mixed number.
15. Reduce $\frac{24}{36}$ to lowest terms.
16. What part of a dollar is 15 cents?
17. Add $\frac{1}{3}$ and $\frac{3}{4}$.
18. From $\frac{1}{2}$ take $\frac{1}{5}$.
19. Reduce $\frac{19}{20}$ to 60ths.
20. Which is largest, $\frac{5}{6}$, $\frac{5}{7}$, or $\frac{5}{8}$?
21. $\frac{3}{4}$ of 84 = ?
22. From $\frac{11}{24}$ take $\frac{7}{24}$.
23. From $\frac{9}{14}$ take $\frac{5}{14}$.
24. From $\frac{15}{16}$ take $\frac{9}{16}$.
25. From $\frac{11}{20}$ take $\frac{3}{20}$.

SUBTRACTION OF MIXED NUMBERS.

464. Sight Exercises.

Subtract:

1. $16\frac{11}{12}$	2. $49\frac{19}{20}$	3. $38\frac{23}{24}$	4. $18\frac{8}{15}$	5. $27\frac{9}{10}$
$13\frac{7}{12}$	$37\frac{13}{20}$	$29\frac{11}{24}$	$14\frac{8}{15}$	$16\frac{1}{10}$
6. $28\frac{7}{16}$	7. $47\frac{7}{9}$	8. $36\frac{11}{14}$	9. $25\frac{15}{16}$	10. $32\frac{17}{18}$
$13\frac{5}{16}$	$29\frac{5}{9}$	$18\frac{5}{14}$	$19\frac{11}{16}$	$18\frac{7}{18}$

465. Slate Exercises.

11. $35\frac{3}{4}$	12. $63\frac{1}{2}$	13. $27\frac{3}{8}$	14. $55\frac{5}{6}$	15. $105\frac{3}{10}$
$-8\frac{2}{5}$	$-9\frac{1}{14}$	$-17\frac{1}{3}$	$-25\frac{3}{4}$	$-8\frac{1}{4}$
16. $120\frac{5}{6}$	17. $39\frac{2}{5}$	18. $13\frac{5}{8}$	19. $99\frac{5}{6}$	20. $67\frac{3}{4}$
$-84\frac{7}{9}$	$-38\frac{1}{4}$	$-7\frac{5}{12}$	$-21\frac{4}{5}$	$-59\frac{1}{5}$
21. 480	22. 375	23. 200	24. 873	25. 1,000
$-97\frac{1}{2}$	$-83\frac{1}{4}$	$-90\frac{1}{8}$	$-757\frac{2}{3}$	$-998\frac{5}{6}$
26. 275	27. 676	28. 999	29. 132	30. 23
$-250\frac{3}{7}$	$-76\frac{5}{8}$	$-138\frac{2}{3}$	$-46\frac{3}{10}$	$-18\frac{1}{11}$

466. Oral Exercises.

$1-\frac{1}{2}=?$ $1\frac{1}{4}-\frac{1}{2}=?$ $10-\frac{1}{2}=?$ $10\frac{1}{4}-\frac{1}{2}=?$ $10\frac{1}{4}-1\frac{1}{2}=?$

467. From $197\frac{2}{3}$ take $68\frac{2}{5}$.

	15	
$197\frac{2}{3}$	9	Reduce the fractions to a common denominator (15), as in addition of fractions. $\frac{10}{15}$ being greater than $\frac{9}{15}$, we find the difference between $\frac{10}{15}$ and $1\frac{9}{15}$, or $\frac{24}{15}$. Writing this difference, $\frac{14}{15}$, we add 1 to 68 and subtract.
$68\frac{2}{5}$	10	
$128\frac{14}{15}$	$\frac{14}{15}$	

468. Subtract:

31.	$8\frac{1}{4}$	32.	$23\frac{1}{2}$	33.	$34\frac{1}{8}$	34.	$16\frac{1}{8}$	35.	$21\frac{1}{4}$
	$5\frac{1}{2}$		$5\frac{3}{4}$		$27\frac{3}{8}$		$8\frac{1}{2}$		$9\frac{3}{4}$
36.	$38\frac{1}{9}$	37.	$33\frac{1}{4}$	38.	$77\frac{1}{10}$	39.	$50\frac{1}{5}$	40.	$99\frac{3}{8}$
	$29\frac{1}{3}$		$7\frac{1}{4}$		$59\frac{1}{5}$		$24\frac{1}{2}$		$88\frac{1}{2}$
41.	$100\frac{3}{10}$	42.	$25\frac{3}{10}$	43.	$93\frac{3}{16}$	44.	$101\frac{20}{21}$	45.	$12\frac{2}{3}$
	$76\frac{2}{5}$		$5\frac{1}{2}$		$24\frac{1}{8}$		$98\frac{3}{7}$		$4\frac{3}{8}$
46.	$23\frac{4}{5}$	47.	$9\frac{5}{18}$	48.	$133\frac{3}{4}$	49.	$16\frac{1}{11}$	50.	$37\frac{1}{6}$
	$16\frac{7}{10}$		$3\frac{1}{3}$		$27\frac{7}{16}$		$3\frac{5}{12}$		$29\frac{1}{2}$
51.	$52\frac{3}{8}$	52.	$64\frac{2}{9}$	53.	$125\frac{7}{10}$	54.	$47\frac{1}{9}$	55.	$72\frac{1}{10}$
	$49\frac{5}{6}$		$18\frac{1}{3}$		$100\frac{3}{5}$		$8\frac{5}{6}$		$50\frac{1}{5}$
56.	$31\frac{3}{5}$	57.	$63\frac{1}{3}$	58.	$3\frac{1}{12}$	59.	$25\frac{2}{3}$	60.	$102\frac{5}{12}$
	$27\frac{5}{12}$		$44\frac{1}{6}$		$1\frac{1}{8}$		$17\frac{1}{2}$		$86\frac{1}{2}$

469. Slate Problems.

1. From a piece of cloth containing $17\frac{1}{2}$ yards, $5\frac{3}{4}$ yards and $4\frac{3}{8}$ yards were sold. How many yards were left?

2. A boarding school uses 3 quarts of milk a day for 7 pupils. If there are 77 pupils in the school, how many gallons of milk will be needed per day? Per week?

3. A man pays \$140.40 for 3 pieces of cloth. What is the length of each piece, if the cloth costs \$1.80 per yard?

4. I buy $12\frac{3}{4}$ pounds coffee at 28 cents per pound, and twice as many pounds at 24 cents per pound. If I give the storekeeper \$10, how much change will I receive?

5. A merchant pays \$30 for 65 vases. He sells 17 of them at 40 cents each, 23 at 60 cents each, and receives 48 cents each for the others. What is his profit?

6. Mrs. Jones buys $\frac{1}{2}$ dozen chairs, a bureau for $17.40, and a mirror for $13.00. She pays for all $38.80. What do the chairs cost apiece?

7. In selling 40 yards of velvet that cost me $1.40 per yard, I gained $24.00. What did I charge per yard?

8. How much heavier is a cheese weighing $40\frac{5}{6}$ pounds than one which weighs $26\frac{3}{4}$ pounds?

9. A farmer sold $36\frac{1}{2}$ dozen eggs to one storekeeper, $5\frac{3}{8}$ dozen to another, $17\frac{3}{4}$ dozen to a third, $8\frac{5}{6}$ dozen to a fourth, and $11\frac{7}{12}$ dozen to a fifth. How much did he receive for them at 12 cents per dozen?

10. A butcher going to market with $174.40, bought 15 sheep at $8 each. If he had had $25.60 more, he could have bought 4 hogs also. What were the hogs worth apiece?

11. A scholar having to multiply a number by 30, mistook the 3 for a 5, and his answer was 600. What is the correct answer?

12. Forty hats were bought for $104. At what price apiece should they be sold to make 40 cents on each hat? At what price per dozen?

13. A farmer had 7 bushels of potatoes. He used 2 bushels and 3 pecks for seed. What would the remainder be worth at 20¢ per peck?

14. A butcher buys an ox weighing 1,200 pounds alive, at 6 cents per lb. When killed and dressed, its weight is $\frac{2}{3}$ of the live weight. What is the butcher's profit, if he sells the meat at an average of 15 cents per lb.?

15. The weight of a tub of butter, including the weight of the tub, is $48\frac{1}{4}$ pounds. The tub weighs $9\frac{1}{2}$ lb. What is the butter worth at 24 cents per pound?

16. One boy had 15 marbles, another had 19, a third had 17, a fourth had 13. What was the average number of marbles for each boy?

17. A teacher divided 200 foreign postage stamps among the eight boys of his class. He gave one-fourth of them to the first boy, one-fifth of the remainder to the second boy, and then divided the rest equally among the other six boys. How many did each receive?

18. A merchant sold $17\frac{3}{4}$ yards of muslin, $14\frac{1}{2}$ yards of silk, and as many yards of calico as of the other two together. How many yards did he sell in all?

19. A dealer mixed $2\frac{1}{2}$ pounds of black tea costing 32 cents per pound with $1\frac{1}{2}$ pounds of green tea costing 40 cents per pound. How much per pound does the mixed tea cost him?

20. A man can do $\frac{1}{8}$ of a certain piece of work in a day; another man can do $\frac{1}{8}$ of the same work in a day. What part of the work can both together do in a day? How long would it take both together to finish the work?

470. Slate Exercises.

Find answers:

1. $36 \times 7\frac{1}{8}$	**2.** $124 \times 19\frac{3}{4}$	**3.** $28\frac{1}{2} \times 8$	**4.** $47\frac{1}{8} \times 12$	**5.** $28 \times 4\frac{3}{7}$
6. $93 \times 106\frac{2}{3}$	**7.** $84\frac{3}{4} \times 4$	**8.** $57 \times 3\frac{1}{2}$	**9.** $69 \times 4\frac{1}{4}$	**10.** $81 \times 15\frac{1}{6}$

471. Divide:

11. $93\frac{3}{4} \div 3$	**15.** $80\frac{5}{8} \div 5$	**19.** $825\frac{2}{25} \div 11$
12. $56\frac{1}{5} \div 4$	**16.** $126\frac{2}{13} \div 6$	**20.** $1{,}728\frac{2}{15} \div 12$
13. $98\frac{7}{8} \div 7$	**17.** $450\frac{9}{10} \div 9$	**21.** $2{,}460\frac{10}{11} \div 10$
14. $28\frac{3}{8} \div 2$	**18.** $360\frac{8}{9} \div 8$	**22.** $3{,}926\frac{13}{15} \div 13$

NOTATION AND NUMERATION.

472. The largest number that can be written with six figures is 999,999.

1,000,000 is called one million.

Write in figures two million. Three million. Four million. Six million. Eight million. Ten million.

473. Read the following:

1. 1,234,567	6. 11,034,065	11. 30,100,021
2. 3,000,560	7. 14,602,500	12. 35,000,600
3. 5,009,008	8. 17,386,925	13. 401,023,160
4. 7,090,070	9. 20,007,316	14. 760,030,020
5. 9,843,000	10. 25,000,005	15. 980,750,000

474. Write in figures:

1. Seventy-eight million, one hundred eight thousand, ninety-six.

2. Three million, eight.

3. Fourteen million, seven thousand, five.

4. Nine hundred eighty-seven thousand, six hundred fifty-four.

5. Twenty million, thirty thousand, forty.

6. Three hundred seven million, nine hundred four thousand, six.

7. Nine hundred ninety-nine million, nine hundred ninety-nine thousand, nine hundred ninety-nine.

8. Four hundred seventy-six million, three hundred thousand.

9. Thirty-four thousand, eighteen.

10. Sixty-four million, thirty-two thousand, sixteen.

Add the foregoing.

475. Review. Slate Exercises.

Read the following numbers. Add each column.

1.	2.	3.
27,083,549	508,900,007	243,576,908
116,908,070	63,487,291	36,200,570
3,006,005	4,629,880	5,987,600
20,080,070	25,936,097	380,070
1,647,893	134,870,603	68,000
206,045	59,009,300	593,056
73,000	7,000,004	2,384,672
180,059	686,909	59,876,004
2,316,045	50,308	123,321,123
54,006,000	9,999	88,888,888

476. Add down and across:

1 +	2 +	3 +	4 = ?
20 +	30 +	40 +	50 = ?
300 +	400 +	500 +	600 = ?
4,000 +	3,000 +	2,000 +	1,000 = ?
50,000 +	40,000 +	30,000 +	20,000 = ?
600,000 +	500,000 +	400,000 +	300,000 = ?
7,000,000 +	8,000,000 +	6,000,000 +	5,000,000 = ?
30,000,000 +	40,000,000 +	50,000,000 +	60,000,000 = ?
? +	? +	? +	? = ?

477. Multiply across. Add multipliers and products.

$10 \times 2 = ?$	$12 \times 7 = ?$
$10 \times 5 = ?$	$12 \times 4 = ?$
$10 \times ? = ?$	$12 \times ? = ?$
$25 \times 2 = ?$	$50 \times 4 = ?$
$25 \times 10 = ?$	$50 \times 20 = ?$
$25 \times ? = ?$	$50 \times ? = ?$
$123 \times 3 = ?$	$560 \times 4 = ?$
$123 \times 20 = ?$	$560 \times 50 = ?$
$123 \times 100 = ?$	$560 \times 600 = ?$
$123 \times ? = ?$	$? \times ? = ?$

478. Add:

1.	2.	3.
\$184,635.87½	\$263,005.95	\$5,503.00
25,904.63	38,462.77	45,837.24
8,756.95	159,076.50	24,328.49
889.57	50,318.92	567,849.60
4,326.98¼	36,485.73	20,486.04
58,030.05	9,860.44	54,200.30
209,508.67	76,035.00	2,578.64
37,654.88	8,900.56	328.99
9,876.54¼	4,056.02	3,468.36
63,250.89	26,465.54	87,243.25
6,505.33	7,826.98	28,376.42

MULTIPLICATION OF MIXED NUMBERS.

483. Sight Exercises.

Give answers:

1. $\frac{2}{5}$ of 9
2. $\frac{3}{4}$ of 7
3. $\frac{5}{6}$ of 11
4. $\frac{2}{3}$ of 17
5. $\frac{3}{5}$ of 15
6. $\frac{3}{7}$ of 20
7. $\frac{7}{9}$ of 13
8. $\frac{1}{3}$ of 14
9. $\frac{9}{10}$ of 19
10. $\frac{4}{11}$ of 30
11. $\frac{5}{12}$ of 17
12. $\frac{4}{13}$ of 10

13. $9 \times \frac{2}{5}$
14. $7 \times \frac{3}{4}$
15. $11 \times \frac{5}{6}$
16. $17 \times \frac{2}{3}$
17. $15 \times \frac{3}{5}$
18. $20 \times \frac{3}{7}$
19. $13 \times \frac{7}{9}$
20. $14 \times \frac{1}{3}$
21. $19 \times \frac{9}{10}$
22. $23 \times \frac{4}{11}$
23. $17 \times \frac{5}{12}$
24. $10 \times \frac{4}{13}$

484. Slate Exercises.

25. $9 \times 1\frac{2}{5}$
26. $7 \times 2\frac{3}{4}$
27. $11 \times 3\frac{5}{6}$
28. $17 \times 4\frac{2}{3}$
29. $15 \times 5\frac{3}{5}$
30. $20 \times 6\frac{3}{7}$
31. $13 \times 7\frac{7}{9}$
32. $19 \times 8\frac{9}{10}$
33. $23 \times 9\frac{4}{11}$
34. $17 \times 10\frac{5}{12}$
35. $12\frac{2}{5} \times 9$
36. $13\frac{3}{4} \times 7$
37. $14\frac{5}{6} \times 11$
38. $15\frac{2}{3} \times 17$
39. $16\frac{3}{5} \times 15$
40. $17\frac{3}{7} \times 20$
41. $18\frac{7}{9} \times 13$
42. $19\frac{9}{10} \times 19$
43. $20\frac{4}{11} \times 23$
44. $21\frac{5}{12} \times 12$
45. $22\frac{1}{13} \times 14$

46. $387 \times 400\frac{3}{4}$
47. $1{,}986 \times 3{,}075\frac{4}{5}$
48. $2{,}340 \times 1{,}607\frac{2}{7}$
49. $698 \times 35\frac{5}{6}$
50. $103\frac{2}{3} \times 275$
51. $4103\frac{3}{4} \times 17$
52. $31{,}509\frac{4}{5} \times 19$
53. $2{,}308\frac{3}{7} \times 1{,}560$
54. $6{,}089\frac{5}{6} \times 3{,}520$

55. $2{,}345\frac{3}{5}$ × 3,004

57. $1{,}357\frac{3}{5}$ × 2,060

59. $3{,}579\frac{1}{16}$ × 4,300

56. 4,060 × $2{,}050\frac{3}{5}$

58. 3,050 × $2{,}060\frac{4}{5}$

60. 2,469 × $4{,}987\frac{2}{11}$

DIVISION OF MIXED NUMBERS.

488. Oral Exercises.

How many times is $\frac{1}{2}$ of a dollar contained in \$1? How many times is $\frac{1}{2}$ of a pint contained in 1 pint? $\frac{1}{2}$ of a gallon in 1 gallon?

How many times is $\frac{1}{2}$ of a dollar contained in \$2? In \$3? In \$5?

How many times is $\frac{1}{2}$ of a dollar contained in \$1.50? In \$2.50? In \$3.50? In \$4.50?

How many times is 1 half contained in 3 halves? In 5 halves? In 7 halves? In 9 halves?

$\frac{3}{2} \div \frac{1}{2} = ?$ $\frac{5}{2} \div \frac{1}{2} = ?$ $\frac{7}{2} \div \frac{1}{2} = ?$ $\frac{9}{2} \div \frac{1}{2} = ?$

How many times is $\frac{3}{2}$ contained in $\frac{3}{2}$? In $\frac{9}{2}$? In $\frac{15}{2}$? In $\frac{21}{2}$?

Divide $1\frac{1}{2}$ by $1\frac{1}{2}$. $4\frac{1}{2}$ by $1\frac{1}{2}$. $7\frac{1}{2}$ by $1\frac{1}{2}$. $10\frac{1}{2}$ by $1\frac{1}{2}$.

Divide 3 by $1\frac{1}{2}$. 6 by $1\frac{1}{2}$. 9 by $1\frac{1}{2}$. 12 by $1\frac{1}{2}$. 15 by $1\frac{1}{2}$.

Divide 5 by $1\frac{1}{4}$. $6\frac{1}{4}$ by $1\frac{1}{4}$. 10 by $1\frac{1}{4}$. $11\frac{1}{4}$ by $1\frac{1}{4}$. 15 by $1\frac{1}{4}$.

Divide $\frac{5}{4}$ by $\frac{5}{4}$. $\frac{6}{4}$ by $\frac{5}{4}$. $1\frac{2}{4}$ by $\frac{5}{4}$. $1\frac{1}{2}$ by $\frac{5}{4}$. $2\frac{1}{4}$ by $\frac{5}{4}$. 3 by $\frac{5}{4}$. $3\frac{3}{4}$ by $\frac{5}{4}$.

489. Slate Exercises.

Divide 250 by $12\frac{1}{2}$.

$12\frac{1}{2}$ = 25 halves.

250 = 500 halves.

500 halves ÷ 25 halves = 500 ÷ 25 = 20, *Ans.*

Divide $1{,}387\frac{1}{2}$ by $18\frac{3}{4}$.

$18\frac{3}{4} = 75$ fourths.

Change $1{,}387\frac{1}{2}$ to fourths by multiplying by 4.

$1{,}387\frac{1}{2} \times 4 = 5{,}550$; that is, $1{,}387\frac{1}{2} = 5{,}550$ fourths.

75 fourths is contained in 5,550 fourths 74 times.

Ans. 74

$75)5550$

300

00

490. Divide. Prove the correctness of the answer by multiplying the quotient by the divisor.

1. $75 \div 12\frac{1}{2}$
2. $150 \div 12\frac{1}{2}$
3. $75 \div 6\frac{1}{4}$
4. $150 \div 6\frac{1}{4}$
5. $62 \div 15\frac{1}{2}$
6. $105 \div 17\frac{1}{2}$
7. $69 \div 5\frac{3}{4}$
8. $93 \div 7\frac{3}{4}$
9. $100 \div 33\frac{1}{3}$
10. $150 \div 16\frac{2}{3}$

491.

11. $62\frac{1}{2} \div 12\frac{1}{2}$
12. $187\frac{1}{2} \div 12\frac{1}{2}$
13. $81\frac{1}{4} \div 6\frac{1}{4}$
14. $193\frac{3}{4} \div 6\frac{1}{4}$
15. $77\frac{1}{2} \div 15\frac{1}{2}$
16. $192\frac{1}{2} \div 17\frac{1}{2}$
17. $97\frac{3}{4} \div 5\frac{3}{4}$
18. $193\frac{3}{4} \div 7\frac{3}{4}$
19. $166\frac{2}{3} \div 33\frac{1}{3}$
20. $133\frac{1}{3} \div 16\frac{2}{3}$
21. $60 \div \frac{1}{2}$
22. $60 \div 1\frac{1}{2}$
23. $60 \div \frac{1}{3}$
24. $60 \div 1\frac{1}{3}$
25. $60 \div \frac{1}{4}$
26. $60 \div 1\frac{2}{3}$
27. $60 \div \frac{2}{3}$
28. $60 \div 2\frac{1}{2}$
29. $60 \div \frac{3}{4}$
30. $60 \div 3\frac{1}{3}$

492.

31. $60 \div 3\frac{3}{4}$
32. $60 \div 7\frac{1}{2}$
33. $60\frac{3}{4} \div 3$
34. $60\frac{1}{2} \div 2$
35. $62\frac{1}{2} \div 6\frac{1}{4}$
36. $87\frac{1}{2} \div 6\frac{1}{4}$
37. $15\frac{3}{4} \div 1\frac{3}{4}$
38. $24\frac{1}{2} \div 1\frac{3}{4}$
39. $87\frac{1}{2} \div 8\frac{3}{4}$
40. $42\frac{1}{2} \div 4\frac{1}{4}$

493.

41. $75 \div 3\frac{1}{8}$
42. $125 \div 3\frac{1}{8}$
43. $53\frac{1}{8} \div 3\frac{1}{8}$
44. $190\frac{5}{8} \div 3\frac{1}{8}$
45. $100 \div 11\frac{1}{9}$
46. $100 \div 1\frac{1}{9}$
47. $12\frac{1}{2} \div 2\frac{1}{12}$
48. $18\frac{3}{4} \div 2\frac{1}{12}$
49. $33\frac{1}{3} \div 2\frac{1}{12}$
50. $29\frac{1}{6} \div 2\frac{1}{12}$
51. $32\frac{1}{2} \div 3\frac{1}{4}$
52. $50\frac{2}{3} \div 4\frac{1}{3}$
53. $51\frac{3}{8} \div 5\frac{1}{6}$
54. $61\frac{3}{7} \div 6\frac{1}{7}$

FEDERAL MONEY.

494. Slate Exercises.

Add the following without placing the amounts in columns.

1. \$8.34, \$40.39, \$638.27, \$594.38, \$1.97.
2. \$0.03, \$8.05, \$600.00, \$38.72, \$198.52, \$0.63.
3. \$432.84, \$96.25, \$3.64, \$782.46, \$800.06, \$6.50.
4. \$3.60, \$40.05, \$91.86, \$350.48, \$84.00, \$287.63.
5. \$98.27, \$0.60, \$600.39, \$8.09, \$37.38, \$503.07.
6. \$202.97, \$42.23, \$453.60, \$7.18, \$63.54, \$0.37.
7. \$8.43, \$0.54, \$2.57, \$85.13, \$425.31, \$8.27.
8. \$486.54, \$84.62, \$1.96, \$8.13, \$35.84, \$236.49.
9. \$83.61, \$523.00, \$23.04, \$0.86, \$35.82, \$584.60.
10. \$34.80, \$93.54, \$200.41, \$324.86, \$50.14, \$8.75.

The foregoing examples may be added directly from this book or from the blackboard, the pupils writing on their slates nothing but the answers.

495. Subtract the following without re-arranging them. Find the sum of the minuends, the sum of the subtrahends, and the sum of the remainders.

11.	$1,000.00	− $876.49	=
12.	$549.37	− $99.89	=
13.	$345.93	− $76.04	=
14.	$1,786.08	− $1,097.19	=
15.	$345.00	− $187.23	=
16.	$3,545.37	− $966.38	=
17.	$82.46	− $7.59	=
18.	$5,074.02	− $4,987.63	=
19.	$77.84	− $9.88	=
20.	$4,680.35	− $4,679.46	=
21.	?	− ?	= ?

496. Multiplication.

Find the cost of:

1. 197 barrels of flour, at $5.66 per barrel.
2. 486 bushels of wheat, at $1.04 per bushel.
3. 237 tons of plaster, at $6.72 per ton.
4. 809 tons of hay, at $11.45 per ton.
5. 74 car-loads of bran, at $20.62½ per load.
6. 208 sheep, at $4.65 per head.
7. 673 barrels of mackerel, at $10.60 per barrel.
8. 984 bushels of onions, at $1.09 per bushel.
9. 99 pounds of butter, at 24 cents per pound.
10. 208 pounds of coffee, at 28 cents per pound.

497. What will be the cost of 157 pounds of sugar, at 5¢ per pound.

157
5
785

At 5¢ per pound 157 pounds will cost 157 times 5¢. In practice, however, we multiply 157 by the smaller number 5.

Ans., $7.85.

11. 1,376 yd. of muslin, at $6\frac{3}{4}$¢.

12. 2,084 bu. of corn, at $47\frac{1}{2}$¢.

13. 1,864 lb. of beef, at $5\frac{1}{2}$¢.

14. 988 lb. of turkeys, at $13\frac{1}{2}$¢.

15. 296 bu. of potatoes, at $47\frac{1}{2}$¢.

16. 1,272 lb. of dried apples, at $2\frac{3}{4}$¢.

17. 488 lb. of lard, at $10\frac{7}{8}$¢.

18. 2,240 lb. of sugar, at $4\frac{5}{8}$¢.

19. 5,176 lb. of wool, at $30\frac{3}{4}$¢.

20. 4,892 bu. of wheat, at $99\frac{3}{4}$¢.

FRACTIONAL PARTS OF A DOLLAR.

499. What will be the cost of 16 base-balls at 25 cents each?

Multiplying 25 cents by 16, we get the answer $4.00. A shorter way is to multiply $\frac{1}{4}$ of a dollar by 16 which gives 16 quarter-dollars, or 4 dollars.

500. Oral Exercises.

At 25 cents per lb., yd., doz., etc., what will be paid for:

1. 32 base-balls?
2. 52 lb. of coffee?
3. 48 straw hats?
4. 84 yd. of ribbon?
5. 36 second readers?
6. 56 vases?
7. 37 doz. lemons?
8. 25 bu. of tomatoes?
9. 41 panes of glass?
10. 33 packages of candy?
11. 49 Roman candles?
12. 60 bars of soap?

501. At 50 cents, give the cost of:

13. 46 lb. of tea.
14. 28 pairs of scissors.
15. 38 penknives.
16. 84 third readers.
17. 44 lb. of candy.
18. 32 caps.
19. 76 grammars.
20. 57 boxes of pens.
21. 49 picture books.
22. 83 dolls.
23. 27 games.
24. 75 feather dusters.

25. What part of a dollar is 75 cents?

26. If I pay 3 quarter-dollars each for sleds, how many quarter-dollars will I pay for a dozen sleds? How many dollars?

502. Give the cost of the following at 75 cents per yd., etc.

27. 16 pairs of skates.
28. 11 yd. of silk.
29. 24 bu. of peaches.
30. 15 gal. of syrup.
31. 9 pairs of gloves.
32. 18 hats.
33. 21 bu. of rye.
34. 22 shad.
35. 13 lb. of tea.
36. 30 gal. of ice-cream.
37. 7 mats.
38. 17 concert tickets.

39. How many cents in one-eighth of a dollar?

40. At one-eighth of a dollar each, what will be the cost of 24 bars soap?

503. Give the cost of the following at $12\frac{1}{2}$ cents per lb., etc.

41. 16 lb. of meat.
42. 48 doz. eggs.
43. 72 qt. of plums.
44. 64 gal. of oil.
45. 80 jars of jelly.
46. 96 cans of condensed milk.
47. 104 yd. of sheeting.
48. 88 lb. of currants.

49. How many cents in one-third of a dollar?

50. At one-third of a dollar each, what will be the cost of a dozen bottles of cologne?

504. Give the cost, at $33\frac{1}{3}$ cents per yd., lb., etc., of:

51. 36 yd. of ribbon.
52. 63 pairs of cuffs.
53. 48 lb. of butter.
54. 27 bu. of oats.
55. 54 pk. of walnuts.
56. 72 doz. oranges.

505. Parts of a Dollar.

$6\frac{1}{4}$ cents $= \frac{1}{16}$ of \$1
$8\frac{1}{3}$ cents $= \frac{1}{12}$ of \$1
$12\frac{1}{2}$ cents $= \frac{1}{8}$ of \$1
$16\frac{2}{3}$ cents $= \frac{1}{6}$ of \$1
25 cents $= \frac{1}{4}$ of \$1
$33\frac{1}{3}$ cents $= \frac{1}{3}$ of \$1
$37\frac{1}{2}$ cents $= \frac{3}{8}$ of \$1
50 cents $= \frac{1}{2}$ of \$1
$62\frac{1}{2}$ cents $= \frac{5}{8}$ of \$1
$66\frac{2}{3}$ cents $= \frac{2}{3}$ of \$1
75 cents $= \frac{3}{4}$ of \$1
$87\frac{1}{2}$ cents $= \frac{7}{8}$ of \$1

506. Oral Exercises.

Give the cost of 72 articles at:

57. $12\frac{1}{2}$ cents each.
58. $37\frac{1}{2}$ cents each.
59. $62\frac{1}{2}$ cents each.
60. $87\frac{1}{2}$ cents each.
61. $33\frac{1}{3}$ cents each.
62. $66\frac{2}{3}$ cents each.
63. $8\frac{1}{3}$ cents each.
64. 25 cents each.
65. 50 cents each.
66. 75 cents each.

507. Multiply:

67. $6\frac{1}{4}$ cents $\times$ 16
68. $8\frac{1}{3}$ cents $\times$ 24
69. $12\frac{1}{2}$ cents $\times$ 88
70. $16\frac{2}{3}$ cents $\times$ 54
71. 25 cents $\times$ 240
72. $33\frac{1}{3}$ cents $\times$ 66
73. $37\frac{1}{2}$ cents $\times$ 48
74. 50 cents $\times$ 186
75. $62\frac{1}{2}$ cents $\times$ 32
76. $66\frac{2}{3}$ cents $\times$ 33
77. 75 cents $\times$ 128
78. $87\frac{1}{2}$ cents $\times$ 88
79. $16\frac{2}{3}$ cents $\times$ 246
80. $33\frac{1}{3}$ cents $\times$ 156

81. $37\frac{1}{2}$ cents $\times$ 80
82. 75 cents $\times$ 160
83. $87\frac{1}{2}$ cents $\times$ 24
84. $62\frac{1}{2}$ cents $\times$ 64
85. $6\frac{1}{4}$ cents $\times$ 160
86. $12\frac{1}{2}$ cents $\times$ 168
87. $\$1.33\frac{1}{3} \times$ 24
88. $\$1.12\frac{1}{2} \times$ 16
89. $\$2.25 \times$ 12
90. $\$3.75 \times$ 12
91. $\$4.37\frac{1}{2} \times$ 8
92. $\$5.16\frac{2}{3} \times$ 6

508. Find the cost of:

93. 86 neckties, at 50 cents each.
94. Six dozen handkerchiefs, at 25 cents apiece.
95. 32 yards of silk, at $\$1.12\frac{1}{2}$ per yard.
96. 64 arithmetics, at 75 cents each.
97. 84 geographies, at $\$1.25$ each.
98. 96 lb. of tea, at 75 cents per pound.
99. 84 pairs of gloves, at $\$1.50$ per pair.
100. 72 yards of cloth, at $\$2.12\frac{1}{2}$ per yard.

509. Blackboard Exercises.

Write answers.

Do not write quantities or prices on the slate.

1. 687 pounds, at 4¢.
2. 976 yards, at 6¢.
3. 2,504 dolls, at 25¢.
4. 352 yards, at $12\frac{1}{2}$¢.
5. 1,728 hats, at 50¢.
6. 933 yards, at $33\frac{1}{3}$¢.
7. 938 coats, at $\$7$.
8. 695 pounds, at 20¢.
9. 248 pounds, at 75¢.
10. 186 pounds, at $66\frac{2}{3}$¢.
11. 12 bushels, at $\$1.43$.
12. 9 tons, at $\$22.75$.
13. 8 barrels, at $\$16.37\frac{1}{2}$.
14. 11 sheep, at $\$7.47$.

15. 16 gallons, at $3.62½.
16. 13 sacks of salt, at $1.11.
17. 124 bushels, at $1.50.
18. 96 pounds, at $1.25.
19. 120 gallons, at $2.33⅓.
20. 64 sacks, at $1.12½.

510. Oral Exercises.

At 50 cents each, how many penknives can be bought for $1? For $2? For $3? For $10? For $20?

At 25 cents each, how many readers can be bought for $1? For $2? For $3? For $10? For $20?

At 12½ cents per yard, how many yards can be bought for $1? For $2? For $3? For $10? For $20?

At 33⅓ cents per pound, how many pounds can be bought for $1? For $2? For $3? For $10? For $20?

511. Oral.

At 25 cents each:

1. How many base-balls can be bought for $9?
2. Straw hats, for $12?
3. Roman candles, for $18?
4. Readers, for $15?
5. Vases, for $21?
6. Bars of soap, for $3¼?
7. Packages of candy, for $4½?
8. Yards of ribbon, for $5.75?
9. Bushels of tomatoes, for $10.50?
10. Pounds of coffee, for $12.75?

512. At 50 cents:

11. Pounds of tea, for $43?
12. Penknives, for $20.50?
13. Pounds of candy, for $94?
14. Third readers, for $17.50?

15. Caps, for $21?
16. Grammars, for $37?
17. Boxes of pens, for $72?
18. Dolls, for $64?
19. Pairs of scissors, for $19?
20. Feather dusters, for $26.50?

513. At 12½ cents:

21. Gallons of oil, for $8?
22. Dozen of eggs, for $11?
23. Pounds of meat, for $21?
24. Quarts of plums, for $1⅛?
25. Jars of jelly, for $¼?
26. Yards of sheeting, for $1½?
27. Cans of milk, for $2¼?
28. Pounds of currants, for $3.12½
29. Whisk brooms, for $4.37½?
30. Collars for $5.62½?

514. At 33⅓ cents:

31. Yards of ribbon, for $6?
32. Pairs of cuffs, for $12?
33. Pounds of butter, for $18?
34. Bushels of oats, for $32?
35. Pecks of walnuts, for $1⅓?
36. Dozen of oranges, for $1⅔?
37. Straw hats, for $2.33⅓?
38. Dolls, for $3.66⅔?
39. Penknives, for $4.33⅓?
40. Pounds of candy, for $5.66⅔?

515. At $16\frac{2}{3}$ cents:

41. Collars, for \$4?
42. Pounds, for \$21?
43. Yards, for $\$\frac{1}{3}$?
44. Ounces, for $\$\frac{1}{2}$?
45. Packages, for $66\frac{2}{3}$ cents?
46. Quarts, for $\$1.16\frac{2}{3}$?
47. Gallons, for \$1.50?
48. Pecks, for $\$2\frac{2}{3}$?
49. Feet, for $\$3.33\frac{1}{3}$?
50. Yards, for $\$4.66\frac{2}{3}$?

516. Blackboard Exercises.

Divide at sight:

51. \$24.50 by 50 cents.
52. \$12.25 by 25 cents.
53. \$26 by $33\frac{1}{3}$ cents.
54. \$14.50 by $12\frac{1}{2}$ cents.
55. \$17 by $16\frac{2}{3}$ cents.
56. \$18.75 by 25 cents.
57. $\$11.87\frac{1}{2}$ by $12\frac{1}{2}$ cents.
58. $\$13.33\frac{1}{3}$ by $33\frac{1}{3}$ cents.
59. \$37.50 by 50 cents.
60. $\$13.33\frac{1}{3}$ by $16\frac{2}{3}$ cents.

DIVISION OF FEDERAL MONEY.

517. Oral Exercises.

How often is 1 quart contained in 1 gallon? 1 pt. in 1 qt.? 2 qt. in 1 gal.? 1 inch in 1 foot? 2 in. in 1 ft.? 3 in. in 1 ft.? 4 in. in 1 ft.? 6 in. in 1 ft.? 6 in. in 2 ft.? 8 in. in 2 ft.? 1 ounce in 1 pound? 1 oz. in 2 lb.? 4 oz. in 2 lb.? 1 fourth in 1 half? 1 third in 1?

How often is 1 cent contained in \$1? 2 cents in a dollar? 4 cents in 2 dollars? 25 cents in 25 dollars?

518. Give answers at sight:

1. $\$4 \div 10¢$
2. $\$5 \div 5¢$
3. $\$12 \div 4¢$
4. $\$36 \div 6¢$
5. $\$63 \div 3¢$
6. $\$7 \div 25¢$
7. $\$20 \div 33\frac{1}{3}¢$
8. $\$36 \div 3¢$

9. $\$40 \div 50¢$
10. $\$9 \div 10¢$
11. $\$1 \div \frac{1}{2}¢$
12. $\$3 \div \$\frac{1}{4}$
13. $\$84 \div 50¢$
14. $\$1 \div 16\frac{2}{3}¢$
15. $\$16 \div 16¢$
16. $\$16 \div 16\frac{2}{3}¢$
17. $\$16 \div 33\frac{1}{3}¢$
18. $\$16 \div 25¢$
19. $\$16 \div 50¢$
20. $\$12 \div 20¢$

519. At 36 cents each, how many spellers can be bought for $27?

$$\begin{array}{r} 75 \\ 36\overline{)2700} \\ \underline{180} \\ 0 \end{array}$$

$27 = 2,700 cents. Since 1 speller costs 36 cents, for 2,700 cents we can buy $\frac{2700}{36}$ spellers. *Ans.* 75 spellers.

520. Slate Problems.

1. At $2.75 per day, how long will it take a man to earn $110?

$$11{,}000 \div 275$$

2. How many yards of muslin, at 12 cents per yard, can be bought for $126?

3. A farmer spent $140 for sheep at $5.60 each. How many did he buy?

4. A grocer pays $74.50 for tea at $\frac{1}{2}$ of a dollar per pound. What is the weight of the tea?

5. When rye is worth 87 cents per bushel, how many bushels can be purchased for $261?

6. At $12\frac{1}{2}$ cents per pound, how many pounds of meat will cost $175.25?

7. If 75 spellers cost $27, what is the price of 1 speller?

8. A woman paid $24 for 36 yards of dress goods. What did she pay per yard?

9. At 6 for a dollar, how many rabbits can be bought for $87?

10. The cost of 13 houses was $36,887.50. What was the price of each?

521. Sight Exercises. Approximate Answers.

1. What will be the cost of $39\frac{7}{8}$ lb. butter at 20¢ per lb.?
(Nearly 40 lb. at 20¢. The cost is nearly what?)

2. A man has 4,200 pounds of flour which he wishes to put into barrels containing 196 lb. each. About how many barrels will he need?
(Each barrel contains nearly how many pounds?)

3. A merchant bought a hogshead of molasses, containing $47\frac{3}{4}$ gallons, at 50 cents per gallon. About how much did it cost?

4. How many lots at $1,975 each can be bought for $12,000?

5. Sold 3 pieces of cloth, 33 yd. to the piece, at $1.95 per yd. Give the approximate amount of the bill.

6. $28\frac{5}{16} + 37\frac{1}{20}$ = nearly what?

7. $175\frac{1}{3} \div 24\frac{9}{10}$ = nearly what?

8. $18\frac{4}{5} \times 9\frac{7}{8}$ = nearly what?

9. $87\frac{1}{16} - 49\frac{8}{19}$ = nearly what?

10. $4\frac{7}{8} \times 4\frac{8}{9} \times 4\frac{9}{10}$ = nearly what?

525. Learn the following tables:

TIME.

60 seconds (sec.)	1 minute (min.)
60 minutes	1 hour (hr.)
24 hours	1 day (da.)
7 days	1 week (wk.)

DRY MEASURE.

2 pints (pt.)	1 quart (qt.)
8 quarts	1 peck (pk.)
4 pecks	1 bushel (bu.)

AVOIRDUPOIS WEIGHT.

16 ounces (oz.)	1 pound (lb.)
2000 pounds	1 ton (T.)

The hundredweight (100 pounds) is written cwt.

LIQUID MEASURE.

2 pints (pt.)	1 quart (qt.)
4 quarts	1 gallon (gal.)

A gill (gi.) is equal to one-fourth of a pint. It is very rarely used.

DENOMINATE NUMBERS.

526. Slate Exercises.

1. How many hours in $7\frac{1}{2}$ days?

2. How many hours in 7 days 12 hours?

3. How many seconds in 2 hours?

4. A man buys 12 bu. and 3 pk. apples @ $1 per bu. What is the cost?

5. What will be the cost of 3 pk. 7 qt. chestnuts @ 8¢ per qt.?

6. How many pints are there in 5 gallons of ice cream?

7. How many half-pints are there in 10 gallons of ice cream?

8. How many 4-ounce packages can be made from 5 pounds of pepper?

9. A boy pays $1.50 for 1 gallon and 2 quarts of ice cream. What is the price per quart?

10. How many gallons of lemonade will be needed to give 96 people $\frac{1}{2}$ pt. each?

11. How many seconds in 5 hours?

12. How many minutes in 1 week?

13. Change 13 hours and 20 minutes to minutes.

14. Change 15 bu. and 4 pk. to pecks. To quarts.

15. How many ounces in 47 lb. 5 oz.?

16. How many pounds and ounces in 237 ounces?

17. Change 1,494 minutes to hours and minutes.

18. Find the number of hours in 6 weeks.

19. How many hours are there in January?

20. How many inches are there in 2 yd. 2 ft. and 2 in.?

21. How many ounces in 4 tons?

22. Reduce 5 cwt. and 80 lb. to oz.

23. How many pounds in $\frac{3}{5}$ of a ton?

24. In $\frac{3}{4}$ of a ton how many cwt.?

25. What will 400 lb. of coal cost at $5 per T.?

26. What fraction of a ton is 1,500 lb.?

27. How many days and hours in $\frac{2}{3}$ of a week?

28. Find the number of yards in 3 pieces of cloth averaging 16 yd. 2 ft. each.

29. When coal is $5.50 per ton, how much will I have to pay for 3,000 pounds?

30. A store-keeper charged 70 cents for a roll of butter weighing 2 lb. 3 oz. What was the price per ounce? What was the price per pound?

31. Change 60 lb. to the fraction of a hundredweight.

32. How many pints are there in a barrel of oil that contains $43\frac{1}{2}$ gallons?

SPECIAL DRILLS.

528. Give sums:

$56 + 25$	$21 + 32 + 43$	$750 + 190$	$225 + 54$
$47 + 47$	$30 + 20 + 18$	$390 + 120$	$315 + 21$
$22 + 68$	$40 + 13 + 26$	$480 + 150$	$437 + 60$
$39 + 31$	$24 + 31 + 30$	$620 + 180$	$540 + 55$

529. Give remainders:

$81 - 56$	$750 - 190$	$750 - 560$	$279 - 54$
$94 - 47$	$510 - 120$	$510 - 390$	$386 - 63$
$60 - 28$	$630 - 150$	$630 - 480$	$457 - 37$
$72 - 39$	$820 - 160$	$820 - 660$	$568 - 25$

530. Give products:

310×9	99×2	26×7	18×25
420×4	65×3	24×8	16×25
630×3	49×4	22×9	13×25
740×2	37×5	18×11	11×25

531. Give quotients:

$196 \div 4$	$196 \div 49$	$450 \div 25$	$2{,}790 \div 9$
$190 \div 5$	$190 \div 38$	$375 \div 25$	$1{,}680 \div 4$
$192 \div 6$	$192 \div 32$	$225 \div 25$	$1{,}890 \div 3$
$196 \div 7$	$196 \div 28$	$350 \div 25$	$1{,}480 \div 2$

532. Give answers:

$2\frac{1}{2} + 1\frac{1}{3}$	$1\frac{1}{3} - \frac{1}{2}$	$\frac{2}{3}$ of 66	$12\frac{1}{2} \div \frac{1}{2}$
$2\frac{1}{4} + 1\frac{1}{3}$	$2\frac{1}{4} - 1\frac{1}{3}$	$84 \times \frac{3}{4}$	$8\frac{1}{4} \div \frac{3}{4}$
$2\frac{1}{5} + 1\frac{1}{2}$	$3\frac{1}{5} - 2\frac{1}{2}$	$\frac{4}{5}$ of 100	$5\frac{1}{3} \div \frac{2}{3}$
$2\frac{1}{6} + 1\frac{1}{3}$	$4\frac{1}{6} - 3\frac{1}{3}$	$186 \times \frac{5}{6}$	$3\frac{1}{3} \div \frac{4}{5}$

533. Oral Problems.

1. Paid 59¢ for muslin and 25¢ for trimming. How much was paid in all?

2. A boy had 75¢. How much had he after spending 25¢ for a knife and 15¢ for a ball?

3. If 8 lb. of raisins cost $1.04, what is the price per pound?

4. At $1.89 per yard of silk, what will be the cost of 1 foot?

5. If 32 lb. of flour cost 96 cents, how many pounds can be bought for 60 cents?

6. One girl has 16 cents, another has 24 cents, a third has 8 cents. How many dolls at 16 cents each can be bought with their money?

7. What will be the weight of 3 bushels corn, weighing 56 pounds per bushel?

8. How many ounces in 9 pounds?

9. How many pounds in 8 packages, each weighing 10 ounces?

10. Find the cost of 3 lb. and 2 oz. of butter at 32 cents per lb.

11. Bought 4 pounds of 6-cent sugar and a pound of butter at 36 cents. How much change from $1?

12. Four boys have 144 marbles among them. If they were equally divided, how many would each have?

13. A man earns $100 per month, and spends $76. How much does he save?

14. If a man saves $32 per month, how many months will it take him to save $960?

15. Paid $27.90 for 9 jackets. What did they cost apiece?

16. Mr. B's farm contains 520 acres. How many acres will he have left after selling 180 acres?

17. William's kite string is 435 yards long, John's is 62 yards longer. What is the length of John's string?

18. A farmer raised 168 bushels of grain. He sold $\frac{1}{8}$ of it. How many bushels did he sell?

19. $6\frac{1}{4}$ yards of ribbon are cut into pieces a quarter of a yard long. How many pieces are there?

20. If it takes $18\frac{3}{4}$ yards of cloth to make 3 suits, how many yards does it take for 1 suit?

21. James has 150 marbles, Thomas has $\frac{2}{5}$ as many. How many marbles have both?

22. A newsdealer received \$6.36 for papers sold at 3 cents each. How many papers did he sell?

23. If it takes $4\frac{1}{2}$ days for one man to do a piece of work, how long will it take 2 men to do the same work?

24. A farm is divided into 4 fields, each containing 49 acres. How many acres are there in the farm?

25. From a piece of cloth containing $10\frac{1}{2}$ yards, $5\frac{3}{4}$ yards are sold. How many yards are left?

26. Find the cost of 28 lb. coffee at 25¢ per lb.

27. How much does a farmer receive for 28 cows which he sells at \$25 each?

28. Find the number of hours in a week.

29. How many pieces, each three-quarters of a yard long, can be cut from six yards of wire?

30. 3,600 seconds are equal to how many minutes?

31. If 25 yards of material are needed for a dress, how many yards will be required for 33 dresses?

32. At 7 for a cent what will 98 marbles cost?

534. Sight Exercises.

Divide:

1.	$960 \div 240$	6.	$8{,}400 \div 2{,}100$	11.	$10{,}800 \div 1{,}200$
2.	$780 \div 260$	7.	$8{,}600 \div 4{,}300$	12.	$10{,}400 \div 1{,}300$
3.	$960 \div 480$	8.	$8{,}800 \div 2{,}200$	13.	$6{,}000 \div 1{,}500$
4.	$720 \div 180$	9.	$9{,}600 \div 3{,}200$	14.	$5{,}700 \div 1{,}900$
5.	$1{,}170 \div 130$	10.	$9{,}900 \div 3{,}300$	15.	$12{,}000 \div 2{,}400$

535. Give answers in whole numbers. (Omit remainders.)

16.	$960 \div 241$	21.	$8{,}400 \div 2{,}110$	26.	$10{,}800 \div 1{,}205$
17.	$779 \div 260$	22.	$8{,}500 \div 4{,}300$	27.	$10{,}300 \div 1{,}300$
18.	$959 \div 480$	23.	$8{,}800 \div 2{,}199$	28.	$6{,}100 \div 1{,}550$
19.	$720 \div 181$	24.	$9{,}599 \div 3{,}199$	29.	$5{,}700 \div 1{,}899$
20.	$1{,}030 \div 130$	25.	$10{,}000 \div 3{,}330$	30.	$12{,}020 \div 2{,}410$

536. Multiply:

31.	$1{,}200 \times 6$	36.	$1{,}300 \times 9$	41.	$2{,}100 \times 4$	46.	$1{,}400 \times 8$
32.	$1{,}800 \times 4$	37.	$2{,}300 \times 3$	42.	$1{,}400 \times 6$	47.	$2{,}400 \times 4$
33.	$2{,}500 \times 3$	38.	$3{,}200 \times 2$	43.	$4{,}100 \times 2$	48.	$1{,}300 \times 7$
34.	$1{,}700 \times 5$	39.	$1{,}500 \times 4$	44.	$1{,}600 \times 5$	49.	$1{,}200 \times 9$
35.	$1{,}400 \times 7$	40.	$1{,}200 \times 8$	45.	$2{,}200 \times 3$	50.	$6{,}300 \times 2$

537. Perform indicated operations:

51.	$18 + (30 \times 4)$	57.	$\frac{1}{2}$ of $(240 + 60)$
52.	$7 + (2 \times 8) - 4$	58.	$(7 + 2) \times (8 - 4)$
53.	$[(7 + 2) \times 8] - 4$	59.	$7 + [2 \times (8 - 4)]$
54.	$1 - (\frac{1}{2} + \frac{1}{8})$	60.	$1 - \frac{1}{2} + \frac{1}{8}$
55.	$(6 \times \frac{1}{2}) + \frac{1}{8}$	61.	$6 \times (\frac{1}{2} + \frac{1}{8})$
56.	$\frac{1}{2}$ of $\frac{1}{8}$ of 600	62.	$\frac{3}{4} \times 12 \times \frac{2}{3}$.

538. Place the following upon the board. The pupils will write the answers and nothing else. (Art. 385.)

63. $\frac{582}{97}$	**67.** $\frac{209}{29}$	**71.** $\frac{907}{93}$	**75.** $\frac{524}{76}$	**79.** $\frac{377}{85}$
64. $\frac{617}{77}$	**68.** $\frac{374}{45}$	**72.** $\frac{470}{67}$	**76.** $\frac{430}{86}$	**80.** $\frac{295}{66}$
65. $\frac{299}{36}$	**69.** $\frac{532}{87}$	**73.** $\frac{260}{55}$	**77.** $\frac{310}{54}$	**81.** $\frac{876}{95}$
66. $\frac{458}{57}$	**70.** $\frac{292}{56}$	**74.** $\frac{400}{77}$	**78.** $\frac{865}{94}$	**82.** $\frac{573}{74}$

539. Slate Exercises. Review.

Add:

1.	**2.**	**3.**
$3,947.25	$14.92	$1,094.07
14,816.00	3,120.50	789.14
956.83	18.72	9,870.00
2,469.98	79,841.24	4,009.89
95,783.04	3,972.87	484.78
9,005.79	104.99	9,741.96
6,598.86	19.90	419.74
1.58	19,877.46	4.58
39.99	387.24	999.10
463.27	91.85	23.46
.85	901.09	.98
4.	**5.**	**6.**
$18,477.09	$46.89	$48.34
494.78	.27	875.39
1,489.07	3,538.39	82.76
104.84	468.43	9.87
91.03	56.19	53.49
20,999.99	3.87	835.47
7,583.94	786.49	3,457.96
87.62	5,898.39	85,473.89
3.47	65.40	4,938.78
6,952.83	808.34	453.48

540. Supply missing numbers:

7.	8.	9.
9,256,874	348	7,293
863,052	2,967	82,538
24,635,998	36,847	786,324
7,007,007	243,837	?
?	3,096,846	94,649
85,386,950	?	1,009,765
6,875,634	183,634	256,834
3,987,456	986,246	3,983,387
35,068	8,216	54,619
705	586,237	760,888
139,049,086	6,000,000	10,685,391

10.	11.	12.
3,157,842	749,309	8,352,465
?	980	37,947
1,308,215	9,876,543	40,897,654
930,084	1,234,567	390,784
17,521,938	468,208	?
743,150	63,593,065	1,246,937
9,807	82,389,659	4,373,539
420,985	1,293,714	351,236
73,612	460,045	9,764,318
9,708	?	665,524
65	15,813,477	74,638
35,986,210	253,352,532	100,001,010

541. Multiply:

13. $9,207 \times 3,014$
14. $35,482 \times 798\frac{3}{8}$
15. $5,290 \times 6,075$
16. $9,204 \times 678\frac{7}{8}$
17. $75,074 \times 395$
18. $68,431 \times 924\frac{3}{5}$
19. $95 \times 95 \times 95$
20. $185 \times 19 \times 78$
21. $87\frac{1}{4} \times 23 \times 36$
22. $706 \times 304 \times 509$
23. $48\frac{3}{8} \times 32 \times 74$
24. $538 \times 247 \times 125$

542. Divide:

25.	34,463 ÷ 370	31.	68,703,705 ÷ 12,345
26.	2,823,150 ÷ 1,298	32.	861,420,135 ÷ 56,789
27.	639,712 ÷ 624	33.	70,370,088 ÷ 25,986
28.	21,345,738 ÷ 72,100	34.	4,510,940 ÷ 4,900
29.	1,861,704 ÷ 3,510	35.	34,639,215 ÷ 39,783
30.	20,857,384 ÷ 3,004	36.	12,345,678 ÷ 57,095

543. Slate Problems.

1. The sum of three numbers is 150. Two of the numbers are 68 and 43. What is the third?

$$68 + 43 + ? = 150$$

2. The divisor is 24; the dividend is 264. Find the quotient.

3. The product is 228; the multiplicand is 19. What is the multiplier?

$$19 \times ? = 228$$

4. The minuend is 175; the subtrahend is 87. What is the remainder?

5. The remainder is 92; the subtrahend is 89. Find the minuend.

$$? - 89 = 92$$

6. The minuend is 176, and the remainder is 99. What is the subtrahend?

7. The multiplier is 15; the multiplicand is 46. What is the product?

8. The multiplier is 16; the product is 272. What is the multiplicand?

9. The dividend is 300; the divisor is 17. Find the remainder.

10. The quotient is 15; the remainder is 3; the divisor is 8. What is the dividend?

$$8\overline{)\ ?}$$
$$15\tfrac{3}{8}$$

11. The dividend is 273; the quotient is 21. What is the divisor?

$$?\overline{)273}\quad 21$$

12. The dividend is 267; the quotient is 13; the remainder is 7. What is the divisor?

$$?\overline{)267}\quad 13\tfrac{7}{?}$$

544. Oral Problems.

1. What will be the cost of 8 pounds of meat at 15 cents per pound?

2. Paid $12.90 for 3 pieces of lace. How much did each cost?

3. Gave $1 in payment for a 25-cent ball, and 4 ten-cent bats. How much change did I receive?

4. If 3 straw hats cost 63 cents, what will be the cost of 5?

5. At the rate of 3 oranges for 5 cents, what will be the cost of a dozen oranges?

6. A gross is 12 dozen. How many pens in $\frac{1}{8}$ gross?

7. How many inches in 4 yards?

8. How many ounces in $6\frac{1}{2}$ pounds?

9. At 5 cents per pint, how much would be paid for a bushel of chestnuts?

10. A person used 2 gal. and 3 qt. of milk in one week, and 3 gal. and 1 qt. the next week. How many gallons are used in the two weeks?

11. I sold $3\frac{1}{2}$ yards of silk and $2\frac{3}{4}$ yards of velvet. How many yards in all did I sell?

12. A man had $\$6\frac{1}{4}$ dollars, and he spent $\$3\frac{1}{2}$. How much money had he left?

545. Slate Problems.

1. A man works 9 months, 26 days per month, and receives $702. What are his daily wages?

2. A merchant buys 136 vases for $272. If 36 are broken, how much must he charge apiece for the others to gain $28 on all?

3. On Monday, the receipts of a store are $180; on Tuesday, $30 less; on Wednesday, $110 less than the total of Monday and Tuesday. What are the receipts for the three days?

4. The yearly rent of a house is $480. What is the rent for 2 years 4 months?

5. A mechanic works 300 days per year, at $4 per day. If his daily expenses for 365 days average $3, how much money does he save each year?

6. A woman pays $5.20 for 3 lb. of tea and 56 lb. of sugar. What is the price per lb. of the sugar, if the tea costs 80¢ per lb.?

7. A man had $7,500. He paid $\frac{2}{5}$ of it for a house, $575.60 for repairs, and $387.75 for furniture. How much money had he left?

8. How much hay will be required to feed 18 horses a year of 366 days, if each horse receives 15 pounds a day?

9. A person pays a debt of $576, giving 40 ten-dollar bills, 30 two-dollar bills, 6 one-dollar bills, and the remainder in five-dollar bills. How many of the last did he give?

10. A drover buys 64 sheep for $400. He sells $\frac{1}{2}$ of them at $7 each, and the remainder at $8 each. What is his profit?

11. A merchant sells 56 yards of cloth for $84, gaining $14. What did it cost him per yard?

12. A package of coffee, costing 60 cents, was sold for 75 cents, the profit on each pound being 5 cents. What was the selling price per pound?

13. How many yards of cloth, at $1.75 per yard, can be bought for $105?

14. A tailor buys a piece of cloth for $50. From it he makes 4 pairs of trousers, which he sells at $7 per pair, and 4 coats, for each of which he receives $15. Thread, buttons, lining, etc., cost him $16. How much does he get for his labor?

15. A man sold a certain number of papers for 50 cents. If he had sold 9 more, he would have received 95 cents. How many papers did he sell?

BILLS.

546. BROOKLYN, *July* 31, 1894.

MRS. H. J. SHORT

Bought of ABRAHAM & STRAUS.

		1¼ yd. Plaid	$1.00			
		16 yd. Cambric	.05			
		12 pr. Socks	.25			
		1 Wrapper		1	98	
		4 yd. Silk	.65			
		1 pr. Gloves		2	25	
		2 spools Silk	.08			
						$

1. Copy the above, filling in the cost of each item and the total.

2. Robert J. Wildes buys of Caswell & Donaldson 64 lb. of sugar @ 4½¢; 28 lb. of lard @ 7½¢; 24 lb. of coffee @ 25¢; 1 bbl. flour @ $5.75; and 12 gal. of molasses @ 25½¢. Make out the bill.

3. Make out a bill for 10 pairs of men's shoes at $4.75; 4 pairs of boys' shoes at $1.47½; 6 pairs of slippers at $.87½; 9 pairs of girls' shoes at $2.43; 8 pairs of women's shoes at $3.37½.

4. Make out a bill for 8½ lb. of ham at 14¢ per lb.; 3¼ lb. of beefsteak at 24¢; 9 lb. of corned beef at 12¢; 10¼ lb. of chicken at 30¢; 12 lb. of roast beef at 18¢.

5. Make out a bill for 14 doz. collars, at $1.50 per dozen; 6 doz. pairs of cuffs, at $2.75 per dozen pairs; 4 doz. shirts, at $9.00 per dozen; 3 dozen ties, at $2.25 per dozen; 17 doz. pairs of socks, at $2.50 per dozen pairs.

DECIMALS.

547. Oral Exercises.

In the number 25, what does the 2 stand for?

In the number 467, what does the 4 represent? The 6? The 7?

In the number 33,333, give the value of the first 3 (commencing at the left). Of the second. Of the third. Of the fourth. Of the fifth.

The last 3 is what part of the number represented by the fourth 3? The third 3 is what part of the second? Each 3 is what part of the 3 to its left?

The value of each 3 in this number depends upon what?

In the number XXXIII, what is the value of the first X? Of the second? Of the third?

548. When we write $784.56, the 7 stands for seven times how many dollars? The eight for 8 times how many dollars? The 4 for four times how many dollars? The 5 stands for five times what part of a dollar? The 6 stands for six times what part of a dollar?

Hundreds.	Tens.	Units.	Decimal Point.	Tenths.	Hundredths.
7	8	4	.	5	6

This is read 784 *and* 56 hundredths.

37.5 is read 37 *and* 5 tenths.

6.492 is read 6 *and* 492 thousandths.

.0005 is read 5 ten-thousandths.

.01234 is read 1,234 hundred-thousandths.

56.000246 is read 56 *and* 246 millionths.

$497.625 is read 497 dollars and 62 cents 5 mills.

NOTE. — Cent means hundredth. Mill means thousandth.

NOTATION AND NUMERATION.

549. NOTE. In reading a number containing an integer and a decimal, the word *and* may be placed between the two, as is shown above. To avoid mistakes, the word *units* should be used after the integer in reading such numbers as 200.005, 3000.0075, etc. Say: Two hundred units and five thousandths, three thousand units and seventy-five ten-thousandths.

550. Read the following:

1. .7	8. 32.4	15. .005
2. 34.9	9. 1.025	16. 1.348
3. .36	10. .35	17. 32.6
4. .95	11. 100.025	18. 100.25
5. 3.275	12. .125	19. 627.009
6. 9.624	13. .99	20. .099
7. .09	14. .38	21. 23.23

551. Express in decimals:

1. 7 tenths.
2. 36 and 47 thousandths.
3. One hundred twenty-five thousandths.
4. One hundred units and twenty-five thousandths.
5. 47 hundredths.
6. Four hundred units and six tenths.
7. Four hundred six thousandths.
8. 3 and 56 hundredths.
9. Twenty-eight.
10. 65 tenths.
11. 6 and 5 tenths.
12. 465 and 8 hundredths.
13. 375 hundredths.
14. 4,000 tenths.
15. 887 and 72 hundredths.
16. 345 tenths.
17. 6,054 hundredths.
18. 6,000 and 54 hundredths.
19. 5 millionths.
20. 562 millionths.

ADDITION OF DECIMALS.

554. Add:

1.	2.	3.	4.
.7	3.84	28.978	5.6
4.18	68.075	.28	.387
.005	.5	5.375	26.93
5.67	24.698	18.758	8.754
10.555	97.113		

5. .027 + 1.39 + 48.6 + 72.978
6. 234.96 + .675 + 50.4 + 6.02 + 1.001
7. 3.047 + 54.79 + .097 + .76 + .862
8. .8 + .38 + .479 + 27.87 + 375
9. .445 + 34.75 + 306.973 + .004 + 48.56
10. .81 + 12.654 + 234.79 + 8.6 + .603 + 42.96
11. 45.78 + .237 + 6.987 + 18 + 372.003 + 37.5
12. 4.745 + 36.58 + 725.894 + 9.87 + 75.357 + 86.74
13. 59.3 + 83 + 9.64 + 48.565 + 6.98 + 8.795 + 963.826
14. 13.387 + 72.563 + .7 + .603 + 7.245 + .483 + 9.25
15. 8.3 + 2.576 + 3.42 + 1.5 + 6.279 + .003 + 1.417
16. .2 + 2.356 + 1.14 + 2 + 4.96 + 3.272 + .7 + 3.54
17. 4.7 + 1.198 + .35 + 763.5 + 1,423 + 157.24 + .437 + 9.5
18. 7.369 + 1.72 + 32.948 + .429 + .7 + 3.14 + 695 + 7.005
19. 3.87 + 2.694 + .8 + 251.47 + 9.3 + 1.916 + 41.5 + 751.006
20. .37 + 6.3 + .008 + 9.63 + 96 + 47.82 + 637.46 + 1.923

SUBTRACTION OF DECIMALS.

555.

From	37.	182.01	1.	28.6
Take	3.7	4.624	.009	1.003
	33.3	177.386	.991	27.597

556. Find answers:

21. $1 - .057$

22. $1 - .245$

23. $6 - 3.324$

24. $4 - 2.491$

25. $3 - 1.568$

26. $7 - 4.736$

27. $3.587 - 1.34$

28. $91.352 - 72.456$

29. $42.007 - 17.658$

30. $68.217 - 39.4$

31. $9.34 - 5.672$

32. $45.268 - 23.068$

33. $219.843 - 187.95$

34. $681.38 - 94.572$

35. $1000 - 465.874$

36. $30.053 - 18.7$

37. $2{,}568.91 - 1{,}925$

38. $1.234 - .825$

39. $473.5 - 298.572$

40. $57.083 - 44.95$

MULTIPLICATION OF A DECIMAL BY AN INTEGER.

557. Three times 3 tenths equals how many tenths?

$.3 \times 3 =$ what?	$.3 \times 4 = ?$	$4 \times .3 = ?$		
1.7	7.5	.275	.35	.0036
$\times 8$	$\times 4$	$\times 12$	$\times 20$	$\times 110$
13.6	30.0	3.300	7.00	.3960

558. The products are 13.6, 30, 3.3, 7, and .396; the ciphers to the right of the decimals, having no value, are omitted in giving the answers.

559. Multiply:

41. $.36 \times 3$
42. 57.2×7
43. 6.4×122
44. 122×6.4
45. $.67 \times 4$
46. $.003 \times 512$
47. $512 \times .003$
48. $.056 \times 987$
49. $97 \times .005$
50. $5{,}430 \times .8$
51. 275×1.2
52. 38.4×25
53. $.048 \times 375$
54. 12.67×300
55. 6.57×9
56. $748 \times .97$
57. 8.76×43
58. $964 \times .347$
59. $570 \times .11$
60. $360 \times .005$

560. Sight Exercises.

Give products:

1. 684×10
2. 68.4×10
3. 3.28×10
4. 5.71×100
5. $5.71 \times 1{,}000$
6. $.961 \times 100$
7. $.57 \times 1{,}000$
8. $.09 \times 1{,}000$
9. $.026 \times 100$
10. 5.17×10

561. Give quotients:

11. $932 \div 100$
12. $86 \div 1{,}000$
13. $328 \div 10$
14. $9 \div 1{,}000$
15. $48 \div 1{,}000$
16. $684 \div 100$
17. $57.6 \div 10$
18. $24.3 \div 100$
19. $8.75 \div 10$
20. $932.5 \div 100$

DIVISION OF A DECIMAL BY AN INTEGER.

562. Sight Exercises.

1. $8.64 \div 2$
2. $48.24 \div 4$
3. $.465 \div 3$
4. $8.40 \div 5$
5. $8.4 \div 5$
6. $.846 \div 6$
7. $.048 \div 8$
8. $.81 \div 9$
9. $.12 \div 5$
10. $.34 \div 4$

563. Where it is necessary, ciphers may be annexed to the right of the decimal in the dividend.

$$8\overline{).12} \quad .015 \qquad 15\overline{).06} \quad .004 \qquad 64\overline{)120.} \quad 1.875$$

$$125\overline{)1.5} \quad .012 \qquad 21\overline{)8.673} \quad .413$$

250		27		480	
0		63		320	
				0	

564. Divide:

11. $25\overline{)1.00}$
12. $4\overline{)21.80}$
13. $8\overline{).2}$
14. $13\overline{)3.913}$
15. $12\overline{)48.12}$
16. $11\overline{)70.07}$
17. $24\overline{)36.6}$
18. $18\overline{).576}$
19. $25\overline{)11.1}$
20. $32\overline{)62.000}$
21. $\frac{1}{4} =$
22. $\frac{1}{8} =$
23. $\frac{4}{5} =$
24. $\frac{4}{50} =$
25. $\frac{3}{8} =$
26. $\frac{7}{125} =$
27. $\frac{100}{32} =$
28. $\frac{180}{75} =$
29. $\frac{5000}{64} =$
30. $\frac{1}{16} =$

565. Slate Problems.

31. A franc is 19.3 cents. Find the cost in United States money of goods amounting to 1,250 francs.

32. A merchant bought 1,800 meters of silk. How many yards did he buy, a meter being 39.37 inches?

33. A kilogram is 2.2046 pounds. What is the difference in weight between the English ton of 2,240 lb. and a French ton of 1,000 kilograms?

34. A cubic foot of water weighs 1,000 ounces. How many pounds does a cubic foot of gold weigh, gold being 19.4 times as heavy as water?

35. There are 128 cubic feet in a cord. How many tons of 2,000 lb. are there in a cord of pine wood, the latter being .66 times as heavy as water?

36. A man buys three plots of ground containing 35.27, 17.8, and 40.375 acres, respectively. Find the total cost at $36 per acre.

37. How many pints are there in 2.375 gallons?

38. What decimal of a peck is a quart?

39. What will be the cost of carrying 468 tons of coal at $0.125 per ton?

40. A farmer sold one-eighth of his farm of 224.2 acres at $62.50 per acre. How much did he receive for it?

41. How long is a post which is $5\frac{1}{2}$ feet above water, one-half of its length in the water, and one-fourth of its length in the mud? (Diagram.)

42. Eight pounds of black tea costing 35¢ per lb. are mixed with twelve pounds of green tea costing 50¢ per lb. What is the cost of a pound of the mixed tea?

43. How many bushels, pecks, and quarts, are there in 1,449 pounds of corn weighing 56 lb. per bushel?

MEASUREMENTS.

569. Preliminary Exercises.

Draw a square each side of which is one inch. This is called a square inch.

Draw a rectangle two inches long, one inch wide. How many square inches will it contain?

Draw a rectangle three inches long, two inches wide. Divide it into one-inch squares. Count them. How many are there? How many square inches in the rectangle?

How many square inches in a rectangle 6 inches long, 3 inches wide?

How many square inches in a rectangle 4 inches long, 4 inches wide?

About how long is your slate? About how wide? About how many square inches are there in its surface?

How many square inches are there in a rectangle 12 inches long, 3 inches wide? In a rectangle 1 foot long, 3 inches wide? In a rectangle 1 foot 1 inch long, 4 inches wide?

570. Slate Exercises.

How many square inches in each of the following rectangles?

1. 13 in. by 14 in.
2. 17 in. by 9 in.
3. 18 in. by 7 in.
4. 23 in. by 15 in.
5. 21 in. by 19 in.
6. 37 in. by 14 in.
7. 13 in. by 42 in.
8. 27 in. by 31 in.
9. 18 in. by 22 in.
10. 64 in. by 29 in.
11. 1 ft. by 7 in.
12. 1 ft. 1 in. by 11 in

571. NOTE. Change each dimension to inches before multiplying.

13. 1 ft. 3 in. by 12 in.
14. 1 ft. by 1 ft.
15. 1 ft. 4 in. by 1 ft.
16. 2 ft. 6 in. by 1 ft.
17. 2 ft. 6 in. by 1 ft. 3 in.
18. 3 ft. 7 in. by 2 ft. 9 in.
19. 4 ft. 11 in. by 1 ft. 8 in.
20. 5 ft. 3 in. by 2 ft. 11 in.

572. Oral Exercises.

How many square feet in a rectangle 2 feet long, 1 foot wide? 6 feet long by 5 feet wide? 9 feet long by 7 feet wide?

573. Slate Exercises.

Find the square feet in the following:

1. 12 ft. by 14 ft.
2. 15 ft. by 17 ft.
3. 19 ft. by 11 ft.
4. 23 ft. by 15 ft.
5. 18 ft. by 16 ft.
6. 29 ft. by 12 ft.
7. $15\frac{1}{2}$ ft. by 12 ft.
8. 15 ft. 6 in. by 12 ft.
9. $18\frac{3}{4}$ ft. by 16 ft.
10. 18 ft. 9 in. by 16 ft.

574. NOTE. Change the inches to fractions of a foot.

11. $23\frac{1}{2}$ ft. by 18 ft.
12. 24 ft. 8 in. by 18 ft.
13. 19 ft. 3 in. by 16 ft.
14. 24 ft. by 17 ft. 9 in.
15. 24 ft. by 16 ft. 1 in.
16. 36 ft. by 23 ft. 5 in.
17. 13 ft. by $24\frac{3}{4}$ ft.
18. 13 ft. 4 in. by 24 ft.
19. 26 ft. 8 in. by 15 ft.
20. $12\frac{1}{2}$ ft. by 12 ft.

575. Suggestive Examples.

Measure the top of the desk, and calculate the surface in square inches. (Do not include fractions of an inch.)

Measure the blackboard, and find how many square feet in its surface. (Do not include fractions of a foot.)

Calculate the number of square inches in a pane of glass in the school-room window.

Find the number of square feet in the floor of the class-room.

Find the number of square feet in the class-room ceiling.

Estimate the height of the class-room, and calculate the number of square feet in the front wall. In the rear wall. In the right-hand wall. In the left-hand wall.

576. Slate Problems.

SUGGESTION. When the surface is required in square inches, change each dimension to inches; when required in square feet, express each dimension in feet, or in feet and the fraction of a foot; when required in square yards, etc., express each dimension in yards, etc.

1. How many square feet are there in the surface of a field 125 feet long, 87.5 feet wide?

2. A rug is 2 yards long, $1\frac{3}{4}$ yards wide. How many square yards does it contain?

3. How many square yards are there in a strip of carpet 6 yards long, 27 inches ($\frac{3}{4}$ yd.) wide?

4. Find the number of square meters in a room 12 meters long, 9.75 meters wide.

5. At 50 cents per square yard, what will be the cost of the oil-cloth needed to cover a floor 18 feet (6 yd.) long, 15 feet (5 yd.) wide?

6. What will be the cost, at \$1.50 per square yard, of carpeting a room $6\frac{1}{2}$ yards long, 15 feet wide?

7. At 3 cents a square foot, how much must be paid for 10 boards, each 16 feet long, $\frac{1}{2}$ foot wide?

8. A field is 30 rods long and 24 rods wide. How many square rods will it contain after a strip 24 rods long and 2 rods wide is taken from it for a road?

9. How many square yards of plastering will be required for a ceiling 18 feet long, 15 feet wide?

10. If a roll of wall paper is 24 feet long and 18 inches wide, how many square yards does it contain?

CHAPTER VII.

FRACTIONS. — DECIMALS. — BILLS. — DENOMINATE NUMBERS. — MEASUREMENTS.

ADDITION OF FRACTIONS.

577. Slate Exercises.

1. Find the sum of $10\frac{3}{8}$, $6\frac{4}{9}$, $5\frac{7}{8}$.

Since 9 is a multiple of 3, any multiple of 9 will be a multiple of 3. Omitting the latter number, we find the least common multiple of 8 and 9, 72, which is the least common denominator of the fractions.

2. Add $2\frac{1}{3}$, $\frac{3}{4}$, $5\frac{2}{7}$, $4\frac{1}{8}$

Omitting 4, which is a factor of 8, we find the least common multiple of 3, 7, 8.

$$3 \times 7 \times 8 = 168$$

3. $6\frac{3}{5} + 4\frac{4}{15} + 7\frac{3}{8} + \frac{2}{3} = ?$

L. C. M. $= 15 \times 8$ Omit 5 and 3.

4. $17\frac{3}{8} + 4\frac{5}{9} + 26\frac{3}{7}$

$$8 \times 9 \times 7 = 504$$

	504
$17\frac{3}{8}$	189
$4\frac{5}{9}$	280
$26\frac{3}{7}$	432
	$\frac{901}{504} = 1\frac{397}{504}$

5. $2\frac{1}{15} + 8\frac{1}{6} + 27\frac{3}{4} + 9\frac{1}{5}$

L. C. M. of 15 and 6 $= ?$

6. $29\frac{3}{10} + 45 + 16\frac{4}{15}$

7. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6}$

8. $1\frac{1}{2} + 5\frac{3}{5} + 13\frac{5}{12} + 8\frac{2}{5} + 19\frac{5}{8}$

9. $8\frac{3}{10} + 37\frac{3}{4} + 28\frac{4}{5} + 9\frac{9}{20} + 19\frac{1}{2}$

10. $\frac{7}{10} + 2\frac{31}{100} + 81\frac{7}{20} + 15\frac{9}{25}$

SUBTRACTION OF FRACTIONS.

578. Find answers:

11. $12\frac{1}{6} - 3\frac{1}{4}$
12. $23\frac{5}{12} - 16\frac{7}{8}$
13. $47\frac{9}{40} - 8\frac{3}{5}$
14. $102\frac{3}{8} - 84\frac{5}{6}$
15. $39\frac{4}{15} - 19\frac{7}{15}$
16. $39\frac{9}{20} - 23\frac{7}{30}$
17. $92\frac{7}{10} - 68\frac{5}{12}$
18. $830\frac{1}{12} - 807\frac{7}{60}$
19. $12{,}345 - 6{,}082\frac{17}{24}$
20. $320\frac{9}{16} - 120\frac{7}{12}$

REVIEW.

579. Perform the operations indicated:

21. $16 \times (2\frac{7}{8} + 1\frac{3}{4})$
22. $(8\frac{1}{3} - 3\frac{1}{2}) - 1\frac{5}{8}$
23. $18\frac{1}{2} - (3\frac{1}{3} - 1\frac{5}{8})$
24. $(24\frac{1}{2} - 15\frac{2}{3}) \times 36$
25. $(46\frac{3}{10} - 17\frac{3}{5}) \div 7$
26. $(50\frac{5}{6} \div 5) + 12\frac{1}{2}$
27. $100 - (4 \times 13\frac{3}{7})$
28. $(83\frac{7}{8} + 17\frac{3}{8}) \times 24$
29. $83\frac{7}{8} + (17\frac{3}{8} \times 24)$
30. $\dfrac{7 \times 2}{15 \times 3} - \dfrac{4}{15}$

FACTORS AND MULTIPLES.

580. Oral Exercises.

Give two factors of:

4 *Ans.* 2, 2		6 *Ans.* 2, 3	
1. 9	6. 25	11. 39	16. 57
2. 10	7. 26	12. 46	17. 58
3. 14	8. 34	13. 49	18. 62
4. 21	9. 35	14. 51	19. 65
5. 22	10. 38	15. 55	20. 69

581. Sight Exercises.

Give three factors of:

1. 8	6. 28	11. 50	16. 70
2. 12	7. 30	12. 52	17. 75
3. 18	8. 42	13. 63	18. 78
4. 20	9. 44	14. 66	19. 98
5. 27	10. 45	15. 68	20. 99

PRIME NUMBERS.

582. A number that has no factors is a *prime number.*
1, 2, 3, 5, 7, etc., are prime numbers.

583. 1. Name the prime numbers between 10 and 20.

2. Between 20 and 30.
3. Betwsen 30 and 50.
4. Between 50 and 70.
5. Between 70 and 100.

584. Sight Exercises.

Give the prime factors, commencing with the smallest.

1. 15	6. 40	11. 64	16. 80
2. 16	7. 48	12. 72	17. 81
3. 24	8. 54	13. 74	18. 82
4. 32	9. 56	14. 76	19. 84
5. 36	10. 60	15. 77	20. 85

585. Slate Exercises.

1. 86	6. 92	11. 100	16. 576
2. 87	7. 93	12. 120	17. 840
3. 88	8. 94	13. 210	18. 1,152
4. 90	9. 95	14. 240	19. 1,728
5. 91	10. 96	15. 360	20. 2,016

GREATEST COMMON DIVISOR.

586. A common factor of two or more numbers is a number that will divide each of them without remainder.

The largest number that is a factor of two or more numbers is called the *greatest common divisor.*

587. Sight Exercises.

Find the greatest common divisor of:

1. 27 and 48
2. 25 and 35
3. 36 and 54
4. 26 and 39
5. 42 and 63
6. 34 and 51
7. 32 and 48
8. 45 and 75
9. 40 and 65
10. 54 and 69

588. Slate Exercises.

Reduce the following fractions to lowest terms:

1. $\frac{144}{168}$
2. $\frac{275}{400}$
3. $\frac{144}{216}$
4. $\frac{182}{176}$
5. $\frac{275}{350}$
6. $\frac{28}{112}$
7. $\frac{81}{99}$
8. $\frac{51}{69}$
9. $\frac{512}{640}$
10. $\frac{824}{482}$
11. $\frac{576}{840}$
12. $\frac{186}{204}$

LOWEST TERMS.

589. How can you tell that a number is divisible by 2? By 5?

A number is divisible by 3 when the sum of its digits (figures) is divisible by 3; it is divisible by 9, when the sum of its digits is divisible by 9.

A number is divisible by 4, when the number expressed by its last two figures is divisible by 4.

When is a number divisible by 25?

590. A fraction is reduced to lowest terms by dividing the numerator and the denominator by their greatest common divisor.

591. Reduce to its lowest terms $\frac{169}{1001}$.

In this example, it is not easy to ascertain by inspection any number that will divide both terms. In such cases, the greatest common divisor is found by dividing the denominator by the numerator. The remainder is divided into the numerator, and each subsequent remainder is divided into the corresponding divisor until there is no longer a remainder. This last divisor is the *greatest common divisor* of the two numbers.

$$\begin{array}{r} \quad 5 \\ 169\overline{)1001}\quad 1 \\ 156\overline{)169} \\ 13\overline{)156} \\ 12 \end{array}$$

13 is the greatest common divisor.

$$\frac{169 \div 13}{1{,}001 \div 13} = \frac{13}{77} \text{ lowest terms.}$$

592. In reducing fractions to lowest terms, the above method of finding the greatest common divisor should not be resorted to if it is possible to get along without it.

593. Reduce to lowest terms:

1. $\frac{123}{300}$

A look at both terms shows that 3 is a common factor. This reduces the fraction to $\frac{41}{100}$. 41 is a prime number, and is not a factor of 100, so that $\frac{41}{100}$ cannot be reduced to lower terms.

2. $\frac{432}{621}$

$$4+3+2=9; \quad 6+2+1=9$$

Since the sum of the digits of each term is divisible by 9, this number is a common factor, and reduces the fraction to $\frac{48}{69}$, etc.

3. $\frac{420}{735}$

5 is clearly a common factor, etc.

4. $\frac{42}{99}$
5. $\frac{111}{222}$
6. $\frac{49}{91}$
7. $\frac{52}{65}$
8. $\frac{217}{248}$
9. $\frac{78}{221}$
10. $\frac{68}{153}$
11. $\frac{77}{182}$
12. $\frac{51}{68}$
13. $\frac{119}{187}$
14. $\frac{115}{161}$
15. $\frac{216}{240}$

LEAST COMMON MULTIPLE.

594. Sight Exercises.

Give the least common multiple of:

1. 16 and 24
2. 12 and 15
3. 3, 9, 11
4. 4, 12, 16
5. 2, 3, 4, 5, 6
6. 2, 3, 5, 9, 10
7. 2, 3, 5, 6, 9, 10
8. 3, 6, 9, 12, 4, 18
9. 2, 7, 14, 3, 9
10. 5, 10, 20, 25, 50

595. Slate Exercises.

Add $14\frac{1}{3}$, $73\frac{2}{9}$, $6\frac{5}{7}$, $\frac{9}{14}$, $23\frac{1}{6}$, $101\frac{3}{14}$, $58\frac{1}{2}$, $9\frac{7}{12}$.

Here we have to find the least common multiple of 3, 9, 7, 14, 6, 14, 2, 12.

Rejecting 3, 6, 2 because they are factors of 12; 7, a factor of 14; and one 14, we have to find the least common multiple of

9 14 12

Divide these numbers by a prime number that is exactly contained in any two of them, bringing down the numbers that are not multiples of the divisor.

Taking 2 as a divisor, bring down 9, and write quotients 7 and 6.

2	9	14	12
3	9	7	6
	3	7	2

3 being a factor of two of the three numbers, 9, 7, 6, is taken as the next divisor. 3 is written as a quotient, 7 is brought down, 2 is a quotient.

As there is no factor common to any two of the numbers, 3, 7, 2, we find the least common multiple by multiplying together the two divisors and these three numbers.

$2 \times 3 \times 3 \times 7 \times 2 = 252$ L. C. M.

252

$14\frac{1}{3}$
$73\frac{2}{9}$
$6\frac{5}{7}$
$\frac{9}{14}$
$23\frac{1}{6}$
$101\frac{3}{14}$
$58\frac{1}{2}$
$9\frac{7}{12}$

Find the sum.

596. Find the least common multiple of the denominators of the fractions $\frac{3}{4}$, $\frac{11}{20}$, $\frac{5}{6}$, $\frac{17}{30}$, $\frac{23}{45}$, $\frac{7}{12}$.

2	20	30	45	12	Omit 4 and 6.
2	10	~~15~~	45	6	Strike out 15, a factor of 45.
	~~5~~		45	~~3~~	Strike out 5 and 3, factors of 45.

L. C. M. $= 2 \times 2 \times 45 = 180$

Add the fractions.

597. Find the L. C. M. of:

1. 4, 6, 3, 5, 8, 20. Strike out 4, 3, 5.
2. 9, 15, 15, 4, 4, 12, 25. Strike out one 15 and two 4's.
3. 2, 3, 5, 7, 5, 14, 10, 12, 24
4. 2, 3, 5, 6, 8, 10, 15, 16, 80
5. 20, 30, 40, 50
6. 2, 3, 4, 6, 8, 12, 16, 24
7. 24, 12, 5, 3, 10, 18
8. 11, 3, 7, 77, 33
9. 18, 5, 9, 40, 16
10. 10, 12, 15, 21

ADDITION AND SUBTRACTION OF FRACTIONS.

598. In adding or subtracting fractions, they must be reduced to a common denominator. The *least common denominator* is the least common multiple of the denominators.

In the following examples, determine the least common denominator by *inspection*, if possible.

599. Add:

1. $8\frac{4}{7}$, $5\frac{1}{4}$, $3\frac{1}{8}$
2. $45\frac{3}{5}$, $20\frac{1}{2}$, $8\frac{3}{4}$, $9\frac{1}{8}$
3. $32\frac{1}{4}$, $19\frac{3}{5}$, $6\frac{1}{2}$, $81\frac{2}{25}$
4. $2\frac{1}{8}$, 20, $3\frac{3}{4}$, $\frac{5}{12}$, $5\frac{1}{3}$
5. $8\frac{1}{3}$, $45\frac{5}{6}$, $2\frac{17}{20}$, $4\frac{1}{2}$, $\frac{21}{40}$
6. $\frac{3}{8}$, $\frac{3}{5}$, $\frac{1}{10}$, $\frac{3}{20}$, $\frac{16}{25}$
7. $63\frac{1}{10}$, $3\frac{3}{5}$, $2\frac{1}{8}$, $5\frac{4}{5}$, $7\frac{7}{12}$
8. $4\frac{3}{10}$, $7\frac{9}{100}$, 84, $6\frac{7}{1000}$, $\frac{23}{100}$
9. $5\frac{1}{12}$, $18\frac{1}{10}$, $3\frac{1}{3}$, $7\frac{1}{8}$, $8\frac{1}{4}$
10. $17\frac{9}{1000}$, $\frac{27}{100}$, $6\frac{3}{10}$, $\frac{14}{1000}$, $\frac{3}{100}$

600. Subtract:

11. $65\frac{11}{24} - 57\frac{5}{16}$
12. $18\frac{17}{40} - 9\frac{23}{60}$
13. $100\frac{41}{48} - 15\frac{13}{72}$
14. $102\frac{3}{16} - 27\frac{17}{20}$
15. $208\frac{11}{12} - 128\frac{19}{20}$
16. $251\frac{8}{21} - 27\frac{17}{35}$
17. $755\frac{19}{36} - 283\frac{25}{27}$
18. $100\frac{4}{100} - 89\frac{78}{1000}$
19. $123\frac{13}{24} - 80\frac{13}{32}$
20. $67\frac{1}{10} - 58\frac{41}{1000}$

601. Perform the operations indicated:

21. $\dfrac{21+5}{25+5} - \dfrac{21}{25}$
22. $\dfrac{21}{25} - \dfrac{21-5}{25-5}$
23. $(37\frac{5}{6} - 11\frac{3}{5}) - (28\frac{7}{15} - 19\frac{7}{12})$
24. $14\frac{5}{8} - 8\frac{1}{2} - 3\frac{3}{5} + 4\frac{1}{4}$
25. $(8\frac{3}{16} + 6\frac{2}{3}) - (8\frac{3}{16} - 6\frac{2}{3})$
26. $4\frac{3}{4} \times 16 \times 8\frac{7}{8}$
27. $(2\frac{2}{9} + 5\frac{1}{6}) \div (1\frac{1}{3} + 2\frac{1}{2} + 3\frac{1}{6})$
28. $(8\frac{1}{2} + 4\frac{1}{4}) \div (2\frac{1}{2} + 1\frac{3}{4})$
29. $(3\frac{5}{12} \times 36) \times 8\frac{3}{7}$
30. $4\frac{2}{3} + 3\frac{1}{2} - 6\frac{5}{7} + 17\frac{1}{6} - 9\frac{4}{5}$

602. Reduce to lowest terms:

31. $\frac{25}{100}$
32. $\frac{40}{100}$
33. $\frac{26}{1000}$
34. $\frac{125}{1000}$
35. $\frac{875}{1000}$
36. $\frac{36}{100}$
37. $\frac{625}{1000}$
38. $\frac{64}{1000}$
39. $\frac{875}{1000}$
40. $\frac{8125}{10000}$
41. $\frac{88}{1000}$
42. $\frac{625}{10000}$

603. Oral Problems.

1. A person traveling from New York to Albany (140 miles apart) has gone 93 miles. How much farther has he to go?

2. There are 196 pounds of flour in a barrel. How many pounds in $\frac{1}{4}$ bbl.?

3. How many fourths in $24\frac{3}{4}$?

4. Reduce $\frac{40}{56}$ to lowest terms.

5. Change $\frac{100}{7}$ to a mixed number.

6. Add $\frac{1}{2}$, $\frac{1}{8}$, and $\frac{1}{4}$.

7. From a chest of tea containing $45\frac{1}{4}$ lb., $14\frac{3}{8}$ lb. are sold. How many pounds remain?

8. From $\frac{1}{8}$ of a dollar take $10\frac{3}{8}$ cents.

9. How many cents in $\frac{1}{4} + \frac{1}{8} + \frac{3}{10}$ of a dollar?

10. A farmer has $60\frac{3}{4}$ acres of pasture and $20\frac{3}{8}$ acres of woodland. How many acres in both?

11. The circumference of a circle is about $3\frac{1}{7}$ times its diameter. If the diameter is 8 feet, about what is the circumference?

12. Mary is 12 years and 7 months old; Jane is 3 years and 11 months older. How old is Jane?

13. In a year and a half William will be 7 years 2 months old. How old is he now?

14. What number multiplied by 3 equals 231?

15. What number between 7 and 12 is a prime number?

16. A boy received 9 marks in arithmetic, 8 in penmanship, and 7 in reading. What was his average mark?

17. $\frac{4}{7}$ of a class consists of boys. How many girls in the class if it contains 49 pupils?

18. If July 1 falls upon Tuesday, what will be the date of the third Tuesday of July?

19. If July 1 falls upon Thursday, upon what day will the first of August fall?

20. A man bought $20\frac{1}{2}$ lb. of sugar; he sold $10\frac{3}{4}$ lb. at one time and $6\frac{1}{2}$ lb. at another. How much had he left?

21. If 3 qt. 1 pt. oil cost 7 cents, what will 1 gal. 1 qt. cost?

22. How much will have to be paid for 7 cows at $50 each, and 4 horses at $150 each?

23. $\frac{3}{4}$ = how many hundredths?

24. What are the two factors of 87?

25. Find the G. C. D. of 36 and 54.

604. Slate Problems.

1. A merchant sold one firkin of butter at $19\frac{3}{4}¢$ per lb., a second at $20\frac{3}{4}¢$ per lb., a third at $16\frac{7}{8}¢$ per lb. What was the average price per lb., each firkin containing 56 lb.?

2. If eggs are sold at the rate of 21 for 25 cents, how much will be paid for $5\frac{1}{4}$ dozen?

3. If a train travels 45 miles per hour, how far will it go from half-past 9 in the morning to a quarter of 7 in the evening?

4. To the sum of $6\frac{7}{8}$ and $19\frac{3}{8}$ add their difference, and find how often the greater number is contained in this total.

5. A mechanic has deposited in the savings bank $35 per month for 11 months, and $20 the twelfth month. His expenses have averaged $3 each day of the year. What are his daily wages for the 300 days he has worked?

6. A merchant sold 4 pieces of cloth containing $27\frac{1}{2}$ yd., $26\frac{3}{8}$ yd., $29\frac{5}{8}$ yd., and $28\frac{1}{8}$ yd., respectively. How much did he receive for the cloth at 96 cents per yard?

7. Reduce $\frac{189}{252}$ to lowest terms.

8. A man has $8\frac{5}{16}$ bu. of peanuts. He puts them into bags holding $\frac{1}{64}$ bu. How many bags does he fill?

9. A 160-acre farm consists of 5 fields. The first contains 17.38 acres, the second 29.4 acres, the third 35.073 acres, the fourth 25.875 acres. How many acres are there in the fifth field?

10. How many seconds in 7 hours 15 minutes?

11. Find the total cost of 2 doz. rockets at $7.50 per gross, 3 dozen Roman candles at $9.60 per gross, and 24 doz. pin wheels at $1.35 per gross. (1 gross = 12 doz.)

12. From a piece of silk that contained $28\frac{1}{2}$ yd., there were sold $2\frac{1}{2}$ yd., $6\frac{1}{8}$ yd., and $13\frac{3}{4}$ yd. Find the value of the remainder at $1.20 per yd.

13. Three pieces of cloth bought at $2 per yard cost $150. The first piece measures $23\frac{1}{2}$ yd., the second measures $30\frac{3}{8}$ yd. How many yards in the third piece?

14. Three lots of tea were sold for $330. The second contained twice as much as the first, and the third three times as much as the first. The third lot contained 330 pounds. Find the selling price of the tea per pound.

15. What part of a person's income remains after he spends $\frac{1}{8}$, $\frac{4}{15}$, and $\frac{1}{5}$ of it?

16. A boy loses $\frac{1}{2}$ of his marbles, and he gives away $\frac{1}{4}$ of them. If he has 17 marbles left, how many had he at first?

17. A cask of molasses contained 80 gallons. One-fourth of it leaked out. If the molasses cost 60 cents per gallon, what price must be charged for the remainder so that there will be no loss.

18. A dealer sells $1\frac{3}{4}$ gross, $3\frac{7}{8}$ gross, $4\frac{11}{36}$ gross, $9\frac{1}{18}$ gross, and $8\frac{2}{3}$ gross of lead pencils at 36 cents per dozen. How much does he receive for all?

19. There are four towns, A, B, C, and D, on a certain railroad running east and west. A is $41\frac{1}{2}$ miles west of C; D is $39\frac{1}{4}$ miles east of B; B is $22\frac{1}{2}$ miles west of C. How many miles from A to D? Make a diagram.

20. If $12\frac{1}{2}$ dozen rockets cost $5.75, what will 15 dozen cost?

SPECIAL DRILLS.

605. Give sums:

$59+75$	$22+34+18$	$560+390$	$225+154$	$65+86$
$67+83$	$19+47+30$	$270+280$	$315+421$	$59+47$
$94+38$	$25+35+26$	$640+260$	$437+260$	$88+22$
$66+56$	$17+18+19$	$430+480$	$540+355$	$61+39$

606. Give remainders:

$134-75$	$750-290$	$131-65$	$279-154$	$124-89$
$150-83$	$510-220$	$123-84$	$386-263$	$112-56$
$132-94$	$630-380$	$156-78$	$457-237$	$180-89$
$122-56$	$820-560$	$164-97$	$668-325$	$100-61$

607. Give products:

49×2	123×3	46×6	47×25	48×8
47×3	431×2	34×6	25×36	37×8
48×4	122×4	38×7	33×25	46×9
43×5	332×3	49×7	25×27	35×9

608. Give quotients:

$141\div3$	$111\div37$	$925\div25$	$266\div7$	$266\div38$
$192\div4$	$192\div48$	$875\div25$	$296\div8$	$296\div37$
$215\div5$	$215\div43$	$750\div25$	$414\div9$	$414\div46$
$276\div6$	$276\div46$	$625\div25$	$360\div8$	$360\div45$

609. Give answers:

$\frac{1}{2}+\frac{2}{3}$	$1\frac{1}{2}-\frac{2}{3}$	$66\times1\frac{2}{3}$	$\frac{1}{2}\div\frac{2}{3}$	$18\frac{2}{3}\div2\frac{2}{3}$
$\frac{2}{3}+\frac{3}{4}$	$1\frac{2}{3}-\frac{3}{4}$	$84\times1\frac{3}{4}$	$\frac{2}{3}\div\frac{3}{4}$	$21\frac{3}{5}\div2\frac{2}{5}$
$\frac{3}{4}+\frac{1}{5}$	$1\frac{2}{5}-\frac{1}{2}$	$100\times1\frac{4}{5}$	$\frac{3}{4}\div\frac{1}{5}$	$10\frac{4}{7}\div\frac{2}{7}$
$\frac{2}{5}+\frac{3}{5}$	$1\frac{3}{5}-\frac{2}{5}$	$126\times1\frac{5}{6}$	$\frac{2}{3}\div\frac{3}{5}$	$10\frac{5}{9}\div2\frac{1}{9}$

610. Oral Problems.

1. Find the cost of 1 lb. of tea at 75 cents, and a piece of ham at 56 cents.

2. A farmer sold 58 sheep from his flock of 121 sheep. How many remained?

3. What will be paid for 8 lb. of coffee at 35¢ per lb.?

4. A laborer received $4.88 for four days' work. How much did he earn per day?

5. At $40 each, how many cows can be purchased for $2,000?

6. Bought 20 lb. of sugar at 5¢ per lb., and 2½ lb. of butter at 30¢. What was the amount of my bill?

7. A piece of cloth measuring 31½ yards was divided into 2 equal parts. Find the length of each.

8. How many weeks in a year of 366 days?

9. If I pay 25 cents for 3 pounds of cherries, how many pounds can I buy for $1.25?

10. Find the cost of a bushel and a peck of peanuts at the rate of 5 cents per quart.

11. A farmer had 164 acres of land. How much had he left after selling 87 acres?

12. Find the total number of pounds in 3 tubs of butter weighing respectively 25 pounds, 34 pounds, and 36 pounds.

13. At 60¢ per lb., how much tea can be bought for $5.85?

14. A drover paid $219 for oxen, at an average price of $73. How many did he buy?

15. What will be the cost of 20 bu. of wheat at $1.04½ per bu.?

16. At 24¢ per lb., how many ounces of butter can be bought for 18¢?

17. A woman pays $540 per year for a house. What is the rent per month?

18. How many weeks in 294 days?

19. At 72¢ per yard, what will be the cost of 2 ft. 11 in. of lace?

20. How much does a grocer receive for a barrel of flour, 196 lb., retailed at 3 cents per lb.?

21. If 47 men can do a piece of work in 4 days, how long will it take 1 man to do the same work?

22. Find the cost of 36 acres of land at $25 per acre.

23. If it takes $3\frac{1}{2}$ yards of cloth to make a coat, how many coats can be made from $24\frac{1}{2}$ yards?

24. How much will be paid for 84 yards of silk at $\$1\frac{3}{4}$ per yard?

25. If a certain quantity of provisions will last one man 215 days, how long will it last 43 men?

26. How many square yards are there in a rectangular field 36 yards long and 25 yards wide?

613. Sight Exercises.

Add:

1. $3\frac{1}{2}+5\frac{1}{8}$
2. $4\frac{1}{2}+8\frac{3}{4}$
3. $9\frac{3}{8}+7\frac{5}{8}$
4. $11\frac{7}{8}+4\frac{1}{2}$
5. $7\frac{3}{5}+9\frac{7}{10}$
6. $5\frac{3}{4}+2\frac{3}{8}$
7. $8\frac{3}{4}+6\frac{7}{8}$
8. $15\frac{3}{8}+8\frac{1}{2}$
9. $9\frac{5}{8}+5\frac{3}{8}$

614. Subtract $8\frac{1}{2}$ from 10.

$(10\frac{1}{4}-8\frac{1}{2})$ is how much greater than $(10-8\frac{1}{2})$?

To subtract $8\frac{1}{2}$ from $10\frac{1}{4}$ mentally, we find the difference between $8\frac{1}{2}$ and 10, which is $1\frac{1}{2}$, and to this add $\frac{1}{4}$. The answer is $1\frac{3}{4}$.

10. $11\frac{3}{8}-6\frac{3}{8}=4\frac{1}{8}+\frac{3}{8}=?$
11. $14\frac{3}{7}-9\frac{1}{2}=4\frac{1}{2}+\frac{3}{7}=?$

615. Subtract at sight:

12.	$23\frac{3}{5} - 19\frac{7}{10}$	18.	$14\frac{1}{4} - 8\frac{3}{10}$
13.	$16\frac{3}{8} - 9\frac{3}{4}$	19.	$27\frac{1}{3} - 7\frac{1}{8}$
14.	$18\frac{1}{4} - 3\frac{3}{8}$	20.	$35\frac{1}{8} - 3\frac{1}{3}$
15.	$15\frac{1}{8} - 8\frac{3}{4}$	21.	$11\frac{2}{7} - 6\frac{1}{2}$
16.	$9\frac{1}{2} - 2\frac{7}{8}$	22.	$43\frac{3}{10} - 8\frac{4}{5}$
17.	$10\frac{1}{12} - 5\frac{1}{4}$	23.	$50\frac{1}{6} - 4\frac{3}{4}$

616. Slate Exercises.

Divide without writing products (Art. 385):

$$\begin{array}{r} 495\frac{17}{36} \\ 36\overline{)17837} \\ \underline{343} \\ 197 \\ \underline{} \\ 17 \end{array}$$

1.	$4{,}320 \div 32$	6.	$42{,}387 \div 123$
2.	$5{,}387 \div 51$	7.	$73{,}690 \div 345$
3.	$8{,}795 \div 75$	8.	$105{,}261 \div 624$
4.	$10{,}246 \div 84$	9.	$423{,}958 \div 1{,}008$
5.	$21{,}321 \div 97$	10.	$867{,}293 \div 2{,}534$

CANCELLATION.

617. Slate Problems.

1. If 17 horses cost \$4,387, what will I pay for 51 at the same rate?

618. Problems consisting of multiplication and division can be sometimes shortened by indicating the operations, and then canceling. To solve the above, we indicate the cost of one horse, $\frac{\$4387}{17}$, and of 51 horses, $\frac{4387}{17} \times 51$ or $\frac{4387 \times \cancel{51}^{3}}{\cancel{17}}$. Since 17 is contained in 51 three times, we cancel both numbers by drawing a line through them, and we write a 3 above the 51. The answer is $\$4{,}387 \times 3 = \$13{,}161$.

2. If 15 eggs cost 25 cents, what will 10 dozen cost?

$$\frac{25 \times 10 \times 12}{15}$$

We indicate price of one egg by dividing by 15. Multiplying this by 10 times 12, we get the price of 10 dozen.

In this case, 15 is not contained in any number above the line. We divide 15 and 10 by 5, canceling them and writing quotients 3 and 2 alongside. 3 is contained in 12 4 times; so we cancel 3 and 12. Our answer now is 25 cents $\times$ 4 $\times$ 2 = 200 cents, or $2.

$$\frac{25 \times \overset{2}{\cancel{10}} \times \overset{4}{\cancel{12}}}{\underset{3}{\cancel{15}}}$$

3. Eighteen men can do a piece of work in 26 days. How long will it take 13 men to do the same work?

One man will take 26 days $\times$ 18.

4. Seventeen barrels of flour, 196 lb. each, were put into bags holding 49 pounds each. How many bags of flour were put up?

5. At the rate of 23 cents for 7 pounds, how much would be paid for 91 pounds of flour?

6. A bank pays $4 interest a year on every $100. How much interest will be paid for 3 years on $650?

7. At $45 per thousand for bricks, what must I pay for 63,200 bricks?

8. If flour is $6 per barrel (196 lb.), what must be paid for a 49-pound bag?

9. A grocer buys 84 dozen eggs, and sells them at the rate of 21 eggs for 25 cents. What does he receive for them?

10. A miller buys 9,840 pounds of wheat at 90 cents per bushel of 60 pounds. How much does he pay for it?

11. What will be the cost of 64 sheep, if 18 cost $198?

12. If 18 men can do a piece of work in 42 days, how long will it take 21 men to do the same work?

13. What will be the cost of 66 doz. pens at 42 cents per gross of 12 doz.?

14. A certain quantity of hay feeds 15 horses 56 days. How long will it feed 14 horses?

15. A merchant bought 9 pieces of cloth, each containing 24 yards, for $189. What was the price per yard?

MULTIPLICATION OF FRACTIONS.

619. Oral Exercises.

What is $\frac{1}{2}$ of 2 fifths? Of 4 sevenths? Of 6 elevenths?

What is $\frac{1}{2}$ of $\frac{1}{2}$? Of $\frac{1}{3}$? Of $\frac{1}{4}$? Of $\frac{1}{5}$? Show by a diagram.

What is $\frac{1}{2}$ of $\frac{2}{3}$? Of $\frac{3}{4}$? Of $\frac{4}{5}$? Of $\frac{5}{6}$?

What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{2}{3}$ of $\frac{1}{2}$? $\frac{3}{3}$ of $\frac{1}{2}$?

What is $\frac{1}{3}$ of $\frac{3}{4}$? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{3}{3}$ of $\frac{3}{4}$?

What is $\frac{1}{4}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{2}{3}$? $\frac{3}{4}$ of $\frac{1}{3}$? $\frac{3}{4}$ of $\frac{2}{3}$?

What is the half of $1\frac{1}{2}$? Of $2\frac{1}{2}$? Of $3\frac{1}{2}$? Of $4\frac{1}{2}$?

What is one-third of $1\frac{1}{2}$? $\frac{2}{3}$ of $1\frac{1}{2}$? $\frac{1}{3}$ of $2\frac{1}{2}$? $\frac{2}{3}$ of $2\frac{1}{2}$?

620. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

This means to find $\frac{4}{5}$ of $\frac{2}{3}$.

Since $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$, $\frac{1}{3}$ of $\frac{4}{5} = \frac{4}{15}$, and $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$.

$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$; that is, the product of the numerators is divided by the product of the denominators.

NOTE. — Cancel when possible.

621. Multiply $\frac{2}{3}$ by $\frac{9}{10}$.

$$\frac{1}{3} \text{ of } \frac{9}{10} = \frac{3}{10} \qquad \frac{2}{3} \text{ of } \frac{9}{10} = 2 \times \frac{3}{10} = \frac{6}{5}$$

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{9}^{3}}{\cancel{10}_{5}} = \frac{3}{5}$$

Show by a diagram that 2×3 tenths $= 3$ fifths.

622. Multiply $12\frac{1}{7}$ by $3\frac{4}{15}$.

Reduce the mixed numbers to improper fractions.

$$\frac{\overset{17}{\cancel{85}}}{\underset{7}{\cancel{7}}} \times \frac{\overset{7}{\cancel{49}}}{\underset{3}{\cancel{15}}} = \frac{119}{3} = 39\tfrac{2}{3}$$

623. Slate Exercises.

Multiply:

1. $\frac{3}{8}$ by 96
2. 128 by $\frac{3}{4}$
3. $\frac{4}{5}$ by $\frac{3}{7}$
4. $\frac{5}{6}$ by $\frac{7}{8}$
5. $\frac{3}{5}$ by $\frac{5}{8}$
6. $3\frac{5}{12}$ by 72
7. $24\frac{2}{3}$ by 18
8. $69\frac{3}{7}$ by 32
9. $111\frac{4}{15}$ by 28
10. 67 by $\frac{15}{67}$
11. $2\frac{1}{2}$ by $3\frac{3}{5}$
12. $\frac{9}{16} \times 2\frac{2}{3}$
13. $17\frac{1}{4}$ by $6\frac{2}{3}$
14. $6\frac{1}{3} \times \frac{4}{7}$
15. $4\frac{1}{4}$ by $8\frac{4}{5}$
16. $\frac{5}{14} \times 8\frac{2}{5}$
17. $3\frac{3}{7}$ by $12\frac{1}{4}$
18. $\frac{7}{8} \times 4\frac{12}{25}$
19. $\frac{5}{7}$ by $\frac{3}{8}$ by $\frac{14}{15}$
20. $\frac{3}{40}$ of $\frac{28}{37}$ of $\frac{5}{7}$
21. $\frac{11}{12} \times \frac{5}{7} \times \frac{9}{22}$
22. $\frac{12}{35}$ of $\frac{14}{15}$ of $\frac{25}{27}$
23. $\frac{7}{8}$ of $65\frac{5}{6}$
24. $\frac{3}{4}$ of $55\frac{5}{6}$
25. $6\frac{1}{2} \times 7\frac{3}{5}$
26. $4\frac{1}{4} \times 5\frac{1}{5}$
27. $\frac{2}{3}$ of $4\frac{1}{2} \times 3\frac{2}{5}$
28. $\frac{5}{7}$ of $3\frac{1}{5} \times 4\frac{1}{11}$
29. $1\frac{1}{2} \times 2\frac{1}{3} \times 3\frac{1}{4}$
30. $2\frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2}$

624. Find answers:

31. $\dfrac{3\frac{1}{2} + 6\frac{1}{4}}{3}$
32. $(3\frac{1}{3} - 2\frac{1}{5}) \times \frac{5}{6}$
33. $\frac{1}{4}$ of $(5\frac{1}{3} - 3\frac{2}{3})$
34. $(24\frac{1}{3} + 16\frac{1}{6}) \div 8$
35. $(3\frac{1}{4} + 2\frac{1}{3}) \times (3\frac{1}{4} - 2\frac{1}{3})$
36. $(8\frac{2}{3} \times 21) - \dfrac{15\frac{5}{11}}{5}$
37. $5\frac{1}{2} + 6\frac{2}{3} + 7\frac{3}{4}$
38. $18\frac{5}{6} - 3\frac{3}{5} - 7\frac{1}{3}$
39. $\frac{2}{3}$ of $\frac{3}{4}$ of $(3\frac{1}{2} + 1\frac{3}{4})$
40. $(18\frac{1}{3} - 6\frac{2}{3}) \div 11$

DIVISION OF FRACTIONS.

625. Oral Exercises.

3 fourths is contained in 4 fourths how many times?

4 fourths $\div$ 3 fourths = ?

$1 \div \frac{3}{4}$ = how many thirds?

626. Give answers in improper fractions.

$1 \div \frac{2}{3} = ?$ $\quad 1 \div \frac{5}{6} = ?$ $\quad 1 \div \frac{3}{5} = ?$

Can you show by a diagram that $1 \div \frac{2}{3} = 1\frac{1}{2} = \frac{3}{2}$?

627. Divide $\frac{4}{7}$ by $\frac{3}{5}$.

We can divide by reducing both fractions to a common denominator:

$$\frac{4}{7} \div \frac{3}{5} = \frac{20}{35} \div \frac{21}{35} = 20 \div 21 = \frac{20}{21}$$

The following is another method:

$\frac{3}{5}$ is contained in 1, $\frac{5}{3}$ times. In $\frac{4}{7}$ of 1, it is contained $\frac{4}{7}$ of $\frac{5}{3}$ times $\frac{20}{21}$.

628. To divide by $\frac{3}{5}$, we multiply by what fraction?

629. Divide $\frac{3}{4}$ by $\frac{9}{16}$.

$\frac{9}{16}$ is contained in 1, $\frac{16}{9}$ times. In $\frac{3}{4}$, it is contained $\frac{3}{4}$ of $\frac{16}{9}$ times.

Canceling, $\frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}}}{\underset{3}{\cancel{9}}}$, we have $\frac{4}{3} = 1\frac{1}{3}$. *Ans.*

630. To divide by $\frac{9}{16}$, we multiply by what?

631. Can you give a rule for dividing by a fraction?

632. N. B. Change mixed numbers to improper fractions.

633. Slate Exercises.

Divide:

1. $8 \div \frac{4}{5}$
2. $\frac{2}{3} \div 4$
3. $10 \div \frac{5}{7}$
4. $\frac{9}{16} \div \frac{3}{20}$
5. $\frac{3}{7} \div 10$
6. $\frac{3}{20} \div \frac{9}{16}$
7. $1\frac{3}{4} \div 5$
8. $5 \div 1\frac{3}{4}$
9. $8\frac{2}{3} \div 11$
10. $4\frac{5}{11} \div 17$
11. $24\frac{1}{6} \div 20$
12. $6\frac{3}{4} \div 9$
13. $\frac{4}{11} \div \frac{3}{11}$
14. $\frac{11}{15} \div \frac{4}{15}$
15. $\frac{3}{11} \div \frac{4}{11}$
16. $\frac{4}{15} \div \frac{11}{15}$
17. $3\frac{3}{5} \div \frac{3}{5}$
18. $\frac{4}{7} \div 3\frac{3}{7}$
19. $\frac{4}{11} \div 6\frac{2}{11}$
20. $\frac{11}{24} \div 8\frac{5}{24}$
21. $\frac{1}{6} \div \frac{3}{5}$
22. $\frac{4}{5} \div \frac{5}{6}$
23. $\frac{3}{15} \div \frac{1}{7}$
24. $\frac{9}{16} \div \frac{3}{11}$
25. $8\frac{7}{15} \div 3\frac{4}{7}$
26. $9\frac{7}{9} \div 3\frac{4}{5}$
27. $18\frac{1}{2} \div 11\frac{2}{3}$
28. $15\frac{4}{11} \div 13\frac{3}{5}$
29. $16\frac{1}{4} \div 13\frac{1}{3}$
30. $23\frac{1}{2} \div 6\frac{5}{7}$

634. Perform operations indicated:

31. $(3\frac{2}{3} \times 4\frac{1}{2}) - 10\frac{1}{4}$
32. $(13\frac{2}{3} - 7\frac{2}{3}) \times \frac{3}{4}$
33. $(20 \times \frac{3}{4}) \div \frac{7}{5}$
34. $(20 \div \frac{7}{5}) \times \frac{3}{4}$
35. $20 \div (\frac{7}{5} \times \frac{3}{4})$
36. $(20 \div \frac{7}{5}) \div \frac{3}{4}$
37. $(14\frac{3}{4} \times 7) - (9 \times 10\frac{7}{8})$
38. $(1\frac{5}{16} \times \frac{24}{25}) + (4\frac{4}{5} \times 6\frac{7}{8})$
39. $34\frac{5}{8} - 17\frac{5}{6} + 20\frac{17}{24}$
40. $18\frac{5}{9} + 24\frac{7}{12} - 36\frac{17}{36}$
41. $\dfrac{5\frac{2}{3} \times 9 \times 7\frac{3}{5}}{4\frac{3}{5} \times \frac{5}{6}}$
42. $\dfrac{5\frac{1}{2} \times 7\frac{3}{4} \times 3\frac{1}{6} \times 6\frac{1}{4}}{2\frac{3}{4} \times 4\frac{1}{5} \times 31}$

635. Oral Problems.

Give analysis of each:

1. If base-balls are worth $\frac{3}{4}$ of a dollar each, what will be the cost of 16 base-balls?

2. Paid \$12 for base-balls, at $\frac{3}{4}$ of a dollar each. How many were bought?

3. What is the cost of 2 feet of ribbon at 30 cents per yard?

4. Find how much a yard of ribbon is worth, if $\frac{2}{3}$ yard costs 20 cents.

5. If it takes $\frac{2}{3}$ yard of material to make one waist, how many can be made from a piece containing 24 yd.?

6. If 18 jackets require 24 yards of cloth, how much is needed for 1 jacket?

7. A man had 60 acres of land. How many acres had he left after selling $\frac{2}{5}$ of his land?

8. After spending $\frac{2}{3}$ of his money, a person had \$36 remaining. How much money had he at first?

9. If tea is worth $\frac{3}{4}$ of a dollar per pound, how much can be bought for $\frac{1}{2}$ of a dollar?

10. When tea is \$.50 per pound, how much can be bought for \$.75?

11. When silk is selling at \$.75 per yard, how much can be bought for one-fourth of a dollar?

12. Find the cost of a gallon of milk at the rate of 9 cents for 3 pints.

13. $\frac{3}{5}$ of a gal. of milk costs 9¢. What is the price per gal.?

14. $\frac{2}{3}$ of what number is 12?

15. 1 yard and 1 foot of wire cost 16 cents. How much must be paid for a yard?

16. A man bought some cows at $35 each, and the same number at $45 each. What was the average price?

17. A girl received 100 per cent in three studies, and 80 per cent in the fourth. What was her average?

18. A square floor contains 144 square feet. How many feet long and wide is it?

19. Mr. Brown mixed 3 pounds of black tea worth 40 cents a pound with 1 pound of 60-cent green tea. What is the mixed tea worth a pound?

640. Slate Problems.

1. A milliner sells 3 pieces of ribbon at 18 cents per yard. They measure $4\frac{3}{8}$ yd., $3\frac{4}{8}$ yd., and $5\frac{5}{12}$ yd., respectively. What is the amount of her bill?

2. How many feet and inches in $\frac{5}{12}$ of a yd.?

3. To make powder, a man mixes $7\frac{1}{4}$ lb. of saltpetre, $1\frac{7}{16}$ lb. of sulphur, and as much charcoal as sulphur. How many pounds of powder will there be?

4. Four men form a partnership; the first furnishes $\frac{1}{3}$ of the capital, the second $\frac{2}{5}$, and the third $\frac{5}{12}$. What fraction of the capital is furnished by the fourth?

5. I pay 15 cents more for a half pound of tea than I pay for a quarter pound of the same tea. What is its price per pound?

6. After doing $\frac{3}{4}$ of a piece of work, a man requires 3 days more to finish it. How many hours does he take to do the whole work if he works 8 hours per day?

7. If 1 lb. 7 oz. coffee cost 46 cents, what will 3 lb. 9 oz. cost?

8. Add 14 days 6 hours 50 minutes and 15 days 17 hours 10 minutes.

9. If a dozen pairs of gloves cost $15.25, what will be the cost of 60 pairs? Cancel.

10. 15 men do a piece of work in $10\frac{2}{3}$ days. How long would it take 5 men to do the same work?

11. To make a cloak 3 yd. of cloth $1\frac{1}{2}$ yards wide are required. How much cloth $\frac{3}{4}$ yd. wide would be required?

12. In 3 years 4 months a gas company manufactures 4,200,000 cubic feet of gas. How many cubic feet are manfactured per year?

13. If $2\frac{2}{3}$ dozen hats cost \$80, what will be the cost of 3 hats?

14. A boy hires a boat at 20 cents per hour. How much has he to pay if he uses it from 20 minutes before 9 A.M. until 10 minutes past 1 P.M.?

15. A and B kill an ox. A takes $\frac{5}{8}$ and B the remainder. If B's share weighs $361\frac{1}{2}$ lb., what is the weight of the ox?

16. A grocer buys 30 dozen eggs at 18 cents per dozen. He sells them at the rate of 15 eggs for 25 cents. What is his profit?

17. How many cents in $\frac{5}{16}$ of a dollar?

18. What fraction of $18\frac{3}{4}$ is $6\frac{2}{3}$? $\frac{6\frac{2}{3}}{18\frac{3}{4}} = ?$

19. A farmer buys a horse for \$140, and sells it at an advance of $\frac{3}{20}$ of the cost. What is the selling price?

20. In 1893, A was 36 years old and B was $1\frac{1}{4}$ times as old. In 1884, B was how many times as old as A?

21. From the sum of $18\frac{7}{10}$ and $25\frac{3}{5}$ take their difference.

22. If $2\frac{3}{4}$ acres of land cost \$220, what will be the cost of $17\frac{1}{8}$ acres? Indicate the work, and cancel.

23. A can do a piece of work in 6 days, B can do it in 6 days, C can do it in 6 days. How long will it take all three working together?

24. Find the value of $\frac{\frac{2}{3} \text{ of } \frac{3}{5} \text{ of } 2\frac{1}{3}}{\frac{1}{2} \text{ of } 3\frac{2}{5}}$.

25. A man sold a horse for $\frac{3}{4}$ of its cost, losing \$40. What did the horse cost him?

FRACTIONAL PARTS OF A DOLLAR.

641. Oral Problems.

1. How many 50-cent base-balls can be bought for $15?

$(15 \div \frac{1}{2})$

2. How many 75-cent base-balls can be bought for $15?

$(15 \div \frac{3}{4})$

3. At 75¢ per lb., how much tea can be bought for $1?

4. How many hats at $1.25 each can be bought for $15?

$(15 \div 1\frac{1}{4})$

5. Paid $16 for coffee at 25¢ per lb. How many pounds were purchased?

6. At $33\frac{1}{3}$¢ per lb., how many pounds of butter can be bought for $32?

7. Find the number of yards of ribbon, at $12\frac{1}{2}$¢ per yd., that will cost $45.

8. At $6\frac{1}{4}$¢ per bar, how many bars of soap will cost $11?

9. If 4 pieces of violet soap are sold for 25¢, how many can be bought for $9?

10. $24 is paid for corn at 75¢ per bu. How many bushels?

11. I spent $30 for lace at $66\frac{2}{3}$¢ per yard. How many yards did I buy?

12. For $36 how many pairs of rubber shoes can be bought at $37\frac{1}{2}$¢ per pair?

13. Oats are $62\frac{1}{2}$¢ per bu. How many bushels will $40 buy?

14. A farmer pays $87\frac{1}{2}$¢ per bu. for seed rye. If his bill amounted to $21, how many bushels did he buy?

15. A store-keeper sold $33 worth of collars, at 16⅔¢ each. How many did he sell?

16. At the rate of 3 for 50¢, how many collars can be bought for $25?

17. Corn is worth 20¢ per can. How many cans will cost $32?

18. Find the cost of 35 yards of cloth, at $1.25 per yard.

19. At $1.25 per yard, how many yards of cloth can be bought for $35?

20. How many pairs of gloves, at $1.75 per pair, will cost $28?

21. When coal is $5.25 per ton, how many tons can be bought for $42?

22. Cost of 16 pairs of shoes at $2.75?

23. 33 jackets at $3.33⅓?

24. 18 yd. cloth at $2.16⅔?

25. Paid $26 for cloth at $2.16⅔ per yard. How many yards did I buy?

26. Find the cost of 16 pairs of skates at $1.87½ per pair.

27. If sheep cost $3.12½ each, how many can I get for $75?

28. How many 25-cent balls can be bought for $8.75?

29. Divide 775 by 25.

30. Divide $8.25 by 75¢.

31. How many square feet are there in a lot 96 ft. long 25 ft. wide?

32. Find the total cost of 32 head of cattle at $75 per head.

33. How much must be paid for 32 cows at $37.50 each?

34. If sheep are worth $3.75 each, how much will a farmer receive for 32 sheep?

35. If a train goes at the rate of 25 miles per hour, how many hours will it take to go 675 miles?

BILLS.

642. NEW YORK, *Oct.* 1, 1894.

MRS. WILLIAM MARTIN,

Bought of GRAY AND WINTER.

	1894					
Aug.	13	44 yd. Carpet	$ .90			
	15	3 Oak Chairs	1.75			
		1 Rocker		12	—	
	19	18 yd. Oil Cloth	.50			
	27	1 Parlor Suit		75	—	
Sept.	19	6 Kitchen Chairs	.75			
		1 Table		4	50	
	26	36 yd. Matting	.33⅓			
						$

1. Copy the above. Supply the missing amounts.

2. John R. Schultz has bought the following goods of Arthur B. Rowe & Co.:

Jan. 3, 1894, 50 lb. of sugar, at 5½¢; 4 lb. of tea, at 62½¢. Jan. 4, 10 lb. of coffee, at 32½¢; 2 bbl. of flour, at $5.75. Jan. 9, 24 bars of soap, at 16⅔¢; 42 lb. of starch, at 8¢.

Make out a bill dated Feb. 1, 1894.

3. Make out a bill for the following articles bought during March and April. Supply the names of buyer and seller, also the dates.

23½ yd. of silk, at 80¢; 1⅜ yd. of lace, at $2.40; 64 yd. of muslin, at 6⅞¢; 8 spools of sewing silk, at 7¢; 4 pr. of stockings, at 65¢; 6 yd. of linen, at 87½¢; ½ doz. collars, at $2.10.

4. Make out a bill for the following goods bought June 15.

3 cases of torpedoes, at $2.20; 12 boxes of fire-crackers, at $1.62½; 3 gross pin wheels, at $1.35; 5 gross sky-rockets, at $3.25; 2 doz. balloons, at $2.25; 45 lanterns, at 9¢.

647. Sight Exercises.

1. $\frac{36 \times 14}{9}$	5. $\frac{42 \times 23}{21}$	9. $\frac{67 \times 36}{18}$	13. $\frac{83 \times 36}{12}$
2. $\frac{37}{8} \times 16$	6. $\frac{4}{23} \times 46$	10. $\frac{32}{41} \times 82$	14. $\frac{15}{16} \times 48$
3. $12 \times \frac{43}{24}$	7. $32 \times \frac{67}{96}$	11. $4 \times \frac{93}{12}$	15. $15 \times \frac{87}{30}$
4. $\frac{25 \times 18}{36}$	8. $\frac{33 \times 12}{99}$	12. $\frac{89 \times 13}{26}$	16. $\frac{44 \times 17}{34}$

648. Visitors to Prospect Park.

Month.	Carriages.	Equestrians.	Pedestrians.	Sleighs.	Totals.
January	43,398	1,953	408,230	6,680	
February	112,140	8,032	485,990	16	
March	128,520	12,027	526,270		
April	120,240	8,827	655,925		
May	359,621	15,805	1,944,353		
June	208,096	14,687	1,233,873		
July	220,860	5,575	1,443,173		
August	260,516	9,578	1,640,651		
September	333,639	11,926	1,704,611		
October	421,220	16,246	1,699,851		
November	316,020	15,324	1,268,101		
December	174,256	12,157	719,569		
Total					
Monthly average . .					
Daily average . . .					

In the foregoing table find the total number of visitors for each month, the number of visitors by carriage for the year, the number of equestrians and pedestrians, the number by sleighs, together with the grand total for the year. Find also the daily and the monthly average.

SHORT METHODS.

649. Oral Problems.

1. Multiply by 25:

16, 19, 21, 23, 25, 29, 33, 36, 42, 48.

2. How many square feet in a lot 84 ft. long, 25 ft. wide?

3. What is the weight of 25 bbl. of flour, each weighing 196 lb.?

4. Find the cost of 25 lb. of coffee at 32¢ per lb.

5. What will a woman have to pay for 25 yd. of silk at $1.60 per yd.?

6. A man sold 25 cows at $44 each. How much did he receive for them?

7. Multiply 64 by $12\frac{1}{2}$.

8. Find the cost of $12\frac{1}{2}$ bu. of wheat at 96¢ per bushel.

9. At $12.50 per bbl., how much would I have to pay for 56 bbl. of pork?

10. How many pens in $12\frac{1}{2}$ gross? (144 to gross.)

11. Find the cost of $12\frac{1}{2}$ lb. of tea at 56¢ per lb.

12. How many square yards in a field 96 yards long 75 yards wide?

650. Blackboard Exercises.

Write only the answers:

1. 837×25	8. $25 \times 2{,}174$	15. $12\frac{1}{2} \times 1{,}084$
2. 763×25	9. 837×250	16. $12\frac{1}{2} \times 2{,}196$
3. 934×25	10. 763×250	17. $12\frac{1}{2} \times 3{,}670$
4. 508×25	11. $864 \times 12\frac{1}{2}$	18. $12\frac{1}{2} \times 6{,}281$
5. 25×686	12. $776 \times 12\frac{1}{2}$	19. 864×125
6. 25×301	13. $236 \times 12\frac{1}{2}$	20. 776×125
7. $25 \times 1{,}039$	14. $404 \times 12\frac{1}{2}$	21. $125 \times 1{,}020$

651. Add $15\frac{2}{3}$ and $8\frac{3}{5}$.

Adding $\frac{2}{3}$ and $\frac{3}{5}$ $\left(\text{or } \frac{10+9}{15}\right)$, we get $1\frac{4}{15}$. Write $\frac{4}{15}$ and carry 1.

22. $34\frac{1}{2} + 15\frac{7}{10}$
23. $42\frac{3}{5} + 19\frac{4}{5}$
24. $84\frac{2}{3} + 18\frac{3}{4}$
25. $40\frac{4}{7} + 16\frac{1}{2}$
26. $15\frac{5}{6} + 32\frac{2}{5}$
27. $35\frac{2}{9} + 20\frac{1}{8}$
28. $12\frac{3}{5} + 41\frac{1}{3}$
29. $25\frac{1}{4} + 46\frac{3}{5}$
30. $57\frac{5}{8} + 17\frac{3}{5}$
31. $29\frac{3}{8} + 84\frac{1}{2}$
32. $68\frac{7}{9} + 13\frac{5}{6}$
33. $74\frac{5}{12} + 18\frac{7}{8}$
34. $37\frac{1}{2} + 60\frac{5}{7}$
35. $13\frac{5}{6} + 81\frac{1}{4}$

652. Write answers:

36. $57\frac{1}{2} - 18\frac{2}{5}$
37. $98\frac{5}{6} - 56\frac{1}{2}$
38. $46\frac{3}{4} - 19\frac{2}{3}$
39. $67\frac{5}{8} - 8\frac{1}{6}$
40. $74\frac{5}{9} - 40\frac{1}{4}$
41. $57\frac{2}{5} - 18\frac{1}{2}$
42. $98\frac{1}{2} - 56\frac{5}{6}$
43. $37\frac{2}{3} - 29\frac{1}{5}$
44. $58\frac{6}{11} - 50\frac{1}{3}$
45. $24\frac{2}{7} - 6\frac{1}{4}$
46. $90\frac{7}{10} - 18\frac{2}{3}$
47. $65\frac{11}{16} - 9\frac{1}{4}$
48. $37\frac{1}{5} - 29\frac{2}{3}$
49. $24\frac{1}{4} - 6\frac{2}{7}$

653. Multiply $18\frac{3}{4}$ by 4.

$\frac{3}{4} \times 4 = 3$. 4 eighths are 32, and 3 are 35 (put down 5). 4 ones are 4 and 3 are 7. *Ans.* 75.

50. $27\frac{1}{2} \times 10$
51. $33\frac{1}{3} \times 12$
52. $16\frac{5}{8} \times 8$
53. $17\frac{3}{4} \times 8$
54. $19\frac{2}{3} \times 6$
55. $15\frac{7}{8} \times 3$
56. $13\frac{4}{5} \times 4$
57. $20\frac{1}{2} \times 11$
58. $40\frac{2}{3} \times 5$
59. $16\frac{3}{4} \times 7$
60. $37\frac{1}{2} \times 3$
61. $45\frac{3}{5} \times 5$
62. $23\frac{1}{3} \times 4$
63. $17\frac{1}{3} \times 6$

654. Do not change dividends to improper fractions.

64. $3)45\frac{3}{7}$

65. $4)56\frac{1}{3}$

66. $12)36\frac{1}{3}$

67. $5)72\frac{1}{2}$

68. $11)83\frac{3}{5}$

69. $8)37\frac{1}{4}$

70. $9)48\frac{1}{4}$

71. $6)25\frac{1}{5}$

72. $7)10\frac{1}{2}$

73. $6)75\frac{3}{5}$

74. $7)97\frac{3}{10}$

75. $10)87\frac{4}{5}$

76. $4)66\frac{3}{8}$

77. $3)94\frac{1}{8}$

MULTIPLICATION OF DECIMALS.

655. Oral Exercises.

3 and a decimal multiplied by 2 and a decimal gives *about* what product?

$44.02 \times 2.05 =$ about what?

656. Slate Exercises.

Multiply:

1. 32×2.5
2. 3.2×25
3. 6.4×4.5
4. 7.2×3.75
5. 12.8×5.7
6. 9.6×1.125
7. 34.9×2.34
8. 5.625×8.4
9. 1.875×12.8
10. 42.36×2.95

657. In multiplying 32 by 2.5, how many decimals are pointed off in the product? In multiplying 3.2 by 2.5, how many are pointed off? How many are pointed off in the product of 9.6 by 1.125?

658. Can you tell the relation the number of decimal places in the product bears to the number in the multiplier and in the multiplicand?

$\frac{7}{10} \times \frac{7}{10} = ?$ $\quad .7 \times .7 = ?$ $\quad .1 \times .1 = ?$ $\quad .1 \times .01 = ?$

11.	1.75×64	**16.**	18.4×20.25
12.	8.375×40	**17.**	11.16×42.40
13.	24.5×18.2	**18.**	$66.6 \times 3.3\frac{1}{3}$
14.	$9.6 \times 12\frac{1}{2}$	**19.**	6.24×1.75
15.	7.43×3.6	**20.**	400.04×39.25

DIVISION OF DECIMALS.

659. Divide 42 by 2.1.

Changing the decimal fraction in the divisor to a common fraction, we have

$$42 \div 2\tfrac{1}{10} = \tfrac{42}{1} \div \tfrac{21}{10} = \tfrac{42}{1} \times \tfrac{10}{21} = \tfrac{420}{21}.$$

$$42 \div 2.1 = 420 \div 21.$$

660. When we change the divisor 2.1 to 21, we have multiplied it by 10, and the same change must be made in the dividend.

661. In the following examples, make each divisor a whole number by removing the decimal point, and make a corresponding change in the dividend.

662. Divide:

21.	$80 \div 2.5$	**30.**	$72 \div .5$
22.	$8 \div 2.5$	**31.**	$960 \div .03$
23.	$840 \div 1.2$	**32.**	$.847 \div .007$
24.	$36 \div \frac{9}{10}$	**33.**	$27 \div .002$
25.	$36 \div .9$	**34.**	$10 \div .8$
26.	$12.6 \div 6.3$	**35.**	$1.263 \div .03$
27.	$48 \div 15$	**36.**	$196.3 \div .013$
28.	$18.36 \div .6$	**37.**	$19.63 \div .013$
29.	$50 \div .25$	**38.**	$1.963 \div .013$

663. Remove the decimal point in the divisor three places to the right, and make a corresponding change in the dividend, adding two ciphers.

To show where the decimal point originally belonged, it may be enclosed in a small circle, instead of being erased.

Ans. 15 100.

$\odot 013.)196\odot 300.$

66

13

00

When the divisor is thus made a whole number, the decimal point in the quotient will be placed under (or over) the new decimal point in the dividend.

$1.736 \div 16$	$17.36 \div .16$	$.01736 \div 1.6$
.1085 *Ans.*	108.5 *Ans.*	.01085 *Ans.*
$16)1.736$	$\odot 16.)17\odot 36.$	$1\odot 6.)\odot 0.1736$
136	136	136
80	80	80

39.	$.504 \div .024$	**47.**	$392 \div 3.2$
40.	$5.04 \div .24$	**48.**	$48 \div 3{,}000$
41.	$50.4 \div 2.4$	**49.**	$92 \div .23$
42.	$504 \div 24$	**50.**	$.875 \div 125$
43.	$168 \div .7$	**51.**	$381.17 \div 8.11$
44.	$36 \div .12$	**52.**	$.624 \div 9.75$
45.	$.875 \div .25$	**53.**	$48.195 \div 3.57$
46.	$123.6 \div .01$	**54.**	$829.31 \div .019$

664. Divide 381.6 by 95.032.

4.015 +

$95\odot 032.)381\odot 600.$

147200

521680

The sign (+) after the last figure of the quotient indicates that there is a remainder.

665. Divide, carrying out the quotient to 3 places of decimals:

55.	$31 \div 13$	**58.**	$7.049 \div 1.6$
56.	$4.5 \div 17$	**59.**	$81.22 \div 3.275$
57.	$920.07 \div 46$	**60.**	$246.3 \div 93.473$

666. Sight Exercises.

Write answers at sight:

1. $.042 \times 200$	5. $40 \times .7$	9. $.121 \times 4{,}000$
2. $.13 \times 300$	6. $25 \times .08$	10. $.061 \times 500$
3. $.014 \times 50$	7. $.234 \times .2$	11. $.03 \times 1{,}000$
4. 8.1×60	8. $.73 \times 30$	12. $.012 \times 700$

667. Remember that $369 \div 1{,}000 = \frac{369}{1000} = 369$ thousandths $= .369$; that $219 \div 100 = 2.19$; and that $6 \div 100 = .06$.

13. $369 \div 3{,}000$	17. $2{,}460 \div 3{,}000$	21. $4.68 \div 20$
14. $219 \div 300$	18. $196 \div 4{,}000$	22. $30.5 \div 500$
15. $48.6 \div 60$	19. $6 \div 500$	23. $18.8 \div 200$
16. $1.89 \div 90$	20. $27.9 \div 300$	24. $248 \div 4{,}000$

668. Slate Exercises.

Cancel the ciphers in the divisor, and remove the decimal point in the dividend a corresponding number of places to the left, prefixing ciphers if necessary.

1. $1{,}728 \div 1{,}200$

$12\cancel{00})17.28_{\odot}$

1.44 *Ans.*

2. $172.8 \div 1{,}200$

$12\cancel{00})1.72_{\odot}8$

$.144$ *Ans.*

3. $1.728 \div 1{,}200$

$12\cancel{00}).01_{\odot}728$

$.00144$ *Ans.*

4. $2{,}436 \div 3{,}000$	10. $45 \div 800$
5. $136.5 \div 1{,}300$	11. $25.2 \div 240$
6. $84.8 \div 80$	12. $345.6 \div 1{,}200$
7. $100.1 \div 700$	13. $4{,}004 \div 110$
8. $1 \div 40$	14. $5.28 \div 60$
9. $2.2 \div 50$	15. $907.5 \div 1{,}500$

SIGHT APPROXIMATIONS.

669. Give approximate answers. Whole numbers.

1. $17\frac{8}{10} \times 3\frac{8}{99}$; or, about $17 \times$ about 4.
2. $25\frac{1}{16} \div \frac{24}{95}$; or, about $25 \div \frac{1}{4}$ nearly.
3. $6\frac{1}{8} \times 6\frac{1}{8}$
4. $300\frac{1}{25} \div 11\frac{86}{87}$
5. $86\frac{2}{3} \times \frac{71}{72}$
6. $35\frac{7}{8} \div 3\frac{19}{20}$
7. $799\frac{31}{32} \div 99\frac{15}{16}$
8. $7\frac{99}{100} \times 7\frac{9}{16}$
9. $7\frac{1}{3} \times 11\frac{1}{13}$
10. $64\frac{1}{86} \times \frac{8}{8}$

670. Give answers in whole numbers:

1. 8.75×9.999; or, 8.75×10 nearly.
2. $24.002 \div .4999$; or, $24 \div$ nearly $\frac{5}{10}$, or $\frac{1}{2}$.
3. 25.125×11.834
4. $36.843 \div 6.105$
5. $86.4 \div .983$
6. 32.04×5.001
7. $799.9 \times .103$
8. 7.999×7.999
9. 7.001×12.003
10. $64.001 \div .249$

671. Give the cost, approximately, of:

1. 49 horses at \$199 each. ($\$200 \times 49$.)
2. 199 yd. 2 ft. 11 in. of cloth at \$2.50 per yard.
3. 3 lb. 15 oz. of butter at 25¢ per lb.
4. 398 coats at \$12 each.
5. 7 bu. 3 pk. 7 qt. potatoes at \$2 per bushel.
6. 798 base-balls at 25 cents each.
7. 19 gal. 3 qt. 1 pt. alcohol at \$2.49 per gallon.
8. 995 lb. tea at $59\frac{3}{4}$ cents per pound.
9. 7 houses at \$4,995 each.
10. 597 pounds of hay at 99 cents per 100 pounds.

672. Slate Problems.

1. Find the cost of 18,756 ft. of lumber at $30 per 1,000 ft.

2. A field is 14.25 rods long by 7.4 rods wide. What is its area in square rods?

3. A rod is 16.5 feet. How many rods are there in 231 feet.?

4. How many marks are there in $100? (A mark is equal to 23.8 cents.)

5. Add 3 and 4 tenths, 96 thousandths, 100 and 5 thousandths, 27 hundredths.

6. From 2,700 take 27 hundredths.

7. Multiply 8 and 4 tenths by 9 and 25 hundredths.

8. Divide 96 and 75 hundredths by 322 and 5 tenths.

9. A load of hay, at 75 cents per 100 pounds, cost $13.98. What was the weight of the hay?

10. The circumference of a circle is 3.1416 times the diameter. How many inches in circumference is a circle whose diameter is 20 inches?

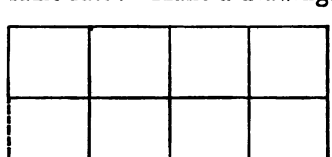
11. Show by a diagram the number of pieces of wire $\frac{2}{3}$ yd. long that can be made from 4 yd. of wire.

12. Show by a diagram that three-fourths of one is equal to one-fourth of three.

13. If $\frac{2}{3}$ of a yd. of material will make an apron, how many half aprons can be made from a yard? Show by a diagram.

14. A boy paid 6 cents for three-eighths of a pie. What would be the cost of the whole pie at the same rate? Make a drawing.

15. Seven-eighths of an acre of land is sold for $140. What is the price of an acre?



REVIEW.

676. In comparing two fractions, reduce both to a common denominator. Change denominate numbers to same denominate unit.

677. Oral Exercises.

1. What part of $\frac{7}{15}$ is $\frac{8}{15}$? (What part of 7 is 3?)
2. $\frac{1}{5}$ is what part of $\frac{7}{15}$?
3. What part of $2\frac{1}{2}$ is $\frac{1}{2}$?
4. 3 pt. is what part of a gallon? (3 pt. is what part of 8 pt.?)
5. What part of a gallon is 1 qt. 1 pt.?
6. Divide $\frac{2}{3}$ by $\frac{3}{5}$. (Divide 10 fifteenths by 9 fifteenths.)
7. Divide $\frac{4}{5}$ by $\frac{2}{3}$.
8. How many sq. ft. in a rectangle 12 ft. long, 13 ft. wide?
9. $\frac{1}{5}$ of a day is how many hours and minutes?
10. 14 ounces is what part of 2 pounds?
11. $\frac{3}{4}$ ft. is what part of a yard?
12. A strip of tape 3 yards long is cut into four equal pieces. How many feet and inches in each piece?
13. At $30 per month, how much rent will I pay in 1 year, 8 months?
14. $2\frac{1}{4}$ months is what part of a year?
15. At $\frac{3}{5}$ of a dollar per lb., how much tea can I get for $1?
16. How many sq. yd. in a room 15 ft. long, 18 ft. wide?
17. A lot is 25 ft. by 100 ft. How many feet of fence will it take to enclose it?
18. 1 pk. 1 qt. is what part of a bushel?
19. 15 is what part of 4 dozen?
20. Reduce $\frac{44}{84}$ to lowest terms.

DENOMINATE NUMBERS.

678. Slate Exercises.

1. Add 4 days 6 hours, 9 days 11 hours, 3 days 7 hours.

2. What part of a week is 1 day 18 hours?

3. If a man receives $60 interest per year, how much will he receive in 3 years $7\frac{1}{2}$ months?

4. Reduce 3 days 18 hours to minutes.

5. How many days and hours are there in 8,100 min.?

6. $\frac{75}{100}$ of a day is how many hours?

7. How many hours and minutes in .4 day?

8. A man receives $1,460 per year of 365 days. What is his salary per week?

9. Find the cost of 1 bu. 1 pk. 1 qt. of potatoes at 8 cents per half-peck.

10. A piece of meat weighing 27 lb. 12 oz. is divided among 6 persons. How many pounds and ounces does each receive?

11. How many bu., pk., and qt., are there in 5 bags, each containing 1 bu. 1 pk. 1 qt.?

12. How many gallons, quarts, and pints of ice-cream will be needed to give a half-pint to each one of 67 persons?

13. Find the cost of 7 lb. 10 oz. of tea at 40 cents per lb.

14. From a pile of 20 bu. wheat there were sold 10 bu. 3 pk. 7 qt. How much remained?

679. Long Measure.

12	inches (in.)	1 foot (ft.)
3	feet	1 yard (yd.)
$5\frac{1}{2}$	yards	1 rod (rd.)
320	rods	1 mile (mi.)

15. How many yards in a mile? How many feet? How many inches?

16. A field is 16 rods long, 12 rods wide. How many square yards does it contain? How many rods of fence will be needed to enclose it? How many feet?

17. How many rails each 30 feet long will be needed for a single track road (two tracks) 40 miles long?

18. A boy steps 33 inches. How many steps will he take in going 2 miles?

19. Dec. 20 the sun rises at Boston at 7.26 A.M. and sets at 4.30 P.M. How long is it between sunrise and sunset? How much longer is the day at Charleston, S. C., where the sun rises at 6.58 A.M. and sets at 4.57 P.M.?

20. On June 21 the sun rises at Boston at 4.23 A.M. and sets at 7.40 P.M. On the same day it rises at Charleston at 4.53 A.M. and sets at 7.11 P.M. What is the length of the day at each place?

Change:

21. 17 lb. and 4 oz. to ounces.

22. 84 tons and 1,560 lb. to pounds.

23. 37 gal. and 3 qt. to quarts.

24. 45 gal. to pints.

25. 63 qt. and 1 pt. to pints.

26. 27 bu. and 3 pk. to pecks.

27. 48 pk. and 7 qt. to quarts.

28. 84 pk. to pints.

29. 27 mi. to yards.

30. 16 rd. and 3 yd. to yards.

31. 15,000 min. to days, etc.

32. 25,124 lb. to tons, etc.

33. 1,650 ft. to rods.

34. 876 pt. to gallons and quarts.

35. 228 in. to yards and feet.

36. 1,650 rods to miles and rods.

37. 864 hours to weeks and days.

38. 296 qt. to bushels and pecks.

39. 315 oz. to pounds and ounces.

40. 743 months to years and months.

41. 3 ft. 6 in. + 9 ft. 5 in. + 12 ft. 3 in.

42. 30 min. 15 sec. + 30 min. 18 sec. + 45 min. 24 sec.

43. 9 yr. 3 mo. + 18 yr. 7 mo. + 22 yr. 2 mo.

44. 19 wk. 4 da. + 7 wk. 5 da. + 8 wk.

45. 9 mi. 169 rd. + 84 rd. + 3 mi. 67 rd.

46. 7 yd. 1 ft. + 33 yd. + 19 yd. 2 ft.

47. 18 gal. 1 qt. + 16 gal. 2 qt. + 15 gal. 3 qt.

48. 5 pk. 3 qt. + 6 qt. + 7 pk. 1 qt.

49. 24 bu. 3 pk. + 24 bu. 3 pk. + 24 bu. 3 pk.

50. 12 qt. 1 pt. + 12 qt. 1 pt. + 12 qt. 1 pt. + 12 qt. 1 pt.

51. 12 qt. 1 pt. × 4.

52. 24 bu. 3 pk. × 3.

53. 5 pk. 3 qt. × 9.

54. 18 gal. 1 qt. × 8.

55. 33 yd. 1 ft. × 6.

56. 9 mi. 25 rd. × 7.

57. 15 wk. 3 da. × 5.

58. 7 yr. 3 mo. × 10.

59. 40 min. 35 sec. × 2.

60. 9 ft. 5 in. × 12.

61. 25 ft. − 18 ft. 7 in.

62. 50 min. 13 sec. − 27 min. 30 sec.

63. 12 yr. 1 mo. − 5 yr. 11 mo.

64. 50 wk. 4 da. − 18 wk. 6 da.

65. 15 mi. − 8 mi. 143 rd.

66. 33 yd. 1 ft. — 18 yd. 2 ft.

67. 240 gal. 1 qt. — 94 gal. 2 qt.

68. 83 pk. 3 qt. — 59 pk. 1 qt.

69. 170 bu. 1 pk. — 85 bu. 2 pk.

70. 135 qt. 1 pt. — 67 qt. 1 pt.

71. 87 qt. ÷ 2.

72. 50 min. 35 sec. ÷ 5.

73. 156 yr. 9 mo. ÷ 9.

74. 73 wk. 2 da. ÷ 3.

75. 50 mi. 135 rd. ÷ 7.

76. 253 yd. 1 ft. ÷ 10.

77. 387 gal. ÷ 6.

78. 222 bu. 3 pk. ÷ 9.

79. 150 qt. ÷ 4.

80. 75 bu. ÷ 8.

81. 87 qt. ÷ 43 qt. 1 pt.

82. 50 min. 35 sec. ÷ 10 min. 7 sec.

83. 78 bu. ÷ 9 bu. 3 pk.

84. 5 lb. 1 oz. ÷ 9 oz.

85. 14 ft. 2 in. ÷ 1 ft. 5 in.

MEASUREMENTS.

680. How many square yards in a room 6 yards long, 5 yards wide?

How many square yards in a room 18 feet long, 15 feet wide?

681. Slate Exercises.

Calculate the number of square yards in the following. First reduce each side to yards.

1. 18 yd. by 21 yd.
2. 54 ft. by 63 ft.
3. 72 in. by 108 in.
4. 19 yd. by 47 yd.
5. 67 yd. by 89 yd.
6. 33 ft. by 36 yd.
7. 27 ft. by 96 ft.
8. 54 ft. by 72 in.
9. 48 ft. by 45 ft.
10. 54 in. by 72 ft.

682. First, indicate the operations; then cancel.

11. Find the number of square yards in a room 18 ft. 4 in. long, 22 ft. 6 in. wide.

$$18 \text{ ft. } 4 \text{ in.} = 18\tfrac{1}{3} \text{ ft.} = \frac{18\frac{1}{3}}{3} \text{ yd.} = \frac{55}{9} \text{ yd.}$$

$$22 \text{ ft. } 6 \text{ in.} = 22\tfrac{1}{2} \text{ ft.} = \frac{22\frac{1}{2}}{3} \text{ yd.} = \frac{45}{6} \text{ yd.}$$

$$\text{Area} = \frac{55}{9} \times \frac{45}{6} \text{ sq. yd.} \quad \text{Canceling,} \quad \frac{55 \times \overset{5}{\cancel{45}}}{\cancel{9} \times 6} = \frac{275}{6} = 45\tfrac{5}{6} \text{ sq. yd.}$$

12. How many square yards in a room 13 ft. 1 in. long, 27 ft. wide?

$$13 \text{ ft. } 1 \text{ in.} = 157 \text{ in.} = \frac{157}{36} \text{ yd.} \quad 27 \text{ ft.} = 9 \text{ yd.}$$

$$\text{Area} = \left(\frac{157}{36} \times 9\right) \text{ sq. yd.} \qquad \frac{157 \times \cancel{9}}{\cancel{36}_{4}} = \frac{157}{4} = 39\tfrac{1}{4} \text{ sq. yd.}$$

13. How many square inches in 12 panes of glass, each 5 inches long, 7 inches wide?

14. A piece of cloth is 48 yards long, 24 inches wide. How many square yards does it contain?

15. A merchant imports 8 pieces of cloth, 36 yards to the piece. How many square yards of cloth are there, if it is 32 inches wide?

16. A board fence 6 feet high surrounds a lot 25 feet front by 100 feet deep. How many square feet of boards in the front fence? In the back fence? In each side fence? In the whole? (Make diagrams.)

17. A room is 18 feet long, 15 feet wide, 12 feet high. How many square feet in the floor?

Draw a rectangle to represent the ceiling. Write the dimensions in their proper places, and write in the centre the number of square feet in its surface. Draw diagrams of the four walls; give dimensions and surface of each.

18. How many faces has a cube? If one edge of a cube measures 4 inches, how many square inches in the entire surface?

Suppose you wish to make a cube out of a single piece of pasteboard. Make a drawing to show the shape of the piece needed, without allowing anything for overlapping parts.

19. The United States government charges a duty of 4¢ per square yard on imported cotton cloth. What duty must the importer pay on a piece containing 24 yards, $\frac{3}{4}$ yd. wide?

20. What will be the cost at $1 per square yard for flagging a sidewalk 12 feet wide and 30 feet long?

CHAPTER VIII.

DECIMALS. — BILLS. — DENOMINATE NUMBERS. — MEASUREMENTS. — PERCENTAGE. — INTEREST.

DECIMALS.

685. Changing Common Fractions to Decimals. Slate Exercises.

Reduce the following common fractions to decimals; *i.e.* perform the indicated division:

1. $1 \div 800$	8. $\frac{1}{32}$	15. $\frac{11}{250}$
2. $1 \div 40$	9. $\frac{21}{4000}$	16. $\frac{1}{625}$
3. $\frac{2}{25}$	10. $\frac{7}{2000}$	17. $\frac{875}{64}$
4. $\frac{25}{32}$	11. $\frac{18}{75}$	18. $\frac{18}{128}$
5. $\frac{17}{64}$	12. $\frac{7}{125}$	19. $\frac{1}{256}$
6. $\frac{3}{16}$	13. $\frac{28}{8}$	20. $\frac{7}{512}$
7. $\frac{3}{500}$	14. $\frac{47}{16}$	21. $\frac{1}{1024}$

686. Changing Decimals to Common Fractions.

What is the denominator of a decimal fraction?

What prime numbers are contained in 10? What are the only factors of 10? The prime factors of 100? Of 1,000?

Can $\frac{7}{1000}$ be reduced to lower terms? Why? Can $\frac{8}{100}$ be reduced to lower terms? Why? Can $\frac{425}{10000}$ be reduced to lower terms? How can we tell by merely looking at a decimal whether or not it can be reduced to a common fraction of lower terms?

687. Slate Exercises.

Reduce the following to common fractions — lowest terms. Do not find the greatest common divisor.

22. .0075
23. .36
24. .0275
25. .44
26. .03125
27. .486
28. .3750
29. .37500
30. .144
31. .0006
32. .27
33. .027
34. .00365
35. .96
36. .0009
37. .816
38. .15625
39. .0375
40. .00625
41. .096

ADDITION OF DECIMALS.

688. Add the following, reducing the common fractions to decimals:

42. $18\frac{3}{4} + 9.084 + 25\frac{1}{20} + 163 + 2.09 + 86\frac{1}{16} + .0975$
43. $275\frac{9}{100} + 58.64 + 8.6796 + 30\frac{1}{8} + 8\frac{2}{5} + 99 + 687\frac{23}{50}$
44. $84\frac{7}{10} + 93\frac{8}{100} + 3\frac{141}{1000} + \frac{219}{10000} + 684.1 + \frac{3}{4} + 17\frac{1}{2}$
45. $250 + 1875.93 + 16\frac{3}{5} + 8\frac{1}{40} + \frac{2}{25} + 608.94 + .0005$
46. $8.6796 + 96.8 + 18\frac{3}{8} + 250\frac{1}{80} + 34\frac{1}{20} + 1876$
47. $40\frac{8}{25} + 7.2832 + 86.3 + 128.46 + 2\frac{1}{25} + 41.5 + 8\frac{4}{5}$
48. $540 + 1.32 + .576 + \frac{33}{10000} + 68\frac{5}{16} + 395\frac{1}{5} + \frac{3}{25} + 7.51$
49. $5.308 + .25 + 567.8 + 8.4825 + 49.795 + 8\frac{3}{40}$
50. $7.08 + 23.04 + 8\frac{7}{10} + .348 + 3\frac{1}{25} + \frac{1}{125} + 7.00019$
51. $8\frac{999}{1000} + 8\frac{4}{5} + 507 + 28\frac{49}{100} + \frac{9}{20} + 6.8819 + 3.1416$

SUBTRACTION OF DECIMALS.

689. Give answers in decimals:

52. $275.3 - 87\frac{2}{50}$
53. $387\frac{3}{8} - 99.0127$
54. $1{,}000 - \frac{1}{1000}$
55. $62.365 - 48\frac{3}{4}$
56. $198\frac{2}{5} - 13.6431$
57. $24\frac{3}{16} - 9\frac{99}{1000}$
58. $2{,}345 - 345\frac{1}{32}$
59. $168\frac{3}{40} - 54.8759$
60. $18.42 - .576\frac{1}{4}$
61. $1{,}847\frac{11}{16} - 344\frac{9}{100}$

MULTIPLICATION OF DECIMALS.

690. Give answers in decimals:

62. $24.75 \times 3\frac{1}{5}$
63. $98\frac{3}{5} \times .00046$
64. $148\frac{1}{40} \times 12.5$
65. $380\frac{1}{2} \times .012$
66. $\frac{5}{32} \times 1.48$
67. $19.5 \times .000484$
68. $1.876 \times 3\frac{3}{5}$
69. 3.48×4.8665
70. $.43\frac{1}{4} \times 1\frac{1}{16}$
71. $192.38 \times .238$

DIVISION OF DECIMALS.

691. Give 3 places of decimals in quotient, exclusive of ciphers. Divide without writing products (Arts. 385, 616):

72. $7.345 \div .29$
73. $340{,}753 \div 4.18$
74. $4.054 \div 18.25$
75. $123.5 \div 384$
76. $471 \div 5.325$
77. $.3126 \div .0134$
78. $12.345 \div .0047$
79. $.8756 \div 4.322$
80. $8 \div 122$
81. $15.3678 \div .9125$
82. $48.45 \div .089$
83. $39562.478 \div 4279$
84. $346.25 \div 64.8$
85. $9.1342 \div 208.3$
86. $1784 \div 29.57$
87. $343.71 \div 1.127$
88. $83.087 \div 5.37$
89. $137.84 \div 7.91$
90. $38.9008 \div .523$
91. $.81074 \div .009157$

692. Solve by short division.

When ciphers are canceled in the divisor, what change must be made in the decimal point of the dividend?

92.	$18.756 \div 300$	**102.**	$48.64 \div 200$
93.	$48.36 \div 4{,}000$	**103.**	$.00531 \div 90{,}000$
94.	$.4824 \div 12{,}000$	**104.**	$96.51 \div 60$
95.	$11.011 \div 700$	**105.**	$87.5 \div 500$
96.	$3.6504 \div 90$	**106.**	$183.275 \div 10{,}000$
97.	$45.63 \div 1{,}500$	**107.**	$1.7632 \div 1{,}600$
98.	$130.13 \div 1{,}100$	**108.**	$1.5639 \div 130$
99.	$.8712 \div 60$	**109.**	$614.4 \div 120$
100.	$3.075 \div 5{,}000$	**110.**	$.576 \div 800$
101.	$.07056 \div 140$	**111.**	$.8008 \div 7{,}000$

MISCELLANEOUS.

694. Slate Exercises.

1. Find the cost of 24,400 bricks @ \$6.25 per M.

Ans. $\$6.25 \times 24.400 = \$6.25 \times 24.4$.

(How do we divide by 1,000?)

2. 760 pounds of hay @ 95 cents per cwt. (100 lb.).
3. 48,600 laths @ \$2.80 per M.
4. 39,250 stamped envelopes @ \$21.30 per thousand.
5. 1,875 pounds of straw @ 68 cents per cwt.
6. 108,745 Philadelphia bricks @ \$22.00 per M.
7. 14,860 oranges @ 75¢ per 100.
8. 2,376 eggs @ $13\frac{1}{2}$¢ per doz.
9. 4,500 cigars @ \$35 per M.
10. 28 doz. wax candles @ \$13.50 per gross (144).

695. Solve by cancellation where possible:

11. 38,648 lb. of wheat @ 90¢ per bu. (60 lb.).

12. 18,964 lb. of coal @ $5 per ton (2,000 lb.).

13. 48,576 lb. of oats @ 36¢ per bu. (32 lb.).

14. 69,104 lb. of rye @ $91\frac{1}{2}$¢ per bu. (56 lb.).

15. 74,816 lb. of corn @ $48\frac{1}{4}$¢ per bu. (56 lb.).

16. 360 meters of cloth @ $1.10 per yd. (1 meter = 39.37 inches).

17. Cost in United States money of 386 hats @ 24 francs each (1 franc = 19.3¢).

18. 480 meters of cloth @ 1.10 marks per meter (1 mark = 23.8¢).

696. Perform indicated operations.

Change divisor to whole number, making corresponding change in the dividend. Cancel.

19. $\dfrac{34.2 \times \overset{7}{\odot\!\!\!

697. Reduce to common fractions — lowest terms:

31. $.3\frac{1}{3}$
32. $.33\frac{1}{3}$
33. $.16\frac{2}{3}$
34. $.142\frac{8}{7}$
35. $.006\frac{1}{4}$
36. $.01\frac{1}{9}$
37. $.06\frac{2}{3}$
38. $.833\frac{1}{3}$
39. $.009\frac{1}{11}$
40. $.04\frac{1}{6}$
41. $.76\frac{12}{18}$
42. $.037\frac{1}{2}$

Change to decimals:

43. $\frac{7}{25}$
44. $\frac{9}{16}$
45. $\frac{3}{64}$
46. $\frac{18}{50}$
47. $\frac{17}{250}$
48. $\frac{11}{160}$
49. $\frac{18}{640}$
50. $\frac{23}{500}$
51. $\frac{23}{128}$
52. $\frac{15}{256}$
53. $\frac{9}{512}$
54. $\frac{4}{125}$

699. Blackboard Exercises.

Write answers at sight.

1. $24\frac{1}{2} + 15\frac{3}{4}$
2. $13\frac{3}{8} \times 6$
3. $42\frac{1}{8} - 13\frac{1}{8}$
4. $31\frac{1}{2} \div 4\frac{1}{2}$
5. $50\frac{2}{3} + 20\frac{1}{6}$
6. $80\frac{1}{6} - 40\frac{1}{4}$
7. $5\frac{1}{2} \times 5\frac{1}{2}$
8. $24\frac{2}{5} \div 2$
9. $36\frac{1}{3} \div 3$
10. $17\frac{1}{8} \div 4$
11. $21\frac{1}{4} \div 5$
12. $48\frac{8}{7} \div 6$
13. $18\frac{2}{3} \div 7$
14. $24\frac{1}{2} \div 8$
15. $36\frac{3}{8} \div 9$
16. $54\frac{1}{2} - 39\frac{3}{4}$
17. $48\frac{8}{9} \div 4$
18. $62\frac{1}{8} + 23\frac{1}{8}$
19. $12\frac{3}{4} \times 6$
20. $66\frac{2}{3} + 33\frac{1}{3}$
21. $33\frac{1}{3} - 16\frac{2}{3}$
22. $1\frac{3}{4} \times 1\frac{3}{4}$
23. $80 \div \frac{5}{6}$
24. $72 \div \frac{3}{4}$
25. $56 \div \frac{7}{8}$
26. $80 \div \frac{4}{5}$
27. $75 \div 1\frac{1}{2}$
28. $90 \div 1\frac{2}{3}$
29. $98 \div 1\frac{3}{4}$
30. $81 \div 1\frac{4}{5}$

MEASUREMENTS.

700. How many *square inches* in each of the following rectangles? First change each dimension to inches.

1. 42 in. by 36 in.
2. 71 in. by 18 in.
3. 3 ft. 1 in. by 4 ft. 2 in.
4. 5 ft. 3 in. by 6 ft. 4 in.
5. 12 ft. by 18 ft.
6. 9 ft. by 11 ft.
7. 27 in. by 30 in.
8. 65 in. by 92 in.
9. 7 ft. 3 in. by 2 yd.
10. 3 yd. by 6 ft. 6 in.

701. How many *square feet* in each of the following rectangles? First change each dimension to feet, or to feet and a fraction.

11. 18 ft. by 24 ft.
12. 36 in. by 4 ft.
13. 6 yd. by 8 yd.
14. 1 yd. by 48 in.
15. $3\frac{1}{2}$ ft. by 4 ft.
16. 3 ft. by $1\frac{1}{3}$ yd.
17. 42 in. by 4 ft.
18. 25 ft. by 17 ft. 6 in.
19. 42 in. by 48 in.
20. 13 yd. by 15 yd.

702. How many *square yards* in each of the following rectangles? Change each dimension to yards, or to yards and a fraction.

21. 18 yd. by 25 yd.
22. 15 yd. by 1 yd. 1 ft.
23. 27 ft. by 36 ft.
24. 54 ft. by 2 ft. 6 in.
25. 24 yd. by 27 in.
26. 36 yd. by 24 in.
27. 17 ft. 6 in. by 32 in.
28. 22 ft. 9 in. by 18 in.
29. 108 in. by 90 in.
30. 180 ft. by 54 in.

SPECIAL DRILLS.

703. Give sums:

$135+89$	$450+690$	$576+76$	$274+815$
$56+256$	$680+350$	$85+646$	$783+306$
$394+77$	$870+260$	$768+48$	$459+740$
$39+461$	$940+480$	$56+575$	$624+535$

704. Give differences:

$224-135$	$1{,}089-274$	$1{,}200-610$	$458-69$
$331-286$	$1{,}197-736$	$1{,}460-780$	$375-87$
$442-369$	$1{,}258-630$	$1{,}320-390$	$672-95$
$554-437$	$1{,}476-823$	$1{,}210-240$	$818-29$

705. Give products:

82×25	98×4	$46\times33\frac{1}{3}$	63×12
25×73	89×5	$39\times33\frac{1}{3}$	54×11
64×25	78×6	$26\times33\frac{1}{3}$	75×9
25×55	67×7	$19\times33\frac{1}{3}$	86×8

706. Give quotients:

$792\div9$	$792\div88$	$975\div25$	$300\div33\frac{1}{3}$
$415\div5$	$380\div76$	$850\div25$	$433\frac{1}{3}\div33\frac{1}{3}$
$693\div11$	$693\div63$	$675\div25$	$666\frac{2}{3}\div33\frac{1}{3}$
$406\div7$	$486\div54$	$825\div25$	$533\frac{1}{3}\div33\frac{1}{3}$

707. Give answers:

$12\frac{3}{4}\times5$	$3\times8\frac{5}{6}$	$4\frac{9}{10}\times7$	$11\times2\frac{1}{10}$
$11\frac{3}{8}\times4$	$4\times7\frac{4}{7}$	$3\frac{8}{11}\times8$	$12\times3\frac{3}{5}$
$10\frac{1}{2}\times3$	$5\times6\frac{7}{8}$	$2\frac{5}{12}\times9$	$11\times4\frac{4}{5}$
$9\frac{4}{5}\times2$	$6\times5\frac{5}{9}$	$1\frac{2}{11}\times10$	$10\times5\frac{4}{7}$

708. Oral Problems.

1. I sold 375 bushels of wheat to one miller and 87 to another. How many bushels did I sell?

2. Bought goods to the amount of $4.29. How much change from a $5 bill?

3. What will be the cost of 89 tons of coal at $5 per ton?

4. If 49 hats cost $147, what is the cost of one hat?

5. 567 marbles are divided among 9 boys. How many does each receive?

6. How many yards in 5 pieces of cloth, each containing $12\frac{3}{4}$ yd.?

7. Divide $29\frac{3}{8}$ by 7.

8. What will be the cost of a barrel of flour at $5.25 and 8 lb. of sugar at 6¢?

9. When silk is 75¢ per yd., how many yards can be bought for $9.75?

10. If $2\frac{5}{8}$ yd. ribbon cost 42 cents, what will $3\frac{1}{4}$ yd. cost?

11. How much must be paid for 55 lb. of raisins, at 8¢ per lb.?

12. Find the cost of 320 lb. of hay at 60¢ per hundred pounds.

13. If eggs are sold at the rate of 18 for 25 cents, what will be the cost of 6 dozen eggs?

14. Three men require 22 days to do a certain piece of work. How long would it take 11 men to do the same work?

15. A father earned $14.60, his son earned $7.80. What were the earnings of both?

16. How many yards of fence will be required to enclose a rectangular field 98 yards long and 50 yards wide?

17. A farmer divides his farm of 425 acres into fields of $12\frac{1}{2}$ acres each. How many fields has he?

18. There are 36 inches in a yard. How many yards are there in 324 inches?

19. The product is 925, the multiplier is 25. What is the multiplicand?

20. What will be the cost of 46 tons of hay, at $\$12\frac{1}{2}$ per ton?

21. What is the weight of 25 firkins of butter, each containing 56 pounds?

22. At $\$1.75$ per yard, how many yards of cloth can be bought for $\$49$?

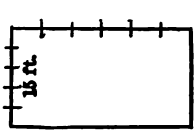
23. What price was paid for 20 sheep, at $\$8.75$ per head?

24. A man saved $\$320$ per year for 5 years. How much more would he require to make $\$2{,}000$?

25. Mr. Jones sold a lot for $\$675$, thereby losing $\$85$. What did he pay for it?

709. Slate Problems.

1. The width of a room is $\frac{5}{6}$ of its length. How many square feet in the floor, if the width is 15 ft.?



2. If 2 lb. 6 oz. of tea cost 95 cents, how many pounds and ounces can be bought for $\$2.35$?

3. What will be the duty on 175 kilograms of wool at 33 ct. per lb.? (1 kilogram = 2.2046 lb.)

4. John and James went out together. John had 38 cents. When one of the boys had spent 18 cents and the other had spent 16 cents, they had 24 cents left between them. Find the amount of money James had.

5. Find $\frac{1}{2}$ of the sum of $\frac{2}{3}$ and $\frac{3}{4}$.

6. What is $\frac{2}{3}$ of the difference between $\frac{4}{5}$ and $\frac{5}{6}$?

7. What fraction added to $\frac{4}{7}$ gives $\frac{3}{4}$?

8. Change $1\frac{3}{11}$ hour to seconds.

9. $\frac{4}{5}$ of what number equals 180?

10. The half of a number added to its fourth part equals $21\frac{3}{4}$. What is the number?

11. A farm is sold for \$5,700, at a loss of $\frac{1}{20}$ of the cost. What was the cost?

12. When it is noon at Philadelphia, it is 15 seconds and 10 minutes past 5 P.M. at Paris. What time is it at Philadelphia when it is noon at Paris?

13. A, B, and C buy a house. A furnished $\frac{1}{2}$ of the cost, B $\frac{1}{3}$, and C \$1,200. What did A and B pay, respectively?

14. A room is $22\frac{1}{2}$ ft. long and 18 ft. wide. What will it cost, at 5¢ per yard, for a strip of moulding around the walls?

15. How many square yards of carpet would be needed for the floor of the above room?

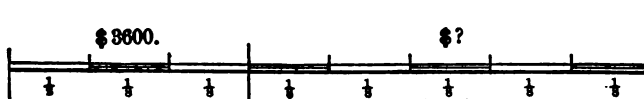
16. How much is the fraction $\frac{3}{4}$ increased or diminished when 2 is added to each of its terms (numerator and denominator)?

17. After James has spent $\frac{2}{3}$ of his money and $\frac{1}{2}$ of the remainder he has but \$1.50 left. How much had he at first?

18. A man buys oranges at \$1.20 per 100. How many would he have to sell, at 25¢ per dozen to gain \$3.18?

19. From a piece of cloth measuring $28\frac{1}{2}$ yards, there have been sold $2\frac{2}{3}$ yd., $6\frac{5}{8}$ yd., $13\frac{3}{4}$ yd. If the remainder is worth \$13.10, what is the value of the whole piece?

20. A man left for charitable purposes \$3,600, which was $\frac{3}{5}$ of his money. The remainder was divided equally among 8 relatives. How much did each relative receive?



REVIEW.

710. Supply missing numbers:

1.	2.	3.
$18,432.65	$26,459.83	$93,259.80
9,876.04	6,087.90	10,059.77
632.95	12,364.58	5,387.04
27.88	3,030.30	.25
5.63	999.99	23.50
.99	6,875.84	681.19
.04	365.93	32,565.88
.87	?	793.20
2.90	6.50	2,684.39
83.15	25.19	?
700.07	308.12	15,909.75
4,862.99	4,321.00	123.40
?	.87	6.15
$50,000.00	$76,543.21	$202,020.20

711. Complete the following table of public school attendance:

	Colored.			White.			Aggregate.
	Male.	Female.	Total.	Male.	Female.	Total.	
Auburn	40	35	75	1,706	1,753	3,459	3,534
Binghamton . .	21	20		2,325	2,397		
Brooklyn . . .	839	797		54,647	54,439		
Cohoes	—	—	—	1,362	1,257		
Elmira	40	61		2,317	2,211		
New York . . .	806	806		98,029	98,304		
Rochester . . .	31	38		8,258	8,597		
Rome	5	5		1,094	992		
Saratoga Springs.	35	34		1,088	1,116		
Yonkers . . .	15	15		1,878	1,716		
	1,832						

SHORT METHODS.

713. Sight Exercises.

1. 68×25	6. 72×25	11. 96×25	16. 88×25
2. 25×49	7. 25×51	12. 25×81	17. 25×97
3. $88 \times 12\frac{1}{2}$	8. $66 \times 33\frac{1}{3}$	13. $48 \times 37\frac{1}{2}$	18. $16 \times 87\frac{1}{2}$
4. 24×75	9. 48×75	14. 92×50	19. $66 \times 66\frac{2}{3}$
5. $82 \times 12\frac{1}{2}$	10. $24 \times 62\frac{1}{2}$	15. $32 \times 33\frac{1}{3}$	20. $16 \times 66\frac{2}{3}$

714. Slate Exercises.

1. $9{,}347 \times 25$	11. $4{,}896 \times 87\frac{1}{2}$
2. 863×75	12. $1{,}284 \times 62\frac{1}{2}$
3. $8{,}123 \times 12\frac{1}{2}$	13. $75 \times 2{,}468$
4. $6{,}483 \times 33\frac{1}{3}$	14. $33\frac{1}{3} \times 3{,}870$
5. $8{,}123 \times 125$	15. $66\frac{2}{3} \times 3{,}456$
6. $9{,}347 \times 250$	16. $16\frac{2}{3} \times 1{,}266$
7. $9{,}347 \times 2\frac{1}{2}$	17. $8{,}408 \times 62\frac{1}{2}$
8. $9{,}347 \times 75$	18. $3{,}875 \times 37\frac{1}{2}$
9. $6{,}483 \times 66\frac{2}{3}$	19. $1{,}925 \times 12\frac{1}{2}$
10. $6{,}488 \times 37\frac{1}{2}$	20. $7{,}314 \times 250$

715. Oral Problems.

1. What will be the cost of 49 lb. of coffee at 25 ¢ per lb.?

2. I paid \$14.75 for eggs at 25 ¢ per doz. How many dozen did I buy?

3. What will be paid for 104 bu. of wheat at $87\frac{1}{2}$ ¢ per bushel?

4. How many bushels of corn at $62\frac{1}{2}$ ¢ per bu. can be bought for \$150?

5. How much will be paid for 99 yd. of dress goods at $33\frac{1}{3}¢$ per yd.?

6. How many yards of carpet at $66\frac{2}{3}¢$ per yard can be bought for $84?

7. Find the cost of 15 doz. collars at $12\frac{1}{2}¢$ each?

8. Paid $24 for cuffs at $16\frac{2}{3}¢$ per pair. How many dozen pairs were bought?

9. What will be the cost of 128 lb. of tea at 75¢ per lb.?

10. A bale of cotton at $6\frac{1}{4}¢$ per lb. cost $25. What was the weight of the cotton?

11. A farmer sold hay at 75¢ per cwt., receiving for it $39. How many cwt. did he sell?

12. How many bbl. of mess pork at $12.50 per bbl. can be bought for $175?

13. What will be the cost of 96 yd. of carpet at $1.25 per yd.?

14. When wheat sells at $1.12\frac{1}{2}$ per bu., how many bushels can be bought for $198?

15. At $3.50 each, what will be paid for 84 coats?

16. Find the cost of 28 hats at $2.75 each?

17. A real estate agent sold 97 lots at $250 each. How much did he receive for them?

18. What will be the cost of 248 horses at $125 each?

19. At $\frac{1}{2}$ cent each, how many pen-holders can I buy for $5.76?

20. Paid $3,675 for cows at $75 each. How many were bought?

716. Slate Exercises.

Multiply 1875 by 21.

3750

39,375 *Ans.*

Do not place the multiplier underneath the multiplicand.

1. $3,456 \times 31$
2. $7,465 \times 81$
3. $2,345 \times 41$
4. $5,432 \times 91$
5. $9,284 \times 51$
6. $1,897 \times 101$
7. $6,736 \times 61$
8. $11,689 \times 111$
9. $4,892 \times 71$
10. $10,754 \times 121$

717. 2468×18.

19744

44,424 *Ans.*

Write the product by 8 one place to the right (above or below).

11. $8,734 \times 13$
12. $4,075 \times 18$
13. $9,485 \times 14$
14. $5,832 \times 19$
15. $576 \times 14 \times 21$
16. $345 \times 15 \times 41$
17. $487 \times 18 \times 81$
18. $207 \times 16 \times 91$

APPROXIMATIONS.

719. Give approximate answers, at sight (Art. 521):

1. $23\frac{7}{8}$ lb. of tea @ $50\frac{1}{8}¢$.
2. 24 horses @ \$124.95.
3. 64 yd. of carpet @ $87\frac{7}{16}¢$.
4. 485 bu. of wheat @ $99\frac{3}{4}¢$.
5. 96 lb. of coffee @ $24\frac{7}{8}¢$.
6. 840 yd. of dress goods @ $33\frac{5}{16}¢$.
7. 360 yd. of oil cloth @ $66\frac{3}{4}¢$.
8. 48 cwt. of straw @ $62\frac{5}{8}¢$.
9. 92 hats @ $\$1.49\frac{3}{4}$.
10. 128 lb. of lard @ $12\frac{3}{8}¢$.

720. Give approximate answers in whole numbers:

11. $\$27 \div 24\frac{15}{16}¢$	21. 17.3×3.98765
12. $\$299.96 \div \$1.49\frac{7}{8}$	22. $256.008 \times .249875$
13. $\$24.05 \div 37\frac{9}{16}¢$	23. 25.1234×15.93
14. $\$15.03 \div 12\frac{5}{8}¢$	24. 6.12×6.12
15. $\$60 \div \$2.49\frac{5}{16}$	25. $86.4 \times .996$
16. $\$32 \div 33\frac{3}{8}¢$	26. 33.333×5.004
17. $\$69.95 \div 87\frac{1}{2}¢$	27. $799.387 \times .125$
18. $\$60 \div 62\frac{7}{16}¢$	28. 7.999×7.99
19. $\$64 \div 66\frac{11}{16}¢$	29. 7.33×11.0083
20. $\$27.95 \div \$1.75$	30. $64.002 \times .3750$

721. Sight Exercises.

Give products:

1. $360 \times .25$	8. $840 \times .075$	15. $400 \times .04$
2. $560 \times .125$	9. $960 \times .005$	16. $165 \times .06\frac{2}{3}$
3. $240 \times .375$	10. $1{,}200 \times .001$	17. $176 \times .06\frac{1}{4}$
4. $400 \times .625$	11. $1{,}500 \times .002$	18. $3{,}300 \times .00\frac{1}{3}$
5. $480 \times .75$	12. $96 \times .3\frac{1}{3}$	19. $880 \times .12\frac{1}{2}$
6. $320 \times .875$	13. $840 \times .02\frac{1}{2}$	20. $105 \times .8$
7. $720 \times .025$	14. $1{,}500 \times .06$	21. $210 \times .10$

722. Give quotients:

1. $240 \div .5$	8. $37 \div .05$	15. $76 \div .04$
2. $360 \div .75$	9. $48 \div .005$	16. $88 \div .00\frac{1}{3}$
3. $45 \div .125$	10. $72 \div .025$	17. $65 \div .12\frac{1}{2}$
4. $23 \div .25$	11. $92 \div .002$	18. $84 \div .8$
5. $360 \div .375$	12. $93 \div .03\frac{1}{3}$	19. $11 \div .06\frac{1}{4}$
6. $100 \div .625$	13. $54 \div .02\frac{1}{2}$	20. $42 \div .6\frac{2}{3}$
7. $154 \div .875$	14. $132 \div .06$	21. $93 \div .5$

723. Give results:

1. $\frac{87 \times 25}{75}$
2. $\frac{74 \times 24}{37}$
3. $\frac{63 \times 19}{21}$
4. $\frac{96 \times 27}{32}$
5. $\frac{23}{8} \times 32$
6. $\frac{47}{15} \times 75$
7. $\frac{65}{14} \times 42$
8. $\frac{39}{11} \times 55$
9. $64 \times 1\frac{5}{8}$
10. $72 \times 1\frac{1}{8}$
11. $38 \times 1\frac{1}{2}$
12. $95 \times 1\frac{1}{5}$
13. $18 \times 2\frac{2}{3}$
14. $16 \times 3\frac{1}{2}$
15. $14 \times 5\frac{1}{7}$
16. $12 \times 9\frac{3}{4}$
17. $22 \div 2\frac{2}{3}$
18. $33 \div 1\frac{3}{5}$
19. $60 \div 2\frac{1}{7}$
20. $40 \div 6\frac{2}{3}$

DENOMINATE NUMBERS.

724. Oral Problems.

1. What will be the weight of 16 hams that average 10 lb. 5 oz. each?

2. From a chest of tea containing 54 lb. there were sold 27 lb. 7 oz. How many lb. remain?

3. Seven bushels of potatoes are divided among 8 persons. How many pecks and quarts does each receive?

4. How many square inches in the surface of a sheet of paper measuring 11 inches by 13 inches?

5. How many feet and inches in $\frac{4}{9}$ yd.?

6. What decimal of a pound is 14 oz.?

7. A man buys a bushel of hickory nuts. After he sells 2 pk. 4 qt., what fraction of the bushel has he left?

8. A dealer puts 30 gal. of milk in cans holding 1 qt. 1 pt. each. How many cans does he fill?

9. At $20 per month, how much rent will a man pay in 1 year and 5 months?

10. 75 hundredths of a pound is how many ounces?

11. How many feet in 5 rods?

12. 1 gal. 3 qt. 1 pt. of milk is divided among 5 people. How many quarts and pints does each receive?

13. What fraction of 2 lb. 3 oz. is 1 lb. 4 oz.?

14. Three-eighths of a ton is how many pounds?

15. Change 9 hours 36 minutes to the fraction of a day.

725. Slate Problems.

1. 32 hams weigh 458 pounds. What is the average weight?

$$
\begin{array}{r}
\text{14 lb. 5 oz.} \\
32\overline{)458 \text{ lb.}} \\
\underline{138} \\
\text{10 lb. remainder.} \\
\underline{16} \\
\underline{160} \text{ oz., new dividend.} \\
0
\end{array}
$$

Ans. 14 lb. 5 oz., average.

2. 595 gal. of oil are put into 14 barrels. How many gal. and qt. does each contain?

3. If there are 42 gal. and 2 qt. in a barrel of oil, how much oil will there be in 15 barrels?

4. In the written number 54,372, the value expressed by the 5 is how many times the value expressed by the 2?

5. A piece of cloth containing 57 yd. is divided equally among six persons. What is the length of each one's share?

6. How many minutes in 1 day, 1 hour, and 1 minute?

7. July 1 is the last school day. How many days' vacation will there be, if school begins Sept. 6?

8. How many hours and minutes are there from half-past 3 Saturday afternoon to a quarter before 9 Monday morning?

9. How many steps, 2 ft. 6 in. long, must a man take in walking 1,200 yards?

10. A man owns a plot of ground 420 ft. long, 240 ft. wide. How many rods of fence will be required to enclose it?

11. A train goes from Jersey City to Washington, 228 miles, in 4 hours 12 minutes. How many miles an hour does it travel? How long does it take the train to go one mile?

12. On Monday a boarding-house uses 3 gal. 2 qt. of milk; on Tuesday, 4 gal.; on Wednesday, 3 gal. 3 qt. 1 pt.; on Thursday, 4 gal. 2 qt.; on Friday, 6 gal.; on Saturday, 5 gal. 2 qt. 1 pt.; on Sunday, 3 gal. 1 pt. How much is used during the week, and what is the average per day?

13. June 21 the sun rises at New York at 4.23 A.M. and sets at 7.40 P.M. How long is the night?

14. From $3\frac{7}{8}$ bu. take $3\frac{7}{8}$ pk.

15. What is the length in rods of a fence surrounding a field 206 ft. 3 in. wide and twice as long?

REVIEW FRACTIONS.

726. Slate Exercises.

Add:

1. $29\frac{3}{5} + 17 + 62\frac{7}{10}$
2. $1\frac{3}{4} + 3\frac{5}{8} + 13\frac{7}{12} + 6\frac{1}{8}$
3. $43\frac{3}{8} + 30 + 63\frac{3}{4} + 8\frac{1}{16} + \frac{7}{8}$
4. $11\frac{4}{15} + 25\frac{3}{10} + 42\frac{5}{6} + 8\frac{3}{8} + 9\frac{1}{4}$
5. $83\frac{5}{32} + 9\frac{11}{16} + 70\frac{3}{80} + 6\frac{7}{24} + 37$
6. $2\frac{4}{5} + 80\frac{1}{8} + \frac{7}{8} + 62\frac{1}{2} + 93\frac{11}{12}$
7. $1\frac{1}{2} + 21\frac{1}{8} + 32\frac{1}{4} + 43\frac{1}{5} + 54\frac{1}{6}$
8. $92\frac{7}{10} + 68\frac{5}{12} + 3\frac{3}{4} + 7\frac{2}{5} + 39$
9. $755\frac{19}{36} + 25\frac{1}{4} + 30\frac{3}{8} + 5\frac{2}{9} + 13\frac{1}{12}$
10. $60\frac{4}{25} + 49\frac{3}{50} + 18\frac{7}{20} + 6\frac{3}{4} + 90\frac{1}{2}$

727. Subtract:

11. $43\frac{1}{2} - 25\frac{3}{5}$
12. $94\frac{3}{5} - 18\frac{4}{7}$
13. $69\frac{7}{10} - 33\frac{13}{25}$
14. $57\frac{1}{15} - 26\frac{17}{60}$
15. $100\frac{3}{24} - 50\frac{11}{36}$
16. $126\frac{6}{35} - 83\frac{2}{7}$
17. $99\frac{3}{16} - 61\frac{7}{20}$
18. $84\frac{7}{36} - 15\frac{7}{27}$
19. $23\frac{1}{20} - 6\frac{1}{12}$
20. $912\frac{19}{30} - 684\frac{23}{45}$

728. Multiply:

21. $43\frac{1}{2} \times \frac{2}{5}$
22. $\frac{32}{45} \times 4\frac{11}{16}$
23. $16\frac{3}{4} \times 12\frac{4}{7}$
24. $4\frac{3}{10} \times 12\frac{2}{9}$
25. $2\frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2}$
26. $\frac{3}{4} \times 1\frac{2}{5} \times 3\frac{4}{21}$
27. $23\frac{1}{4} \times 10\frac{2}{3}$
28. $8\frac{5}{6} \times 9\frac{3}{5}$
29. $\frac{2}{10} \times 1\frac{1}{3} \times 3\frac{4}{7}$
30. $16\frac{2}{3} \times \frac{3}{50} \times 83\frac{1}{3}$

729. Divide:

31. $\frac{7}{10} \div \frac{3}{5}$
32. $\frac{9}{16} \div \frac{3}{20}$
33. $138\frac{7}{15} \div \frac{1}{10}$
34. $17\frac{16}{21} \div 1\frac{11}{12}$
35. $18\frac{3}{4} \div 2\frac{1}{3}$
36. $7\frac{3}{8} \div 3\frac{3}{10}$
37. $37\frac{3}{5} \div 15$
38. $128\frac{4}{17} \div 25$
39. $5 \div 9\frac{1}{6}$
40. $7 \div 12\frac{3}{5}$

730. Perform indicated operations:

41. $(1\frac{1}{3} \div \frac{2}{7}) + (6\frac{3}{5} \div \frac{1}{4})$
42. $\dfrac{(\frac{3}{4} \times 20) - (4\frac{1}{3} \times 2\frac{1}{2})}{\frac{1}{3} + \frac{2}{5}}$
43. $\dfrac{16}{\frac{2}{3} \text{ of } 4\frac{1}{2}} + \dfrac{5\frac{1}{2}}{\frac{1}{3} \text{ of } 1\frac{1}{7}}$
44. $23\frac{3}{4} \div (3\frac{1}{3} + 1\frac{2}{5})$
45. $52\frac{1}{2} \times (1\frac{7}{8} - \frac{15}{16})$
46. $\dfrac{4\frac{1}{2} + 3\frac{1}{3} - 6\frac{7}{8}}{5}$
47. $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{15} \div \dfrac{\frac{5}{8} \text{ of } 2\frac{2}{5}}{8}$
48. $\frac{2}{7}$ of $(3\frac{1}{3} - 2\frac{1}{6} + 9\frac{1}{2})$

DENOMINATE NUMBERS.

732. Slate Exercises.

Change:

1. 48 pounds and 9 ounces to ounces.
2. 34 rods and 3 yards to yards.
3. 2 miles to yards.
4. 3 days and 17 hours to hours.
5. 24 minutes and 15 seconds to seconds.
6. 8 tons and 1675 pounds to pounds.
7. 43 gallons and 8 quarts to quarts.
8. 75 gallons to pints.
9. 19 bushels and 3 pecks to pecks.
10. 19 bushels and 3 pecks to quarts.
11. $\frac{1}{32}$ ton to pounds and ounces.
12. .03125 ton to pounds and ounces.
13. $\frac{7}{9}$ yard to feet and inches.

735. Slate Exercises.

Change:

1. 975 ounces to pounds and ounces.
2. 396 inches to yards.
3. 517 hours to days and hours.
4. 1,694 seconds to minutes and seconds.
5. 9,314 pounds to tons and pounds.
6. 987 pints to gallons, quarts, and pints.
7. 1,485 quarts to pecks and quarts.
8. 185 pecks to bushels and pecks.
9. 840 hours to weeks.

12. If a certain quantity of flour will last 48 persons 57 days, how long will it last 38 persons?

13. Divide $\frac{2}{3}$ of 56 by $1\frac{1}{6}$ times 32.

DENOMINATE NUMBERS.

742. Slate Exercises.

Multiply:

1. 12 lb. 7 oz. × 3
2. 3 hr. 10 min. × 7
3. 4 T. 985 lb. × 11
4. 7 bu. 3 pk. × 9
5. 3 wk. 4 da. × 4
6. 4 yd. 1 ft. × 5
7. 7 min. 18 sec. × 10
8. 9 gal. 3 qt. × 2
9. 2 ft. 9 in. × 8
10. 1 yd. 1 ft. 6 in. × 6

743. Divide:

11. 9 lb. 2 oz. ÷ 2
12. 31 gal. 2 qt. ÷ 9
13. 19 hr. 21 min. ÷ 3
14. 26 bu. 1 pk. ÷ 5
15. 41 min. 44 sec. ÷ 8
16. 18 yd. 2 ft. ÷ 7
17. 19 ft. 2 in. ÷ 10
18. 34 T. 936 lb. ÷ 4
19. 17 wk. 1 da. ÷ 6
20. 52 yd. 0 ft. 9 in. ÷ 11

744. Divide:

21. 18 lb. 4 oz. by 4 lb. 9 oz.
22. 16 yd. by 2 yd. 2 ft.
23. 2 da. 3 hr. 36 min. by 6 hr. 27 min.
24. 47 min. 42 sec. by 5 min. 18 sec.
25. 84 yr. 7 mo. by 12 yr. 1 mo.
26. 19 da. 3 hr. by 2 da. 3 hr.
27. 3 mi. 40 rd. by 125 rd.

28. 103 T. 808 lb. by 8 T. 1,234 lb.

29. 52 gal. 2 qt. by 3 gal. 2 qt.

30. 68 bu. 1 pk. by 5 bu. 1 pk.

31. 30 ft. 8 in. by 1 ft. 11 in.

32. 52 yd. 9 in. by 4 yd. 2 ft. 3 in.

33. 51 wk. 3 da. by 2 wk. 6 da.

MEASUREMENTS.

745. Slate Problems.

Make a diagram in each case:

1. A lot 25 ft. by 100 ft. has on it a house 25 ft. by 55 ft. How many square feet are there left for a yard?

2. How many square feet are there in the floor of a room 24 ft. long, 18 ft. wide?

3. How many square yards are there in the ceiling of the same room?

4. Find the number of square yards of plastering needed for the end wall of a room 18 ft. wide, 9 ft. high, after deducting for two windows each 6 ft. high, $4\frac{1}{2}$ ft. wide.

5. How many square yards of plastering will be needed for the opposite wall of the same room, 18 ft. wide, 9 ft. high, after deducting for a door $7\frac{1}{2}$ ft. high, 6 ft. wide?

6. Calculate the number of square yards of plastering needed for two side walls of a room 24 feet long, 9 feet high, after deducting for a fireplace 6 feet square on one side.

7. A house 30 ft. by 60 ft., with an addition 15 ft. square, is built upon a lot 100 ft. square. How many square feet of ground are covered by the building? How many square feet remain for a garden?

8. Measure the top of a brick and calculate the number of square inches in its surface. How many square inches in the surface of the bottom of the brick? Measure one side, and calculate its surface. How many square inches are there in the surface of the opposite side? How many square inches in each end?

9. Measure a crayon box, and calculate the number of square inches in each face.

10. Calculate the number of square feet in the floor of the class-room. In the ceiling. In each side wall. In each end wall.

PERCENTAGE.

746. *Per cent* means hundredths. Six per cent means six hundredths, $\frac{6}{100}$, or .06. It is written 6%.

747. Oral Exercises.

1. What is $\frac{6}{100}$ of 200?
2. Find .06 of 300
3. 6 per cent of 400
4. 6% of 50
5. 6% of 150
6. 6% of 250
7. 6% of 125
8. 6% of 75
9. 6% of 60
10. 6% of 160
11. 4% of 125
12. 7% of 500
13. 5% of 240
14. 1% of 600
15. $\frac{1}{2}$% of 600
16. $\frac{1}{4}$% of 600
17. $2\frac{1}{2}$% of 600
18. $3\frac{1}{4}$% of 400
19. $\frac{1}{8}$% of 400
20. 9% of 90

748. What fraction equals:

21. 25%
22. $12\frac{1}{2}$%
23. $33\frac{1}{3}$%
24. $16\frac{2}{3}$%
25. 20%
26. 50%
27. $6\frac{1}{4}$%
28. $3\frac{1}{8}$%
29. $6\frac{2}{3}$%
30. $37\frac{1}{2}$%
31. $62\frac{1}{2}$%
32. $87\frac{1}{2}$%

749. Find:

33. 50% of 96
34. 25% of 72
35. $12\frac{1}{2}$% of 120
36. $6\frac{1}{4}$% of 48
37. $33\frac{1}{3}$% of 36
38. $16\frac{2}{3}$% of 126
39. $8\frac{1}{3}$% of 72
40. 100% of 140
41. 200% of 140
42. 300% of 140
43. 150% of 140
44. 250% of 140
45. 125% of 140
46. 1% of 140
47. 1% of 350
48. 2% of 350
49. 3% of 350
50. 4% of 350

750. Slate Problems.

1. A house is valued at $24,500. How much taxes must the owner pay at the rate of $22.40 per $1,000 valuation?

2. A consignee sells a lot of cotton for $1,872.50. He receives 2% of this amount as commission. How much is his commission?

3. I loan $600 at 6% interest per year. How much interest should I receive from Jan. 1, 1892 to Jan. 1, 1894?

4. How much will it cost me to insure goods to the amount of $18,760 at one per cent?

5. A dealer imports books worth $548.40, on which he pays duty to the government at the rate of 25%. What is the amount of the duty?

6. Eighty per cent of a class of 55 pupils are promoted. How many are not promoted?

7. A man buys a house for $16,000 and sells it at a profit of 3 per cent. How much does he gain?

8. A clerk spends for rent 18 per cent of his income of $1,850 per year. What rent does he pay?

9. A girl spelled correctly 95 per cent of 60 words. How many did she miss?

10. Tea costing 40 cents per pound is sold at a profit of 50 per cent. What is the selling price?

751. Oral Problems.

1. What per cent does a boy receive if he solves 16 examples of the 20 given out?

2. What is the interest on $200 at 4% for 2 years?

3. If $2\frac{1}{4}$ yd. of calico cost 22 cents, how many yards can be bought for 60¢?

4. What part of a ton is 125 pounds?

5. How old, Dec. 1, 1892, was a boy born Sept. 1, 1879?

6. What is the cost of 3,500 bricks at $20 per M?

7. How many sheep, at $5 each, should be given in exchange for 12 horses, worth $200 each?

8. Reduce $\frac{1}{80}$ to a decimal.

9. One hundred fifty marbles are divided among a certain number of boys. Each receives 12 and there are 6 remaining. How many boys are there?

10. At 3 oranges for 5 cents, what will be the cost of 4 dozen oranges?

11. 75 men can do a certain piece of work in 9 days. How long will it take 45 men to do the same work?

12. If it takes 24 yards of carpet, a yard wide, to cover a floor, how many yards $\frac{3}{4}$ yd. wide will be needed for the same floor?

BILLS.

753. PHILADELPHIA, *Sept.* 24, 1894.

MR. HARRISON JARVIS

To WM. HART & SON, DR.

	To 50 lb. of Pipe	@	5½¢			
	" 3 Faucets	"	75"			
	" 1 Sink			4	75	
	" 3¼ days' Labor	"	$4.75			
						$

1. Copy and complete the above bill.

2. R. W. Jones has done 3½ days' work, @ $3.50 per day, for Charles Johnston. He charges for 850 ft. lumber at $2 per hundred; 5 lb. of nails at 9¢ per lb.; 3 locks @ 50¢; 2 bolts at 10¢. Make out his bill.

3. A gardener furnishes 3 rose-bushes at 75¢; 4 grape-vines at 50¢; 11 fuchsias at 30¢; 25 pansies at 10¢. He charges $3.25 per day for 2¾ days' labor. Make out his bill.

4. An upholsterer charges $3 per day for repairing some furniture. He supplies 6 lb. of hair at 50¢ per lb.; 17 yd. of plush at $1.75 per yd.; 3 papers of tacks at 10¢; cord, gimp, etc., 47¢. He works 4 days. Make out his bill.

5. Make out and receipt a bill for four articles bought to-day by John R. Brown from Smith and Robinson, grocers.

6. Make out a bill containing ten items bought by Mrs. S. W. Robb, at different times during October, 1894, from Frederick Loeser & Co., dealers in dry goods.

7. Make out a bill for labor done and materials furnished by Anthony Jones, gardener.

INTEREST.

754. Oral Exercises.

1. What will be the interest on $100 for 1 year at 4%?
2. On $200 for a year at 5%?
3. On $300 for a year at 6%?
4. On $400 for a year at 7%?
5. On $250 for a year at 4%?

At 4% per year, what will be the interest:

6. On $200 for 1 year?
7. On $300 for 2 years?
8. On $100 for 3 years?
9. On $200 for $1\frac{1}{2}$ years?
10. On $200 for 1 year 6 months?
11. What will be the interest on $200 for 3 years at 5%?
12. On $300 for 2 years at 6%?
13. On $400 for 6 years at 3%?
14. On $100 for 5 years at 7%?
15. On $250 for 2 years at 4%?
16. On $100 for 1 year 6 months at 6%?
17. On $200 for 3 months at 4%?

At 4% per year, what will be the interest:

18. On $200 for 6 months?
19. On $300 for 4 months?
20. On $400 for 3 months?
21. On $300 for 2 months?
22. On $150 for 1 month?
23. Find the interest on $24 for 1 year at 5%.
24. On $36 for 1 year at 4%.
25. On $67 for 1 year at 3%.

755. Slate Exercises.

Find the yearly interest on:

1. $286.50 at 4%
2. $485 at 6%
3. $375.40 at 5%
4. $379 at 3%
5. $486 at 4½%
6. $186.75 at 4%
7. $199.50 at 2%
8. $636 at 3½%
9. $84.70 at 6%
10. $93.25 at 8%
11. $1,257 at 7%
12. $168 at 3¾%
13. $244 at 5½%
14. $890 at 7$\frac{8}{10}$%
15. $63.75 at 4%
16. $937.50 at 6%
17. $980.40 at 5%
18. $159.60 at 2½%
19. $1,357.37 at 7%
20. $2,146.18 at 4½%

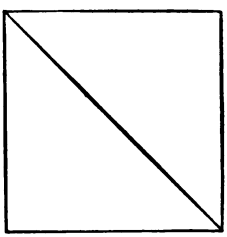
Find the interest on:

21. $290 for 2 years at 4%
22. $1,400 for 3 years at 4½%
23. $2,840 for 4 years at 5%
24. $1,250 at 6% for 3 years
25. $5,360 at 5½% for 2 years
26. $380 at 3% for 4½ years
27. $780 for 1 year 4 months at 6%
28. $2,560 for 2 years 6 months at 5%
29. $1,025 for 3 years 3 months at 4%
30. $1,296 for 7 months at 7%
31. $648 for 5 months at 5%
32. $275 for 4 months at 3%
33. $1,000 for 11 months at 6%

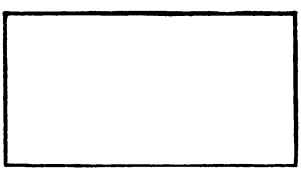
AREAS OF RIGHT-ANGLED TRIANGLES.

756. The square shown in the diagram is divided into two parts by a diagonal. One side of the square measures 10 feet.

1. Mark in each triangle its area.



Square.

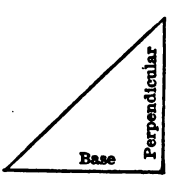


Rectangle.

2. Divide a rectangle 20 ft. by 12 ft. into two parts by a diagonal. Mark in each triangle its area.

3. Draw a right-angled triangle 3 in. by 4 in. Calculate its area in square inches.

4. How many square yards in the surface of a right-angled triangle whose base measures 30 feet, and whose perpendicular measures 22½ feet.



Find the area in *square feet* of the following right-angled triangles. (Change each dimension to feet.)

5. Base 20 yd., perpendicular 30 ft.

6. Base 16 in., perpendicular 3 ft.

7. Base 30 in., perpendicular 1 yd.

8. Base 3 ft. 6 in., perpendicular 5 ft.

9. Base 2 yd. 1 ft., perpendicular 1 yd. 9 in.

10. Base 50 yd., perpendicular 36 yd.

11. Base $112\frac{1}{2}$ ft., perpendicular 30 yd.
12. Base 90 in., perpendicular 2 ft.
13. Base $12\frac{1}{2}$ yd., perpendicular $13\frac{1}{8}$ yd.
14. Base 1 rod, perpendicular $7\frac{1}{2}$ ft.
15. Base $33\frac{1}{8}$ ft., perpendicular 18 ft. 6 in.

SHORT METHODS.

758. Multiply. Do not reduce to improper fractions.

$$\begin{array}{r} 12\frac{3}{4} \\ \times\ 4\frac{1}{8} \\ \hline 51 \\ \quad \frac{4}{4} \\ \hline 55\frac{1}{4} \end{array}$$

4 times $12\frac{3}{4} = 51$; $\frac{1}{8}$ of $12\frac{3}{4} = 4\frac{1}{4}$.

1. $18\frac{3}{8} \times 6\frac{1}{2}$
2. $25\frac{5}{8} \times 8\frac{1}{5}$
3. $16\frac{4}{5} \times 5\frac{1}{4}$
4. $36\frac{1}{3} \times 9\frac{1}{6}$
5. $22\frac{5}{6} \times 6\frac{1}{11}$
6. $16\frac{3}{7} \times 7\frac{1}{5}$
7. $48\frac{3}{4} \times 12\frac{1}{8}$
8. $37\frac{3}{5} \times 10\frac{1}{6}$
9. $26\frac{5}{6} \times 9\frac{1}{8}$
10. $32\frac{3}{8} \times 8\frac{1}{7}$

759. Divide. Short Division.

11. $15)\underline{75\frac{3}{8}}$
12. $13)\underline{169\frac{8}{10}}$
13. $14)\underline{295\frac{5}{8}}$
14. $12)\underline{976\frac{1}{2}}$
15. $16)\underline{195\frac{3}{4}}$
16. $21)\underline{231\frac{1}{2}}$
17. $26)\underline{263\frac{1}{4}}$
18. $24)\underline{504\frac{1}{8}}$
19. $17)\underline{199\frac{3}{5}}$
20. $18)\underline{221\frac{1}{7}}$
21. $12)\underline{865\frac{2}{3}}$
22. $13)\underline{392\frac{1}{2}}$
23. $14)\underline{567\frac{3}{4}}$
24. $23)\underline{702\frac{7}{8}}$
25. $25)\underline{805\frac{2}{3}}$
26. $22)\underline{465\frac{3}{5}}$

REVIEW.

763. Answers in fractions:

1. Simplify $\dfrac{2+5\frac{1}{7}-3\frac{2}{5}}{3+1\frac{1}{7}-2\frac{3}{5}}$.

2. Add $\frac{4}{5}$, $\frac{5}{16}$, $\frac{7}{20}$, $\frac{8}{25}$, $\frac{19}{400}$.

3. Reduce $\dfrac{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}}{1+\frac{1}{2}-\frac{1}{3}-\frac{1}{4}}$ to a simple fraction.

4. Divide $(\frac{1}{2}+\frac{1}{3}+\frac{2}{5})$ by $(\frac{6}{7}\times 1\frac{1}{12}\times \frac{14}{55})$.

5. Simplify $\dfrac{1+\frac{1}{2}}{1-\frac{1}{2}}$ of $\frac{2}{3}$ of $\dfrac{1}{1\frac{1}{2}}$.

6. $\dfrac{7\frac{1}{7}-3\frac{1}{3}}{7\frac{1}{2}}=?$

7. Find the value of $\dfrac{4\frac{1}{5}+(\frac{2}{3}\text{ of }\frac{7}{16})}{(\frac{5}{8}\text{ of }1\frac{1}{8})-\frac{2}{5}}$.

8. $(2\frac{1}{2}+1\frac{1}{8})\div(2\frac{1}{2}+3\frac{1}{8})=?$

9. Find the value of $2\frac{1}{2}$ times the quotient of $(3-2\frac{1}{4})\div(2\frac{1}{2}-\frac{5}{8})$.

10. $3\frac{4}{5}+14-7\frac{2}{3}+5-\frac{2}{15}=?$

764. Answers in decimals:

11. Divide the sum of .736 and 1.2854 by their difference.

12. Divide .1 by .2 and .35 by 35, and find the product of the quotients.

13. Reduce $\frac{7}{5120}$ to a decimal, and divide it by .3125.

14. Divide .12096 by .032.

15. Multiply .00273 by 3,000.456, and divide the product by .08.

16. Divide 12.3125 by .000625.

17. Divide 51.5 by 412, and 412 by 51.5.

18. Multiply 31.5 by 27.9, and divide the product by 9.765.

19. Reduce $\frac{4.25}{3+\frac{1}{8}}$.

20. Find the value of $\frac{.0021 \times 3.004}{.024}$.

765. Reduce to lowest terms:

1. $\frac{69}{253}$
2. $\frac{215}{688}$
3. $\frac{428}{762}$
4. $\frac{851}{888}$
5. $\frac{877}{1189}$
6. $\frac{847}{3003}$
7. $\frac{1728}{6888}$
8. $\frac{8900}{5218}$
9. $\frac{2167}{5782}$
10. $\frac{3828}{6069}$
11. $\frac{3218}{5045}$
12. $\frac{6587}{6785}$

CHAPTER IX.

DENOMINATE NUMBERS. — SURFACES AND VOLUMES. — PERCENTAGE. — INTEREST.

REDUCTION OF DENOMINATE NUMBERS.

766. Reduce 5 gal. 3 qt. 1 pt. to pints.

In 5 gal. there are 20 qt. Adding the 3 qt., we have 23 qt. Multiplying by 2 to reduce to pints, and adding in 1 pt., we get the answer, 47 pints.

5 gal. 3 qt. 1 pt.
4
23 qt.
2
47 pt. *Ans.*

767. Reduction Descending. Slate Exercises.

Reduce to pints:

1. 16 gal. 1 qt. 1 pt.
2. 27 gal. 2 qt.
3. 16 gal.
4. 16 gal. 1 pt.
5. 34 gal. 3 qt. 1 pt.
6. 31½ gal.
7. 9 gal. 2½ qt.
8. 10 gal. 2 qt. 1 pt.
9. 27 gal. 1 pt.
10. 4 gal. 3 qt. 1½ pt.

768. Change 67 pt. to gallons, quarts, and pints.

2 | 67 pt.
4 | 33 qt. 1 pt.
8 gal. 1 qt. 1 pt. *Ans.*

769. Reduction Ascending.

Change to gal., etc.:

11. 156 qt.
12. 79 qt.
13. 408 pt.
14. 1,302 pt.
15. 63 pt.
16. 277 pt.
17. 139 pt.
18. 171 qt.
19. 63 qt.
20. 711 pt.

770. Reduction Descending.

21. Change 17 yd. 1 ft. 9 in. to inches.
22. 4 mi. 100 rd. 4 yd. to yards.
23. 74 bu. 2 pk. 7 qt. to quarts.
24. 156 lb. 11 oz. to ounces.
25. 63 yd. 0 ft. 3 in. to inches.
26. 19 bu. 0 pk. 3 qt. to quarts.
27. 11 rd. $3\frac{1}{2}$ yd. to feet.
28. 63 gal. 3 qt. to pints.
29. 3 bu. 6 qt. to quarts.
30. 17 T. 369 lb. to pounds.
31. 15 hr. 16 min. to seconds.
32. 4 wk. 6 da. 11 hr. to hours.
33. $\frac{5}{7}$ of a week to hours.
34. $\frac{3}{8\frac{1}{2}}$ of a mile to yards.
35. .00125 T. to ounces.

771. Reduction Ascending.

Change:

36. 1,876 in. to yd., etc.
37. 475 oz. to lb., etc.
38. 729 qt. to bu., etc.
39. 8,675 min. to da., etc.
40. 4,972 lb. to T., etc.
41. 972 rd. to mi., etc.
42. 117 pt. to gal., etc.
43. 9,483 sec. to hr., etc.
44. 877 qt. to bu., etc.
45. 1,495 oz. to lb., etc.
46. 373 in. to yd., etc.
47. 216 qt. to gal., etc.
48. 876 rd. to mi., etc.
49. 319 pt. to gal., etc.
50. 3,520 yd. to mi.

772. Oral Exercises.

1. How many hours in $\frac{3}{8}$ of a day?
2. How many hours in $\frac{1}{6}$ of a day?
3. How many minutes in $\frac{4}{5}$ of an hour?
4. How many hours and minutes in $\frac{1}{6}$ of a day?
5. How many quarts and pints in $\frac{3}{8}$ of a gallon?
6. How many hours and minutes in .2 day?
7. How many quarts and pints in .375 gallon?
8. Change .3 day to hours and minutes.
9. Change .625 bu. to pk. and qt.
10. What part of a gallon is 1 pt.?
11. What part of a gallon is 3 pt.?
12. What part of a gallon is 1 qt. 1 pt.?
13. What decimal of a gallon is 1 qt. 1 pt.?
14. What decimal of a gallon is 2 qt. 1 pt.?
15. What part of 2 gallons is 2 qt. 1 pt.?
16. Change .375 bu. to pt. and qt.
17. What decimal of a bu. is 4 qt.?
18. What fraction of a day is 3 hr. 20 min.?
19. Reduce 960 min. to hours.
20. How many minutes in a day?

773. Slate Exercises.

1. What decimal of a ton is 3 lb.?
2. What fraction of a day is 12 min. 30 sec.?
3. Reduce $\frac{1}{32}$ of a day to minutes.
4. Reduce .03125 day to minutes.

5. What decimal of a day is 9 minutes?

6. What will be the cost of 15 T. and 750 lb. coal at $5 per ton?

7. If coal is $5 per ton, how much can be bought for $18.76?

8. If 7 T. 296 lb. coal cost $35.74, how much will I have to pay for 18,748 lb.?

9. A man pays $48.92 for 9 T. 1,568 lb. coal. How many tons, etc., would he receive for $73.11?

10. Change 2 ft. 7 in. to the fraction of a yd.

11. Reduce 3 pk. 4 qt. 1 pt. to the decimal of a bu.

12. How many pk., qt., etc., in .9375 bu.?

13. If .1875 of a gal. of cologne cost $1.125, what will 1 pt. cost?

14. Find the cost of 42 gal. 3 qt. 1 pt. oil, at 16¢ per gal.

15. Reduce $\frac{11}{16}$ of a gal. to qt. and pt.

16. What part of 3 T. is 1 T. 960 lb.?

17. A man raised 139 bu. 2 pk. and 2 qt. of rye. He sold 119 bu. 2 pk. 4 qt. What fraction of his crop did he sell?

18. 10 bu. 1 pk. of seed is packed in 8 bags. How much is there in each bag?

19. What decimal of a day is 21 hr. 14 min. 24 sec.?

20. How many feet in a mile?

COMPOUND ADDITION.

774. Add the following:

1.
17 lb. 8 oz.
4 lb. 9 oz.
23 lb. 12 oz.
15 oz.

2.
18 bu. 3 pk. 7 qt.
9 bu. 2 pk. 4 qt.
14 bu. 1 pk. 6 qt.
2 pk.

3. 16 yd. 2 ft. 9 in.
17 yd. 4 in.
1 ft. 6 in.
11 in.

4. 11 da. 5 hr. 19 min.
23 da. 40 min.
17 hr. 50 min.
5 da. 20 hr. 6 min.

5. 93 gal. 3 qt. 1 pt.
74 gal.
18 gal. 1 qt.
2 qt. 1 pt.

6. 5 hr. 30 min. 20 sec.
45 min. 33 sec.
6 hr. 11 min. 5 sec.
10 hr. 3 min. 30 sec.

7. 12 T. 1,576 lb.
3 T. 980 lb.
476 lb.
1 T. 1,830 lb.

8. 2 wk. 5 da. 12 hr.
6 da. 15 hr.
5 wk. 2 hr.
2 da. 19 hr.

9. 18 mi. 100 rd.
34 rd.
29 mi.
6 mi. 160 rd.

10. 47 yr. 11 mo.
5 yr. 9 mo.
7 mo.
22 yr. 5 mo.

11. 487 T., 316 T. 1,816 lb., 247 lb., 43 T. 811 lb., 19 T. 25 lb.

12. 83 lb. 15 oz., 9 lb. 5 oz., 18 lb., 22 lb. 11 oz., 5 lb. 8 oz., 12 oz.

13. 8 hr. 15 min. 5 sec., 37 min. 52 sec., 5 hr. 48 min., 23 hr. 59 min. 5 sec.

14. 72 gal. 3 qt. 1 pt., 17 gal. 1 qt., 2 qt. 1 pt., 90 gal. 1 pt.

15. 7 yd. 2 ft. 11 in., 19 yd. 6 in., 105 yd., 4 yd. 2 ft. 2 in., 1 ft.

16. 93 mi. 300 rd., 87 mi. 154 rd., 194 rd., 3 mi. 175 rd., 9 mi.

17. 82 yr. 1 mo., 19 yr. 10 mo., 25 yr. 9 mo. 6 da., 8 mo. 15 da.

18. 4 wk. 6 da. 17 hr., 20 wk. 5 da., 4 da. 11 hr., 9 wk. 5 da. 1 hr.

19. 5 hr. 13 min. 23 sec., 16 hr. 27 min. 30 sec., 43 min. 5 sec., 24 sec.

20. 8 bu. 3 pk. 7 qt., 5 qt., 2 pk. 1 qt., 4 bu. 6 qt., 3 bu. 1 pk. 1 qt.

775. Find answers:

21. 19 lb. 12 oz.
+ ?
37 lb. 3 oz.

22. 14 bu. 2 pk. 4 qt.
+ ?
18 bu. 1 pk. 1 qt.

23. 7 yd. 2 ft. 7 in.
+
15 yd. 2 ft. 2 in.

24. 19 da. 14 h. 40 min.
+
30 da.

25. 14 gal. 2 qt. 1 pt.
+
18 gal. 1 qt.

26. 10 hr. 15 min. 30 sec.
+
24 hr.

27. 246 T. 1,876 lb.
+
493 T. 1,552 lb.

28. 9 wk. 6 da. 11 hr.
+
21 wk. 3 hr.

29. 84 mi. 24 rd.
+
100 mi. 15 rd.

30. 13 yr. 9 mo.
+
25 yr.

31. 83 lb. 4 oz. + ? = 100 lb.

32. 16 bu. 2 qt. + ? = 25 bu. 1 pt.

33. 1 ft. 4 in. + ? = 9 yd. 1 in.

34. 47 da. 15 min. + ? = 60 da.

35. 93 gal. 3 qt. 1 pt. + ? = 150 gal.

COMPOUND SUBTRACTION.

776. Subtract:

36. 83 yr. 3 mo.
15 yr. 9 mo.

37. 62 mi. 84 rd.
19 mi. 159 rd.

38. 76 T. 225 lb.
37 T. 1,679 lb.

39. 100 lb.
83 lb. 4 oz.

40. 52 wk.
13 wk. 3 da. 7 hr.

41. 19 gal. 1 pt.
8 gal. 3 qt.

42. 18 hr. 5 min.
40 min. 25 sec.

43. 16 yd. 9 in.
7 yd. 1 ft. 11 in.

44. 100 bu.
42 bu. 3 pk. 7 qt.

45. 45 da. 1 hr. 1 min.
6 da. 6 hr. 6 min.

46. From 27 bu. 1 pk. 5 qt. take 13 bu. 3 pk. 7 qt.

47. From 100 gal. 1 qt. take 83 gal. 2 qt. 1 pt.

48. From 22 hr. 15 min. 20 sec. take 15 hr. 45 min. 40 sec.

49. From 17 lb. 2 oz. take 13 lb. 8 oz.

50. From 100 bu. take 74 bu. 2 pk. 1 qt.

COMPOUND MULTIPLICATION.

777. Add 3 lb. 9 oz.
3 lb. 9 oz.

Add 4 gal. 3 qt. 1 pt.
4 gal. 3 qt. 1 pt.
4 gal. 3 qt. 1 pt.

Multiply 3 lb. 9 oz. by 2.

Multiply 4 gal. 3 qt. 1 pt. by 3.

778. Multiply:

51. 13 bu. 3 pk. 6 qt. by 2.

52. 25 gal. 2 qt. 1 pt. by 3.

53. 7 lb. 10 oz. by 4.

54. 3 hr. 15 min. 15 sec. by 5.

55. 23 bu. 3 qt. by 6.

56. 32 gal. 1 pt. by 7.

57. 25 lb. 4 oz. by 8.

58. 33 min. 33 sec. by 9.

59. 2 pk. 7 qt. by 10.

60. 3 qt. 1 pt. by 11.

61. 4 yr. 6 mo. by 12.

62. 5 wk. 6 da. 12 hr. by 16.

63. 4 T. 250 lb. by 18.

64. 3 yd. 1 ft. 6 in. by 22.

65. 2 mi. 15 rd. by 32.

66. 4 hr. 15 min. 20 sec. by 9.

67. 31 gal. 2 qt. by 42.

68. 4 qt. by 37.

69. 43 sec. by 215.

70. 4 wk. 6 da. 20 hr. by 19.

71. 1 bu. 2 pk. 3 qt. by 13.

72. 4 yd. 2 ft. 9 in. by 15.

73. 21 hr. by 24.

74. 3 yr. 11 mo. by 14.

75. 1 gal. 1 qt. 1 pt. by 30.

COMPOUND DIVISION.

779. Divide:

76. 15 lb. 9 oz. by 3.

77. 2 lb. 3 oz. by 5.

78. 2 gal. 1 qt. by 3.

79. 5 bu. by 4.

80. 7 hr. by 6.

81. 17 lb. 7 oz. by 3.

82. 37 bu. 3 pk. 6 qt. by 2.

83. 67 yd. 2 ft. by 4.

84. 33 da. 15 hr. 57 min. by 3.

85. 563 gal. by 6.

86. 22 hr. 20 min. 20 sec. by 4.

87. 112 T. 125 lb. by 5.

88. 13 wk. by 5.

89. 74 mi. 80 rd. by 4.

90. 69 yr. by 12.

91. 27 bu. 3 pk. 4 qt. by 2.

92. 76 gal. 3 qt. 1 pt. by 3.

93. 5 hr. 1 min. 57 sec. by 9.

94. 33 wk. 3 da. by 12.

95. 36 yd. 6 in. by 7.

96. 45 bu. 6 qt. by 6.

97. 20 da. 13 hr. 4 min. by 16.

98. 15 gal. 3 qt. by 18.

99. 54 yd. 1 ft. 4 in. by 20.

100. 41 wk. 4 da. 1 hr. ÷ 4.

101. 457 hr. 37 min. 30 sec. ÷ 9.

102. 147 gal. 3 qt. 1 pt. ÷ 13.

103. 157 bu. 3 pk. 6 qt. ÷ 7.

104. 175 yd. 2 ft. 6 in. ÷ 10.

105. 188 mi. 12 rd. 2 yd. ÷ 6.

106. 311 da. 21 hr. 36 min. ÷ 12.

107. Divide 180 da. 3 hr. 4 min. by 16.

780. Dividing 180 days by 16, we get 11 days quotient and 4 days remainder. Reducing 4 days to hours and adding 3 hours, the next dividend is 99 hours. This gives 6 hours quotient, 3 hours remainder. Reducing to minutes and adding 4 minutes, the next dividend is 184 minutes. This gives 11 minutes quotient, and 8 minutes remainder. Reducing, we have 480 seconds for next dividend. Dividing, as before, the last quotient is 30 seconds.

11 da. 6 hr. 11 min. 30 sec.
16)180 da. 3 hr. 4 min.
20
4 da. (remainder)
24
99 hr.
3 hr. (remainder)
60
184 min.
24
8 min. (remainder)
60
480 sec.
0

108. Divide 236 gal. 1 qt. by 18.

109. 334 yd. 9 in. by 21.

781.

15 yd. 2 ft. 9 in.
21)334 yd. 0 ft. 9 in.
124
19 yd. (remainder)
3
57 ft.
15 ft. (remainder)
12
189 in.
0

110. 825 lb. by 48.

111. 112 T. by 25.

112. 43 mi. by 32.

113. 84 yr. by 24.

114. 462 bu. by 36.

115. 1,078 yd. by 63.

116. 288 hr. 9 min. by 54.

117. 863 gal. 2 qt. 1 pt. by 47.

118. 33 wk. 1 da. by 72.

119. 1,138 T. 910 lb. by 81.

120. 1,629 yd. 1 ft. by 96.

121. 1,867 gal. 1½ pt. by 125.

782. Oral Problems.

1. How many tons and pounds of coal in 40 bags, each containing 80 pounds?

2. If it takes 3 hr. 20 min. to hoe a row of corn, how long will it take to hoe 3 rows?

3. A man puts up $3\frac{1}{2}$ pounds of tea into 4 oz. packages. How many packages does he make?

4. 3 pecks 3 quarts of apples are divided among 9 children. What quantity does each child receive?

5. What part of a day is 30 minutes?

6. If there are $2\frac{1}{4}$ gallons of wine in 12 bottles, how many pints are there in each bottle?

7. What is the weight of two packages each containing 15 lb. 11 oz.?

8. What part of an hour is 40 seconds?

9. What is the rent of a house for 1 year 9 months at $16 per month?

10. If 3 gal. 2 qt. 1 pt. of milk are taken from a can containing 10 gal., how much is left in the can?

11. 5 hams weigh $61\frac{1}{4}$ lb. What is the average weight?

12. There are on an average 41 pupils in a class. How many are there in 14 classes?

13. At $37\frac{1}{2}$ cents per yard, how many yards can be bought for $6.75?

14. Find the cost of 16 bbl. of flour at $6.12\frac{1}{2}$ each.

15. $1.65 is equally divided among 15 boys. What is the share of each?

16. A floor containing $40\frac{1}{4}$ square yards is 7 yards long. How many yards wide is it?

17. How many ounces in $5\frac{7}{8}$ pounds?

783. Slate Problems.

1. If a watch gains 1 min. 17 sec. per day, how much will it gain during March and April?

2. How many bu., pk., and qt. in 1,449 lb. corn, weighing 56 lb. to the bu.?

3. A chain, 97 yd. 8 in. long, contains 1,000 links. Find the length of one of the links.

4. A farmer sold out of 5 bu. of peas the following quantities: 3 pk. 6 qt.; 4 pk; 4 pk. 3 qt.; 1 bu. 1 pk. 1 qt. How much has he still to sell?

5. A man walks on Monday 15 mi. 161 rd.; Tuesday, 10 mi. 84 rd.; Wednesday, 19 mi. 15 rd.; Thursday and Friday, 12 mi. 121 rd. each day; Saturday, 14 mi. 240 rd. What distance per day does he average?

6. If the sun rises at 5 hr. 10 min. A.M., and sets at 6 hr. 42 min. P.M., how long is the day? How many hours and minutes of night?

7. An iron rod is 12 ft. 6 in. long. From it are cut 73 bolts, each $1\frac{3}{4}$ in. long. How much is left?

8. A man rows a mile in 10 min. 30 sec. How long would he take to row 27 miles at the same rate?

9. A man rows 51 miles in 23 hr. 5 min. and 30 sec. How long does he take to row a mile?

10. If I lost \$50 by selling a lot for two-thirds of its cost, what would I have lost if I had sold it for three-fourths of its cost?

11. At the rate of \$2.75 per day of 10 hours, how much should be given a man that works from a quarter before 8 until 5 minutes past 11?

12. If a railroad train travels 18 miles in 40 minutes, how far will it travel, at the same rate, in $7\frac{1}{2}$ hours?

SPECIAL DRILLS.

784. Give sums:

$163 + 137$	$42 + 35 + 77$	$185 + 546$	$4{,}170 + 470$
$256 + 184$	$63 + 19 + 54$	$668 + 193$	$1{,}260 + 850$
$149 + 312$	$87 + 22 + 48$	$167 + 734$	$2{,}140 + 680$
$458 + 197$	$91 + 63 + 17$	$476 + 155$	$3{,}450 + 390$

785. Give differences:

$400 - 163$	$540 - 384$	$7{,}310 - 6{,}850$	$618 - 495$
$501 - 375$	$361 - 149$	$8{,}610 - 7{,}680$	$455 - 128$
$275 - 137$	$455 - 358$	$5{,}000 - 4{,}670$	$648 - 509$
$650 - 488$	$662 - 176$	$4{,}960 - 4{,}380$	$856 - 147$

786. Give products:

11×15	$48 \times 16\frac{2}{3}$	21×15	28×75
12×14	$32 \times 37\frac{1}{2}$	22×14	$40 \times 87\frac{1}{2}$
13×13	$24 \times 62\frac{1}{2}$	31×13	$39 \times 33\frac{1}{3}$
14×14	$36 \times 66\frac{2}{3}$	41×14	49×25

787. Give quotients:

$165 \div 15$	$616\frac{2}{3} \div 16\frac{2}{3}$	$135 \div 45$	$933\frac{1}{3} \div 66\frac{2}{3}$
$168 \div 14$	$837\frac{1}{2} \div 37\frac{1}{2}$	$136 \div 68$	$975 \div 75$
$169 \div 13$	$733\frac{1}{3} \div 33\frac{1}{3}$	$220 \div 44$	$612\frac{1}{2} \div 87\frac{1}{2}$
$294 \div 14$	$687\frac{1}{2} \div 62\frac{1}{2}$	$196 \div 49$	$925 \div 25$

788. Give answers:

$13\frac{3}{4} \times 5$	$51\frac{5}{6} \div 2$	$24 \div 2\frac{2}{3}$	$21 \times 3\frac{3}{7}$
$14\frac{2}{3} \times 4$	$40\frac{3}{5} \div 7$	$18 \div 4\frac{1}{2}$	$22 \times 4\frac{2}{11}$
$14\frac{3}{10} \times 7$	$34\frac{2}{5} \div 2$	$49 \div 1\frac{3}{4}$	$18 \times 3\frac{5}{6}$
$13\frac{2}{11} \times 8$	$48\frac{2}{3} \div 3$	$35 \div 3\frac{1}{2}$	$17 \times 4\frac{1}{4}$

789. Oral Problems.

1. How many ounces in $11\frac{3}{16}$ lb.?

2. 258 yd. equal how many ft.?

3. A dealer bought 652 tons of coal and sold 476 tons. How much had he left?

4. Sold my wheat for $347 and my oats for $154. How much did I receive for both?

5. $40\frac{3}{8}$ yd. of ribbon are cut into 7 pieces. Find the length of each piece.

6. How many sq. yd. in a floor $5\frac{1}{2}$ yd. long and $5\frac{1}{2}$ yd. wide?

7. What will be the cost of 14 lb. of lard at 14¢ per lb.?

8. At $1\frac{1}{2}$¢ each, how many lead pencils can I buy for 27¢?

9. What part of a 196-lb. barrel of flour is contained in a 49-lb. bag?

10. At 45¢ per yd., how much lace can be bought for $1.35?

11. A woman has saved $833. How much more must she save to have $1,000?

12. What will be the cost of 16 lb. of sugar at $4\frac{3}{8}$¢ per lb.?

13. Spent $2.56 for dry goods and $1.84 for groceries. How much did I spend for both?

14. Find the cost of 3 lb. 10 oz. of butter at 32¢ per lb.

15. At $.375 per yd., how much ribbon can be bought for $.75?

16. If it takes $1\frac{1}{8}$ yd. of cloth to make a jacket, how many can be made from a piece of cloth containing 30 yd.?

17. A boy paid 50¢ for the use of a boat for $3\frac{1}{2}$ hours. What was the price per hour?

18. If 13 pounds of raisins cost $1.69, what is the cost of 1 pound?

TABLE.

FOREIGN COMMERCE OF THE UNITED STATES.

Exports and Imports, 1875–1891.

790. The following table shows the values of the exports and the imports of merchandise during each year from 1875 to 1891, inclusive.

Imports and Exports of Merchandise.

Year ending June 30.	Exports.	Imports.	Excess of exports.	Excess of imports.
1875	$513,442,711	$533,005,436		
1876	540,384,671	460,741,190		
1877	602,475,230	451,323,126		
1878	694,865,766	437,051,532		
1879	710,439,441	445,777,775		
1880	835,638,658	667,954,746		
1881	902,377,346	642,664,628		
1882	750,542,257	724,639,574		
1883	823,839,402	723,180,914		
1884	740,513,609	667,697,693		
1885	742,189,755	577,527,329		
1886	679,524,830	635,436,136		
1887	716,183,211	692,319,768		
1888	695,954,507	723,957,114		
1889	742,401,375	745,131,652		
1890	857,828,684	789,310,409		
1891	884,480,810	844,916,196		
Total.				

Find the excess of exports or of imports for each year.

Find the total exports and the total imports for the 17 years ending June 30, 1891, and the difference between the excess of exports and excess of imports for the same period.

SHORT METHODS.

791.

4,846	7,854
$\times 3\frac{2}{5}$	$27\frac{7}{9}$
14,538 3 times	54,978 7 times
$2{,}907\frac{2}{5}$ $\frac{1}{5}$ of 3 times	$6{,}108\frac{6}{9}$ $\frac{1}{9}$ of 7 times
$17{,}445\frac{2}{5}$ *Ans.*	157,08 20 times
	$218{,}166\frac{6}{9}$ *Ans.*

Multiply by 3, then add $\frac{1}{5}$ of this product.

1. $247 \times 4\frac{4}{7}$
2. $1{,}896 \times 5\frac{5}{8}$
3. $1{,}234 \times 6\frac{6}{11}$
4. $3{,}742 \times 7\frac{7}{10}$
5. $4{,}053 \times 7\frac{7}{9}$
6. $6{,}305 \times 9\frac{9}{11}$
7. $8{,}762 \times 23\frac{3}{5}$
8. $12{,}345 \times 32\frac{2}{3}$
9. $7{,}890 \times 105\frac{5}{7}$
10. $67{,}890 \times 234\frac{4}{5}$

792. Multiply 8,654 by 99.

865,400	Subtract the number from 100 times the number.
856,746 *Ans.*	

11. $7{,}835 \times 99$
12. $9{,}427 \times 99$
13. $6{,}073 \times 99$
14. $5{,}483 \times 99$
15. $2{,}761 \times 999$
16. $8{,}305 \times 999$
17. $999 \times 1{,}234$
18. $999 \times 3{,}859$
19. $9{,}832 \times 990$
20. $7{,}543 \times 990$
21. $2{,}684 \times 25$
22. $9{,}321 \times 33\frac{1}{3}$
23. $8{,}693 \times 12\frac{1}{2}$
24. $4{,}862 \times 66\frac{2}{3}$
25. $3{,}025 \times 37\frac{1}{2}$
26. $3{,}464 \times 62\frac{1}{2}$
27. $4{,}872 \times 87\frac{1}{2}$
28. $860 \times 2\frac{1}{2}$
29. $6{,}318 \times 16\frac{2}{3}$
30. $9{,}204 \times 75$

793. Divide. Do not write products (Arts. 385, 616):

31. 41,874,365 ÷ 9,999 **33.** 300,200,100 ÷ 31,416

32. 123,456,789 ÷ 1,987 **34.** 254,637,809 ÷ 26,543

35. 837,029,456 ÷ 16,074.

794. Write answers: (Art. 385.)

36. $\frac{23146}{7809}$ **38.** $\frac{11223}{1984}$ **40.** $\frac{98643}{28643}$ **42.** $\frac{63336}{16895}$

37. $\frac{68400}{5709}$ **39.** $\frac{72063}{5999}$ **41.** $\frac{87631}{17025}$ **43.** $\frac{72084}{10203}$

795. Avoirdupois Weight. Long Ton.

In selling iron, coal at the mines, ores, etc., and in calculating the duties at the U. S. custom houses upon imported goods, the following table is used:

28 pounds (lb.)	1 quarter (qr.)
4 quarters	1 hundredweight (cwt.)
20 hundredweight	1 ton (T.)
1 cwt. = 112 lb.	1 T. = 2,240 lb.

796. The ton of 2,240 pounds is called a *long* ton. Unless otherwise specified in a problem, the cwt. of 100 lb. and the ton of 2,000 lb. are to be taken.

1. Reduce 25 T. 13 cwt. 2 qr. 25 lb. tc pounds. (Long ton.)

2. Change 100,000 pounds to tons (long), cwt., qr., lb.

3. Find the duty at $1\frac{1}{10}¢$ per lb. on an invoice of tin weighing 33 T. 7 cwt. 20 lb. (Long ton.)

4. What is the total weight in tons (long), etc., of 19 barrels of soda-ash weighing 13 cwt. 2 qr. 10 lb. each?

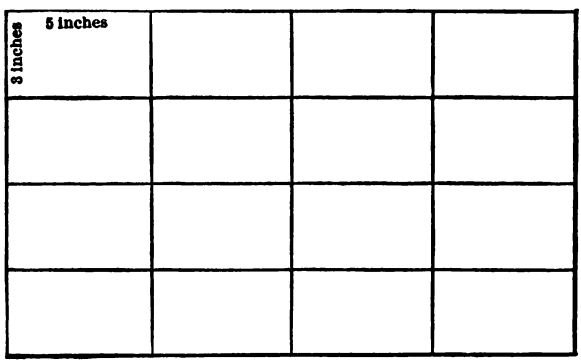
5. Find the weight of the rails required for 100 miles of double track (four rails), the weight of a rail being 18 pounds per running foot.

What will be the cost of the rails at $21 per long ton?

6. A coal dealer buys 175 (long) tons of coal. How much does he receive for it at $5 per ton of 2000 pounds?

MEASUREMENTS.

800. What is the length in inches of a row of four envelopes, each five inches long, placed end to end? What is the length in feet and inches?



1. What is the width in inches of four such rows just touching each other? What is the width in feet?

How many envelopes are there? How many sq. in. in each envelope? How many sq. in. are covered by all of them?

2. How many envelopes 5 inches by 3 inches would cover the top of a table 4 feet 2 inches long and 2 feet 6 inches wide?

3. Draw a rectangle to represent a floor 24 feet long 18 feet wide. Draw rugs 6 feet long, 3 feet wide, and see how many will be needed to cover the floor.

4. How many boards 12 feet long, 6 inches wide will be required for a floor 8 yards long, 6 yards wide?

If the boards run lengthwise, now many boards in length are used? How many boards wide?

5. Given the area in square inches of a surface to be covered with envelopes, and the area in square inches of an envelope, how is the number of envelopes ascertained?

If the area of the surface to be covered is given in square feet, how must we proceed?

6. Given the dimensions of a surface to be covered and the dimensions of the articles to be used for covering, how can we indicate the operations to be performed, without actually doing the work?

How about the denominations used; yards, feet, etc.?

7. How many bricks 8 inches by 4 inches will be needed for a walk 24 yards long, 6 feet wide, making no allowance for waste?

First indicate the work. Cancel.

8. How many paving tiles $\frac{1}{2}$ foot square will cover a hearth 6 feet long, 3 feet wide?

Make a diagram.

9. How many boards 12 feet long, 9 inches wide will be required for a close fence 120 yards long, 6 feet high?

10. Find the number of paving stones 9 inches by 3 inches, in a street 100 rods long, 10 yards wide.

11. A man buys a piece of ground 300 feet long, 150 feet wide. He builds a house, 50 feet by 30 feet, and a shed 12 feet by 13 feet. How many square yards will he have left for a garden?

12. There are 160 square rods in an acre. How many square yards are there in an acre?

13. Give the dimensions, in yards, of a field that will contain just an acre. Of one that will contain two acres.

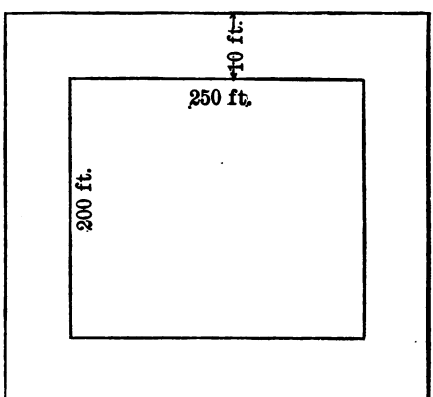
14. Draw a rectangle 2 inches $\times$ 3 inches. Draw one twice the size. What are the dimensions of the latter?

A plot 100 ft. $\times$ 100 ft. is how many times as large as a plot 25 ft. $\times$ 25 ft.?

15. How many square feet are there in a fence 10 feet high around a lot 250 feet long, 200 feet wide?

16. The owner of a piece of ground 250 feet long, 200 feet wide, takes 10 feet from each side to make a gravel walk, and uses the remainder for a garden. Give the dimensions of the garden and its area in square feet? How many square feet are taken up by the walk? How many square feet in the whole piece of ground?

17. How many square feet of flagging would be required for a sidewalk 10 feet wide outside a lot 250 feet long, 200 feet wide?



If 250 running feet of sidewalk, 10 feet wide, were laid on two sides of the lot, and 200 running feet on each of the other sides, would the job be finished?

18. At $80 per acre what is the value of a field 80 rods long, 70 rods wide?

What will it cost to fence the field at 20¢ per running yard?

19. A room is 24 feet long, 18 feet wide, 12 feet high. Draw, touching each other, four rectangles representing the four walls. Write the dimensions of each wall.

What are the dimensions of the large rectangle made up of the four smaller ones? Give the area in square feet. In square yards.

20. Show by a diagram the shape of a piece of paper that when folded will entirely cover a box 12 inches long, 6 inches wide, 4 inches high. Write the dimensions.

801. This is called the "development" of the box.
What is the area of the paper in square inches?

21. Make a diagram of a room 24 feet long, 18 feet wide, 12 feet high, showing the surface that is generally plastered.

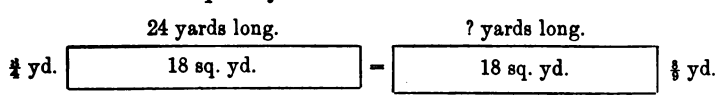
How many square yards of plaster will be needed for the above room, making no allowance for doors, windows, etc.?

22. What is the length of a rectangular field 60 rods long that contains 60 acres?

23. To contain 48 square yards, how many yards long must be a piece of carpet 27 inches wide?

24. I have bought 24 yards of dress goods, 27 inches wide. How many square yards does the piece contain?

How many yards of lining 32 inches wide will contain the same number of square yards?



25. How many square yards are there in 27 rugs, each 63 inches long, 45 inches wide?

TIME BETWEEN DATES.

802. Oral Problems.

1. How many hours from 3 o'clock Saturday afternoon to 9 o'clock Sunday morning?

2. How many days from May 1 to June 1?

3. A boy takes a spoonful of medicine every hour. If he takes the first dose at 2 o'clock, at what hour will he take the sixth? The second? The fourth?

4. A man begins work on the morning of the 6th and ends on the evening of the 11th. How much does he earn at $3 per day?

5. An importer receives some cases of goods numbered consecutively. How many cases are there if the lowest number is 29 and the highest number is 53?

6. How many posts 6 ft. apart will be needed for a fence 120 ft. long? For a fence 6 ft. long? 12 ft. long?

7. Find the time from Jan. 1 to Jan. 31, counting the first and the last day. Omitting both days.

8. How many days from July 4 to August 15, inclusive?

9. How many chapters from the 25th to the 49th, exclusive?

10. A girl begins at the 146th problem and solves all those on two pages. If the last is the 172d problem, how many does she solve?

803. In finding the time between dates, either the first or the last day is excluded; that is, from the 1st to the 21st is considered 20 days.

How many days from March 4 to Sept. 1?

Excluding March 4, there remain in the month 31 − 4, or 27 days. To this add the number of days in April, May, June, July, and August. Since March 4 is excluded, we take 1 day in September, making the total 181 days.

March 4 to March 31,	27	days.
April	30	"
May	31	"
June	30	"
July	31	"
Aug.	31	"
Sept.	1	"
Ans.	181	days.

804. How many days from

11. Jan. 1 to Feb. 19?
12. Oct. 31 to Dec. 30?
13. Sept. 30 to Dec. 16?
14. Nov. 1 to Dec. 19?
15. March 16 to April 25?
16. Feb. 29 to April 1?
17. May 21 to July 4?
18. April 7 to May 27?
19. June 10 to Aug. 1?
20. July 4 to Sept. 1?

805. Slate Problems.

(Take note of leap year.)

How many days from:

1. Feb. 6, 1892, to Oct. 1, 1892?
2. Oct. 14, 1892, to March 3, 1893?
3. Jan. 1, 1892, to April 19, 1892?
4. Dec. 23, 1891, to March 8, 1892?
5. Sept. 3, 1892, to Feb. 1, 1893?
6. March 16, 1892, to Dec. 25, 1892?
7. June 3, 1892, to Nov. 29, 1892?
8. Aug. 17, 1892, to Jan. 3, 1893?
9. April 4, 1892, to July 4, 1892?
10. May 16, 1892, to Oct. 14, 1892?

11. How much wages at $4 per day should a man receive from Monday, Jan. 2, 1893, to Feb. 28, inclusive, no pay to be received for Sundays or Washington's birthday?

12. A man borrowed $100 April 4, and returned it November 25. How many days' interest did he owe? (Do not include both days.)

13. May 1, 1893, fell on Monday. Upon what day of the week did July 4 fall?

14. How many days does vacation last if it begins on the morning of Saturday, July 2, and school commences on the first Tuesday of September?

15. A man borrows some money June 16, and agrees to return it in 60 days. On what date should he pay it?

16. A traveler starts upon a trip Aug. 24, 1894, and reaches home again Feb. 10, 1895. How long is he away?

806. When the difference between dates is more than a year, it is customary to take 30 days to each month. (See Appendix, Part III.)

Find the difference in time between March 3, 1879, and Jan. 1, 1893.

Writing 1893, 1st month, 1st day, we subtract 1879, 3d month, 3d day.

1893	1	1
1879	3	3
13	9	28

Answer, 13 years 9 months 28 days.

17. George Washington was born Feb. 22, 1732. How old was he at the signing of the Declaration of Independence, July 4, 1776?

18. Abraham Lincoln was first inaugurated president, March 4, 1861. How long had he served at his death, April, 15, 1865?

19. The battle of Lexington took place April 19, 1775. The treaty of peace was signed Sept. 3, 1783. How many years, months, and days between the two events?

20. How many years elapsed between the discovery of America by Columbus, Oct. 12, 1492, and the landing of the Pilgrims, Dec. 21, 1620?

21. General Harrison fought the Battle of Tippecanoe Nov. 7, 1811. He was inaugurated president 29 years 3 months 27 days later. Give the date of his inauguration.

22. How long was it after the treaty with England, signed Dec. 24, 1814, that the Mexican treaty was concluded, Feb. 2, 1848?

23. General Taylor died July 9, 1850. How long did he live after the capture of Monterey, Sept. 24, 1846?

24. President Garfield was born Nov. 19, 1831. How old was he at his inauguration, March 4, 1881?

25. The last battle of the Mexican war took place Sept. 14, 1847. The Battle of Bull Run was fought 13 years 10 months 7 days later. What was the date of this battle?

26. Find the time between July 4, 1776, and Jan. 1, 1894.

PERCENTAGE.

807. Oral Exercises.

1. Find 4% of $125.
2. 25% of 16.
3. 6% of 200 cows.
4. 1% of 150 lb.
5. 20% of 65 yd.
6. $33\frac{1}{3}$% of 1 day.
7. $62\frac{1}{2}$% of $12.
8. 9% of $23.
9. 75% of 3 gal.
10. $1\frac{1}{4}$% of $400.

808. Slate Exercises.

1. Find 6% of $576.
2. $4\frac{1}{2}$% of $340.
3. 25% of 1,876 bu.
4. $12\frac{1}{2}$% of 864 cows.
5. 50% of 432 yd.
6. $33\frac{1}{3}$% of 576 soldiers.
7. $16\frac{2}{3}$% of 696 gal.
8. $6\frac{1}{4}$% of $4.96.
9. 25% of $156.
10. 1% of $156.
11. $\frac{1}{4}$% of $156.
12. 50% of 480 hr.
13. $\frac{1}{2}$% of 480 hr.
14. $\frac{1}{8}$% of $1,420.
15. $3\frac{1}{3}$% of $66.
16. $7\frac{1}{2}$% of 360 days.

INTEREST.

809. In computing interest, the year is considered as composed of 12 months of 30 days each.

810. Oral Exercises.

Find the interest on:

1. $90 for 2 mo. at 4%.
2. $60 for 60 da. at 6%.
3. $100 for 2 yr. 6 mo. at 5%.
4. $120 for 30 da. at 5%.
5. $300 for 90 da. at 3%.
6. $100 for 1 yr. 3 mo. at 4%.
7. $50 for 3 yr. at 6%.
8. $100 for 2 yr. 4 mo. at 6%.
9. $60 for 40 da. at 6%.
10. $120 for 120 da. at 5%.

811. Find the interest on \$63 for 4 yr. 5 mo. at 5%.

\$63 is called the principal.

5 = rate.

4 yr. 5 mo. = time.

812. $\text{Interest} = \dfrac{\text{principal} \times \text{rate} \times \text{time (in years)}}{100}$.

4 years 5 months $= 4\frac{5}{12}$ yr. $= \frac{53}{12}$ yr.

$$\overset{.21\odot}{\$\cancel{63}} \times \frac{5}{\cancel{100}} \times \frac{53}{\cancel{12}_{4}} = \frac{\$55.65}{4} = \$13.91+ \quad \textit{Ans.}$$

NOTE. — The divisor 100 is canceled by placing a decimal point before 21.

Find the interest on \$160.50 for 3 mo. 15 da. at 6%.

$$160.50 \times \frac{6}{100} \times \frac{3\frac{1}{2}}{12} = \overset{.8025}{\cancel{1.60}\odot\cancel{50}} \times \frac{\cancel{6}}{\cancel{100}} \times \frac{7}{\cancel{24}_{\cancel{4}_{2}}} = \frac{5.6175}{2} = 2.808+$$

Ans. \$2.81.

Find the interest on \$69.75 for 1 mo. 17 da. at 4%.

$$\overset{.00775}{.0\odot 69\odot 75} \times \frac{\cancel{4}}{\cancel{100}} \times \frac{47}{\cancel{360}_{\cancel{90}}} = .36425. \quad \textit{Ans.}\ 36 \text{ cents.}$$

813. Slate Exercises.

Find the interest on:

1. \$192 for 3 yr. 7 mo. at 5%.
2. \$60 for 2 mo. 12 da. at 4%.
3. \$240 for 1 yr. 1 mo. at 6%.
4. \$14.40 for 5 yr. 5 mo. at 5%.
5. \$36 for 77 days at $4\frac{1}{2}$%.

6. $99 for 21 months at 6%.

7. $192 for 2 yr. 4 mo. at 5%.

8. $600 from Jan. 1 to Jan. 16 at 4%.

9. $1,200 from July 1, 1891, to Jan. 1, 1893, at 6%.

10. $57.60 from Oct. 4, 1890, to Feb. 4, 1894, at 5%.

REVIEW.

814. Oral Problems.

1. What part of $\frac{4}{5}$ is $\frac{3}{4}$? (16 twentieths, 15 twentieths.)

2. 16 is how many hundredths of 64?

3. What per cent of 25 is 5?

4. What part of 2 lb. 1 oz. is 1 lb.? (33 oz., 16 oz.)

5. Divide 4 gal. by 3 pt.

6. How many pencils at 4 mills each can be bought for a dollar?

7. Write $\frac{1}{80}$ as a decimal.

8. Divide 34 by 200.

9. At 20¢ per qt., what will be the cost of 2 gal. 3 qt. 1 pt. of maple syrup?

10. Find the cost of 4 T. 400 lb. of coal at $5 per ton.

11. A man puts 4 lb. 8 oz. of tea into 12 oz. packages. How many packages does he make?

12. 4 pecks 3 quarts of apples are given to some children. If each child's share is 5 quarts, how many children are there?

13. If it takes 3 hours 20 minutes to hoe a row of corn, how many rows can a man do in 2 days of 10 hours each?

14. How many square inches in the surface of a sheet of paper 1 ft. 8 in. long, 11 in. wide?

15. How many pieces of paper 2 inches square will exactly cover a slate 12 inches long, 8 inches wide?

815. Slate Problems.

1. What part of 6 hr. 17 min. 5 sec. is 3 hr. 15 min. 25 sec.?

2. If a man walks at the rate of 3 mi. and 96 rd. per hour, how far will he walk in 3 hr. and 20 min.?

3. What is one-ninth of 28 bu. 3 pk. and 7 qt.?

4. Three men buy a house for $1,200. A furnishes $600; B, $400; C, $200. They sell the house for $1,500. How much money should each receive?

5. If 5 T. and 1,000 lb. of coal cost $30.25, how much will be paid for 7 T. and 320 lb.?

6. At 25¢ per hour, how much should a man receive that works 8 hours and 36 minutes?

7. At $45 per month, what is the rent of a house for 2 yr. 7 mo. and 8 da.?

8. If 2 lb. 4 oz. of tea cost $1.35, what will be the cost of 11 lb. 12 oz.?

9. How many sq. in. in a paving tile 6 in. square? How many sq. in. in a rectangle 4 ft. by 3 ft.? How many paving tiles 6 in. by 6 in. would cover a surface 4 ft. by 3 ft.?

10. A merchant imports 360 yd. of dress goods, 27 inches wide, costing 30¢ per yd. What will the duty be at 8¢ per square yard, and 50 per cent of the cost in addition?

11. A man pays $60 interest per year. How much interest does he pay in 3 yr. 7 mo. 10 da.?

12. Find four-ninths of 28 bu. 3 pk. 7 qt.

APPROXIMATIONS.

816. Give an estimate of the answer (Art. 521):

1. If 3 T. and 1,988 lb. of coal cost $19.97, what will be the cost of 8 T. and 1 lb.? (Nearly 4 tons cost nearly $20.)

2. At $500 per year, what will be the rent of a house for 1 year 11 mo. and 29 da.? (Nearly 2 years.)

3. Find the cost of 5 bbl. sugar, averaging 299 lb. each, at $4\frac{5}{16}¢$ per lb.

4. What is the interest on $199.86 at 6%, for 5 mo. 28 da.?

5. If 11 men and 2 boys can finish a piece of work in $23\frac{1}{2}$ days, how long would it take 23 men and 5 boys?

6. What decimal of 639 acres is 321 acres?

7. What will be the cost of 20,060 bricks at $19.90 per M.?

8. A farmer sells 5,584 lb. of rye at $87¢$ per bu. of 56 lb. How much does he receive?

9. If 9 lb. and 15 oz. of tea cost $7.95, what will be the cost of 21 lb. and 1 oz.?

10. Paid freight on 1,987 lb. at $70¢$ per hundredweight. How much did I pay?

MEASUREMENTS.

817. Make diagrams when necessary.

1. A man has a lot 100 feet by 200 feet. How many square feet will he have left for a garden after he builds a house 25 feet by 60 feet?

2. One wall of a room is 24 feet long and 12 feet high. There is a door in it 8 feet high, $4\frac{1}{2}$ feet wide. How many square yards of plastering will be needed to cover the wall?

3. A brick is 8 inches long, 4 inches wide, 2 inches thick. How many square inches are there in the surface of the widest face? In the surface of one side? In the surface of one end?

4. How many bricks laid on the widest face will be needed for a walk 288 inches long, 96 inches wide?

5. How many bricks laid on the side will be needed for a walk 24 feet long, 8 feet wide?

6. Make a diagram of a piece of paper that when folded will just cover the six faces of a brick $8 \times 4 \times 2$ inches. How many square inches of paper would be needed?

7. How many square feet are there in a roll of wall paper 24 feet long, 18 inches wide?

8. How many rolls 24 feet long, $1\frac{1}{2}$ feet wide, would be required to paper the ceiling of a room 45 feet long, 36 feet wide, making no allowance for matching or waste?

9. The owner of a piece of ground 200 feet wide, 300 feet long, divides it into lots 25 feet by 100 feet. How many lots are there?

10. Make table of:

818. Square Measure.

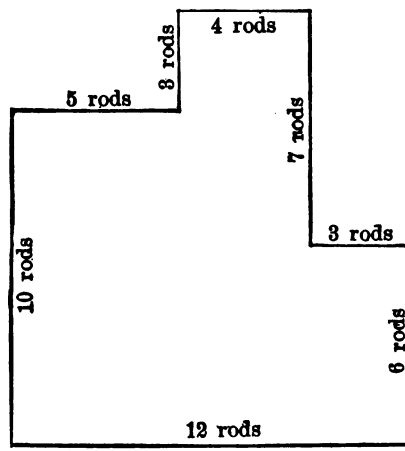
square inches (sq. in.)	1 square foot (sq. ft.)
square feet	1 square yard (sq. yd.)
square yards	1 square rod (sq. rd.)
160 square rods	1 acre (A.)
acres	1 square mile (sq. mi.)

11. The owner of a piece of ground 600 feet long, 150 feet wide, builds a fence 6 feet high around the plot. How many square feet of fence are there?

12. What would be the cost of building 1,800 feet of fence 6 feet high at $1 per square yard?

13. A farm is one mile square. How many 40-acre fields does it contain?

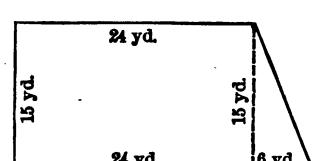
14. How many yards of fence will be needed to enclose the plot of ground shown in the following diagram?



15. The above field was originally a rectangle, but the owner sold one piece 5 rods by 3 rods, and a second piece 3 rods by 7 rods. How many square rods did it contain at first? What is its present area?

16. How many acres in a field in the shape of a triangle whose base and perpendicular measure 40 rods each?

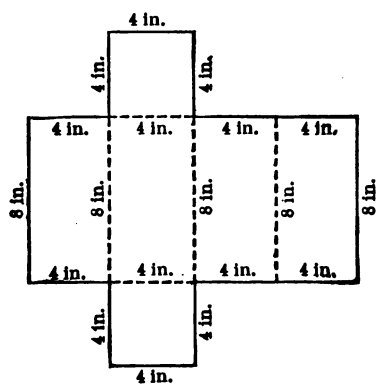
17. Calculate the number of square yards in the field shown in the accompanying diagram.



18. The owner of a field 160 yards long, 121 yards wide sold from one corner a triangular piece 40 yards long, 30¼ yards wide. Find the number of square yards in the part remaining. (Make diagram.)

19. How many acres are there in a triangular plot of ground when the base measures 80 yards and the perpendicular measures 60½ yards?

20. What are the dimensions of the box that can be made of a piece of paper of the size shown in the following diagram? How many square inches of paper are needed for such a box (making no allowance for pasting)?



DENOMINATE NUMBERS. (*Long Measure.*)

819. Slate Exercises.

1. Change 43 yd. to rods and a fraction.
2. Change 43 yd. to rods and yards.
3. Change 43 yd. to rods, yards, and feet.
4. Change 43 yd. to rods, yards, feet, and inches.
5. Change 72 yd. to rods, etc.
6. Change 66 yd. to rods.

820. Change to rods, etc.:

7. 49 yd.
8. 147 ft.
9. 1,764 in.
10. $8\frac{10}{11}$ rd.
11. 50 yd.
12. 51 yd.
13. 153 ft.
14. 1,836 in.
15. 1,837 in.
16. 52 yd.
17. $49\frac{1}{2}$ yd.
18. 49 yd. 1 ft. 6 in.
19. $148\frac{1}{2}$ ft.
20. 1,782 in.

821. Change 1,581 in. to rods, etc.

12)1581 in.
3)131 ft. 9 in.
$5\frac{1}{2}$)43 yd. 2 ft. 9 in.
2 2
11)86
7 rd. 9 half-yards 2 ft. 9 in.
7 rd. $4\frac{1}{2}$ yd. 2 ft. 9 in.

7 rd. 4 yd. 2 ft. 9 in.
\+ 1 ft. 6 in. = $\frac{1}{2}$ yd.
7 rd. 5 yd. 1 ft. 3 in. *Ans.*

822. To find how many rods in 43 yards, we divide by $5\frac{1}{2}$. $5\frac{1}{2}$ yards = 11 half-yards. 43 yards = 86 half-yards. As there are 11 half-yards in a rod, 86 half-yards will be equal to 7 rods and 9 half-yards, or 7 rods $4\frac{1}{2}$ yards. Changing $\frac{1}{2}$ yd. to 1 ft. 6 inches, we obtain the answer as above.

823. Change to rods, etc.:

21. 1,483 in.
22. 984 in.
23. 1,345 in.
24. 2,796 in.
25. 1,121 in.
26. 1,470 in.
27. 3,453 in.
28. 1,278 in.
29. 1,576 in.

30. Change 5 rods to inches.
31. Change 990 inches to rods.
32. How many inches in 7 rods 1 yd.?
33. Change 1,422 inches to rods.

824. Add:

34.
4 rd. 3 yd. 1 ft.
9 rd. 4 yd. 2 ft.
3 rd. 1 ft. 6 in.

35.
7 rd. 5 yd.
3 rd. 1 ft.
2 yd. 2 ft.
4 rd. 1 ft.

36.
5 rd. 4 yd. 2 ft.
5 yd. 1 ft.
6 rd. 1 yd.

37.
11 rd. 3 yd. 1 ft.
4 rd. 2 ft.
5 yd. 1 ft.
6 rd.

38. From 8 rd. 1 ft. take 2 rd. 2 ft.

39. Find the difference between 3 rd. 1 yd. 1 ft. and 16 rd.

40. Multiply 5 rd. 4 yd. 2 ft. by 4.

41. Multiply 11 rd. 2 ft. by 10.

42. Divide 30 rd. 5 yd. 2 ft. by 8.

43. Divide 34 rd. 2 yd. by 9.

SOLID CONTENTS.

825. How many one-inch cubes can be placed on the bottom of a box 3 in. long, 4 in. wide?

1. If the box is one inch high, how many will it hold? If the box is 2 in. high? 3 in. high?

2. A cube one inch long, one inch wide, one inch high contains a cubic inch.

How many cubic inches in a box 3 in. long, 4 in. wide, 1 in. high? In a box 3 in. long, 4 in. wide, 2 in. high? In a box 4 in. long, 4 in. wide, 4 in. high?

3. If you had 24 one-inch cubes, how could you pile them to make a solid with six rectangular faces?

If the pile was 2 inches high, how many cubes would there be in each tier? How many square inches would the lower tier cover?

How could the 24 cubes be arranged to make a pile 3 inches high?

4. Can you give a rule for finding the contents of a box 6 in. long, 8 in. high, 4 in. wide?

5. How many cubic inches of water would a tin box hold, the dimensions of the box being 5 in. by $3\frac{1}{2}$ in. by 4 in.?

6. How many one-inch cubes could be placed in a box one foot long, one foot wide, one foot high?

7. How many cubic inches in a cubic foot?

8. How many one-foot cubes could be placed in a cubical box one yard long, one yard wide, one yard high?

How many cubic feet in a cubic yard? How many cubic inches in a cubic yard?

9. How many cubic inches in a solid, 3 yd. long, 2 ft. wide, 6 in. high? How many cu. ft.? How many cu. yd.?

Indicate the operations necessary in each case to obtain the correct answer.

What should be done with the denominations in each case before beginning the work of obtaining the solid contents?

10. A cord of wood contains 128 cu. ft. If the wood is cut into 4-ft. lengths, what should be the other two dimensions of a regular pile to hold just a cord?

11. How many cubic feet of air in a room 24 ft. long, 18 ft. wide, 12 ft. high?

12. A gallon contains 231 cu. in. Give the dimensions of a tin box that will hold exactly a gallon.

13. Find the solid contents of a piece of timber 25 ft. long, 3 ft. wide, 5 ft. thick. Is it larger or smaller than a piece 4 ft. wide, 4 ft. thick, and 23 ft. 6 in. long?

14. How many cubic yards of earth will have to be removed in digging a cellar 18 ft. wide, 55 ft. long, 6 ft. deep? What will be the cost at 60¢ a load (1 cu. yd.)?

15. Give the width of a wagon body 18 in. high, 6 ft. long, that will hold, when full, a cubic yard.

16. About how many gallons are there in a cu. ft.?

17. A bushel contains 2,150.4 cu. in. About how many cu. ft. in a bushel?

18. Find (by cancellation) the capacity in gallons of a tank 1 ft. 9 in. long, 1 ft. 3 in. wide, 1 ft. 10 in. deep.

19. Find (by cancellation) the capacity in bushels of a bin 1 yd. long, 2 ft. 4 in. wide, 5 ft. 4 in. high.

20. How many cords of wood (128 cu. ft.) are there in a pile 24 ft. long, 4 ft. wide, 12 ft. high?

21. A brick is 8 in. long, 4 in. wide, 2 in. thick. How many bricks are there in a pile 90 bricks long, 60 bricks wide, 30 bricks high? What are the dimensions of the pile?

How many cu. in. in 1 brick? In the pile?

22. Find the number of bricks in a wall 24 ft. wide, 48 ft. high, 1 ft. thick, making no allowance for mortar, etc.

23. How many bricks are there to a cu. ft.?

24. Allowing 20 bricks to a cubic foot when laid in mortar, how many bricks will be needed for a wall 24 ft. wide, 50 ft. high, 20 in. thick?

25. What will be the cost of building a stone wall 40 rods long, 4 ft. high, 1 yd. thick, at $6.40 per perch of $24\frac{3}{4}$ cu. ft.?

APPROXIMATIONS.

826. Give sight answers in whole numbers:

1. If there are about $7\frac{1}{2}$ gal. to a cu. ft., estimate the number of gallons in a tank 5 ft. long, 3 ft. wide, 4 ft. high.

2. If there are about $1\frac{1}{4}$ cu. ft. in a bushel, estimate the contents in bushels of a bin 5 ft. × 3 ft. × 4 ft.

3. Give the dimensions of a tank of 150 gal. capacity.

4. Give the dimensions of a bin that will hold 100 bushels.

5. At 20 bricks laid in mortar to the cu. ft., give the length and the height of a wall 1 ft. thick that can be built with a thousand bricks.

6. At \$1 a load (1 cu. yd.), give the dimensions of an excavation that can be made for \$100.

7. A cu. ft. water (about $7\frac{1}{2}$ gal.) weighs $62\frac{1}{2}$ lb. About what does a gallon weigh? A pint?

8. If iron is about $7\frac{1}{4}$ times as heavy as water, about what does a cubic foot of iron weigh?

9. About what is $49\frac{3}{4}\%$ of \$801?

10. About what will be the interest at 6 per cent on \$100 for 3 yr. 11 mo. 29 da.?

827. Cubic Measure.

1,728	cu. in.	1 cu. ft.
27	cu. ft.	1 cu. yd.
231	cu. in.	1 gallon.
2,150.4	cu. in.	1 bushel.
128	cu. ft.	1 cord.

828. Troy Weight.

In weighing gold and silver, troy weight is used. The following is the table:

24 grains (gr.)	1 pennyweight (dwt.)
20 pennyweights	1 ounce (oz.)
12 ounces	1 pound (lb.)

The abbreviation for pennyweight is also written pwt.

829. The pound apothecaries' weight — used in putting up prescriptions — contains the same number of grains (5,760) as the pound troy.

830. The pound avoirdupois contains 7,000 grains.

1. At $18 per M., what will be the cost of a pile of bricks 20 ft. long, 15 ft. wide, 6 ft. high? A brick measures 8 in. by 4 in. by 2 in.

2. Find the cost of a silver pitcher, weighing 5 lb. 5 oz. 12 pwt. 12 gr., at 48 cents per ounce troy.

3. How many pounds and ounces avoirdupois does the above pitcher weigh?

4. How many 5-grain pills can be made from an avoirdupois pound of quinine? From a pound apothecaries' weight?

5. How many spoons weighing 18 pwt. 16 gr. each, can be made from 11 oz. 4 pwt. of silver?

CANCELLATION.

834. Indicate operations, and cancel where possible. Terms compared should be of the same denomination.

1. If 90 tons of coal cost $472.50, what will be the cost of 132 tons?

2. If 3 lb. and 4 oz. of tea cost $1.95, what will 12 oz. cost?

3. A party of men can build 16 rd. 2 ft. of wall in 20 days. How long will it take them to build 4 yd. and 6 in.?

4. What will be the cost of 3 bu. 2 pk. 7 qt. 1 pt. of oats if 7 bu. 1 qt. cost $4.50?

5. By traveling at the rate of 20 miles a day, a person can complete a journey in 18 days. At what rate must he travel to finish it in 15 days?

REVIEW.

841. Add across:

1. $13\frac{1}{2} + 16\frac{2}{3} + 8\frac{3}{4}$
2. $4\frac{1}{4} + 5\frac{2}{5} + 27\frac{3}{8}$
3. $19\frac{1}{3} + 3\frac{5}{6} + 35\frac{1}{5}$
4. $8\frac{1}{2} + 9\frac{3}{10} + 14\frac{1}{8}$
5. $23\frac{3}{5} + 5\frac{1}{4} + 32\frac{5}{12}$
6. $59\frac{3}{7} + 3\frac{1}{2} + 4\frac{3}{4}$
7. $7\frac{5}{9} + 18\frac{3}{8} + 40\frac{1}{6}$
8. $35\frac{2}{3} + 51\frac{1}{5} + 8\frac{7}{10}$
9. $3\frac{7}{8} + 9\frac{1}{3} + 25\frac{7}{12}$
10. $66\frac{1}{2} + 8\frac{3}{5} + 14\frac{1}{3}$

842. Subtract across:

11. $25\frac{1}{4} - 18\frac{7}{12}$
12. $63\frac{2}{5} - 49\frac{3}{8}$
13. $70\frac{4}{11} - 15\frac{1}{2}$
14. $92\frac{5}{6} - 24\frac{7}{8}$
15. $33\frac{1}{3} - 15\frac{7}{10}$
16. $68\frac{3}{4} - 6\frac{11}{12}$
17. $100\frac{1}{9} - 62\frac{1}{4}$
18. $56\frac{1}{4} - 37\frac{4}{9}$
19. $83\frac{3}{7} - 43\frac{2}{3}$
20. $42\frac{1}{8} - 16\frac{2}{3}$

843. Multiply. Do not reduce to improper fractions:

$$\begin{array}{r} 12\frac{3}{4} \times 5\frac{1}{3} \\ \hline 63\frac{3}{4} \\ 4\frac{1}{4} \\ \hline 68 \end{array}$$

21. $37\frac{1}{2} \times 3\frac{1}{2}$
22. $48\frac{3}{4} \times 4\frac{1}{3}$
23. $64\frac{1}{8} \times 10\frac{1}{3}$
24. $29\frac{2}{3} \times 6\frac{1}{4}$
25. $18\frac{7}{8} \times 5\frac{1}{2}$
26. $13\frac{1}{4} \times 7\frac{1}{5}$
27. $9\frac{3}{4} \times 8\frac{1}{3}$
28. $45\frac{1}{3} \times 2\frac{1}{6}$
29. $50\frac{2}{3} \times 10\frac{1}{7}$
30. $38\frac{3}{4} \times 11\frac{1}{3}$

844. Divide:

31. $13\overline{)205\frac{1}{2}}$
32. $14\overline{)186\frac{1}{3}}$
33. $15\overline{)250\frac{1}{4}}$
34. $16\overline{)198\frac{1}{5}}$
35. $21\overline{)450\frac{1}{3}}$
36. $31\overline{)970\frac{1}{7}}$
37. $24\overline{)553\frac{3}{5}}$
38. $25\overline{)568\frac{1}{2}}$
39. $32\overline{)965\frac{1}{3}}$
40. $36\overline{)722\frac{2}{5}}$

845. Miscellaneous.

1. Find the value of (*a*) $(6.125 + 8.75 - 9.1235) \div .0125$; (*b*) $(1{,}708.4592 \div .00024) \times .003$.

2. Simplify $\dfrac{7 - 3.004}{.2 + 7.3} \times \dfrac{5 - .08}{4.8} \div 1\frac{1}{5}$.

3. $(\frac{3}{5}$ of $\frac{7}{8}) + (\frac{2}{3}$ of $\frac{5}{7}) - (\frac{4}{9}$ of $2) = ?$

4. $\dfrac{\frac{1}{2} \text{ of } 7\frac{1}{2}}{\frac{3}{8} \text{ of } 15} - \dfrac{\frac{9}{10} \text{ of } 4\frac{3}{4}}{1\frac{1}{2} \times 11} = ?$

5. Add $8\frac{5}{7} + \frac{3}{7} + \frac{5}{6} + \frac{8}{11} + \frac{3}{5}$.

6. Find the value of $728 - \frac{3}{4} - \frac{1}{7} - \frac{3}{5} - \frac{1}{3}$.

7. $1\frac{1}{2} \times (\frac{3}{4} \div \frac{2}{3}) \times \frac{1}{3}$.

8. Reduce $\frac{7}{10}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\dfrac{13.5}{7}$ to a decimal.

9. If .1875 of a vessel cost \$273.12½, what is the value of $\frac{5}{12}$ of it at the same rate?

10. A person owning $\frac{6}{10}$ of a factory sells 75 per cent of his share for \$1,710. What is the value of the whole factory?

11. Find $\frac{3}{7}$ of 2 days 5 hours 45 minutes.

12. What is the interest on \$760 for 5 months at 3½%?

13. If a piece of cloth is 20 yd. long and $\frac{3}{4}$ yd. broad, how broad is another piece which is 12 yd. long and contains as many square yards as the first?

14. Simplify $\dfrac{2\frac{1}{2} + 1\frac{1}{3}}{2\frac{1}{2} - 1\frac{1}{3}} \times \dfrac{1\frac{7}{12} - 1}{\frac{1}{4} + \frac{5}{8}}$.

15. A merchant insures property worth \$20,000 for $\frac{3}{4}$ of its value. How much does he pay, the rate being 1¼%?

16. If 7 men can do a piece of work in 10½ days, how long will it take 8 men and 5 boys to do the same work, each boy doing one-half as much as a man?

ANGLES, TRIANGLES, QUADRILATERALS.

846. The following may be drawn free hand, the compasses being reserved for the geometrical problems in Chapter XVI.

1. Draw two lines meeting at a point.

These lines make an *angle*.

2. Draw two lines that will make four angles.

3. Draw two lines so as to make two angles.

Two such angles are called *adjacent* angles.

4. Make two equal adjacent angles.

Equal adjacent angles are called *right* angles. A line making a right angle with another line is said to be *perpendicular* to it.

5. Draw two lines so as to make one right angle.

Is the right angle made by two lines, each 10 ft. long, any larger than a right angle made by two lines, each 1 inch long?

6. What is the smallest number of straight lines that will enclose a space?

Draw a figure enclosed by the smallest possible number of straight lines. What is its name? Why?

7. Make a triangle having one right angle.

8. Can you draw a triangle having two right angles? Why? What name is given to lines that will not meet, no matter how far they are extended?

9. An angle less than a right angle is called an acute angle.

Draw a triangle containing an acute angle.

10. Can you draw a triangle containing two acute angles? Three acute angles?

11. An angle greater than a right angle is called an obtuse angle.

Draw a triangle containing an obtuse angle.

12. Can you draw a triangle containing three obtuse angles? Containing two?

13. Draw a triangle with sides 2 inches, 3 inches, 4 inches, respectively.

A triangle having no two sides equal is called a *scalene* triangle.

14. Draw a triangle having two equal sides.

This is called an *isosceles* triangle. The unequal side is called the base.

15. Draw an isosceles triangle with the base uppermost. With the base on the left. On the right.

16. Draw a triangle having three equal sides (an equilateral triangle).

17. Draw a square. Draw a rectangle 4 in. by 3 in.

How many right angles in each?

18. Draw a four-sided figure having its opposite sides parallel, but containing no right angle (rhomboid).

What kinds of angles does it contain? How many of each? Write name in each angle.

19. Draw a four-sided figure, having all its sides equal, but containing no right angle (rhombus).

20. Draw a quadrilateral (four-sided figure) having only two parallel sides (trapezoid).

21. Draw a quadrilateral having no parallel sides (trapezium).

22. Draw a rhombus, each side 2 inches. A square, each side 2 inches.

What is the difference between them? Which is larger?

23. A parallelogram is a quadrilateral that has its opposite sides parallel.

Name the parallelograms that have four equal sides (equilateral). Those that have four equal angles (equiangular).

24. The height of a parallelogram is called its altitude. Draw a rectangle, base $3\frac{1}{2}$ inches, altitude $2\frac{1}{2}$ inches. Draw a rhomboid, base $3\frac{1}{2}$ inches, altitude $2\frac{1}{2}$ inches. Draw several rhomboids of the above dimensions, all differing in shape.

25. Cut out of paper a rectangle, base 3 inches, altitude 2 inches. Cut out a rhomboid, base 3 inches, altitude 2 inches. Place one upon the other, and see how their areas compare.

26. Can you calculate the number of square inches in a rhomboid whose base is 3 inches and altitude 2 inches?

27. Draw a rectangle, base 4 inches, altitude 3 inches. Divide by a diagonal into two triangles. Mark in each triangle its area.

28. Draw a right-angled triangle, base 4 inches, perpendicular (altitude) 3 inches. Calculate its area.

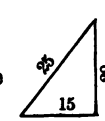
29. Draw a rectangle, base 4 inches, altitude 3 inches. From the middle point of the upper base draw lines to the extremities of the lower base, making three triangles. Mark in each triangle its area.

30. Draw an isosceles triangle, base 4 inches, altitude 3 inches, and calculate its area.

847. Areas of Triangles and Quadrilaterals.

Find the areas of the following:

1. A right-angled triangle whose sides measure 15, 20, and 25 inches, respectively.



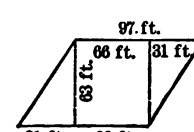
2. A right-angled triangle whose base measures 64 yd., perpendicular 48 yd.

3. A triangle whose base measures 18 rods, altitude 13 rods.

4. A square whose side measures 35 feet.

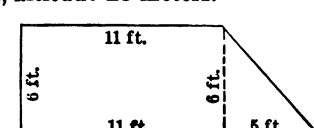
5. A rectangle 42 yd. by 37 yd.

6. A rhombus whose base is 97 feet, altitude 63 feet.

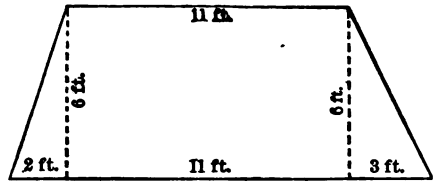


7. A rhomboid, base 33 meters, altitude 28 meters.

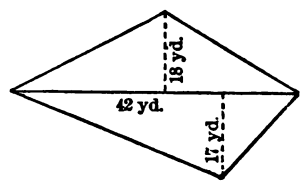
8. A trapezoid whose parallel sides measure 11 and 16 ft., respectively, the perpendicular distance between them being 6 ft.



9. A trapezoid as shown in the accompanying diagram.



10. A trapezium one of whose diagonals measures 42 yards, the perpendiculars to the opposite corners measuring 18 yd. and 17 yd., respectively.



CHAPTER X.

ALGEBRAIC EQUATIONS.

ONE UNKNOWN QUANTITY.

848. Oral Exercises.

1. What number increased by 12 equals 16?

2. 9 added to a number equals 14. Find the number.

3. What number diminished by 7 equals 8?

4. 18 diminished by what number has 10 for remainder?

5. Eight times what number equals 64?

6. What number multiplied by 9 gives 63 for product?

7. Three times a certain number added to twice the same number equals 40. What is the number?

8. 36 is equal to 10 times what number added to 8 times the same number?

9. The difference between 4 times a number and 3 times the same number is 72. What is the number?

10. Twice a number subtracted from 11 times the same number equals 27. What is the number?

849. Sight Exercises.

Give values of x, y, z, etc.:

1. $?+12=16$
2. $9+?=14$
3. $?-7=8$
4. $18-?=10$
5. $8x=64$
6. $9y=63$

7. $3x+2x=40$

8. $10z+8z=36$

9. $4v-3v=72$

10. $11w-2w=27$

11. $3x-2x+5x=54$

12. $10y+8y-4y=42$

13. $2x+4x=52-12$

14. $3z+4z=30-9$

15. $12y-5y=25+10$

16. $6w+6w=16+8$

850. Slate Problems.

1. A horse and a wagon cost together \$600. What is the price of each, if the wagon costs twice as much as the horse?

Let x = cost of horse;
then $2x$ = cost of wagon.
Cost of both $= 2x+x = 600$
$3x = 600$
$x = 200$
$2x = 400$

Ans. Cost of horse, \$200; of wagon, \$400

2. Divide 100 into two parts, one of which shall be four times as large as the other.

Let x = one part;
then $4x$ = the other.
$x+4x = 100.$

3. \$18,000 is divided among three children, the second of whom receives twice as much as the first, and the third of whom receives six times as much as the first. Required the share of each.

$x,\ 2x,\ 6x.$

4. In a class of 54 pupils, there are twice as many boys as girls. How many are there of each?

5. The sum of two numbers is 78. One is five times as large as the other. What are the numbers?

6. 156 is equal to seven times a number added to five times the same number. Find the number.

7. The difference between three times a certain number and nine times the same number is 66. What is the number?

8. $27,000 is divided among three children, the second of whom receives twice as much as the first, and the third of whom receives three times as much as the second. What is the share of each?

9. The sum of two numbers is 72, and the greater is 5 times the other. What are the numbers?

10. John, Henry, and James have 54 marbles. Henry has twice as many as John, and James has as many as the other two. How many has each?

11. The sum of the ages of mother and daughter is 42 years. What is the age of each, if the mother's age is six times that of her daughter?

12. A man paid $96 for an equal number of hats and coats, paying $2 apiece for the former and $10 apiece for the latter. How many of each did he buy?

13. Divide 41 into four parts, the first being twice the second, the second three times the third, and the third four times the fourth.

(Let x = the fourth.)

14. The sum of three numbers is 180. The first is double the second, and the third is three times as large as the sum of the other two. What are the numbers?

15. Mr. Smith paid 81 cents for sugar and flour, the same quantity of each. For the sugar he gave 5¢ per pound, and for the flour 4¢ per pound. How many pounds of each did he buy?

16. The length of a rectangular field is 24 rods, its breadth is x rods, its area is 456 square rods. Find the value of x.

17. It takes 340 feet of fence to enclose a square lot. What are the dimensions of the lot?

18. Mrs. B. divides $120 between her son and her daughter. She gives the latter twice as much as she gives the former. What is the share of each?

19. The earnings of a man and his son during January amounted to $175, both having worked the same number of days. The father's wages were $4 per day, and the son's wages were $3 per day. How many days did they work?

20. The sum of $240 is divided among four children, two boys and two girls. Find the share of each, if each girl's share is double that of each boy.

21. A man worked twice as many days as his son. Their combined earnings amounted to $165. Find the number of days each worked, if the father earned $4 per day and the son three-fourths as much per day.

22. A boy's bank contains 78¢ in dimes, nickels, and cents. There are twice as many nickels as there are dimes, and three times as many cents as there are nickels. How many are there of each?

23. I paid 75¢ more for a roll of 15-cent ribbon than I did for a roll of 12-cent ribbon of the same length. How many yards did each roll contain?

24. A rectangular field whose length is four times its breadth requires 250 rods of fence to enclose it. What are the dimensions of the field? (Make diagram.)

25. A girl paid 60 cents for a speller and a reader, the cost of the former being one-third that of the latter. Find the cost of each.

26. The sum of two numbers is 72, and the smaller is one-fifth of the other. What are the numbers?

27. Mary, Susan, and Jane have 54 hickory nuts. Susan has one-half as many as Mary, and Jane has as many as the other two. How many has each?

CLEARING OF FRACTIONS.

851. Oral Exercises.

1. One-fifth of a number is 4. What is the number?

2. $\frac{1}{5}$ of a number is 8. What is $\frac{2}{5}$ of the number?

3. $\frac{1}{4}$ of a number is 12. What is the number?

4. $\frac{1}{4}$ of a number is 10. What is $\frac{3}{4}$ of the number?

5. If $\frac{3}{4}$ of a number is 30, what is the number?

6. One-half a number added to $\frac{1}{4}$ of the same number equals what fraction of the number?

7. One-half a number added to $\frac{1}{4}$ of the same number equals 30. What is the number?

8. One-third of a number + one-sixth of the number = what fraction of the number?

9. One-third of a number added to $\frac{1}{4}$ of the number = what fraction of the number?

10. $\frac{1}{3}x + \frac{1}{4}x =$ what fraction of x? $\frac{x}{3} + \frac{x}{4} = ?$

852. When $x = 32$, find the value of three-fourths of x; *i.e.*, $\frac{3x}{4}$.

When $\frac{3x}{4}$, ($3x$ divided by 4) $= 24$, what is the value of $3x$? Of x?

Find the value of y, when $\frac{y}{3} = 12$. Of $2y$, when $\frac{2y}{3} = 24$.

Given the equation $\frac{4z}{5} = 20$; by what whole number can we multiply the first member to get rid of the fraction? If we multiply one member of an equation by any number, what must we do to the second member in order to preserve the equality?

853. Sight Exercises.

Give values of x, y, z, etc.:

1. $\frac{x}{5}=4$
2. $\frac{2y}{5}=8$
3. $\frac{z}{4}=7$
4. $\frac{3v}{4}=21$
5. $\frac{w}{2}+\frac{w}{4}=12$
6. $\frac{x}{2}+\frac{x}{3}=5$
7. $\frac{y}{3}+\frac{y}{6}=10$
8. $\frac{z}{3}+\frac{z}{4}=7$
9. $\frac{v}{5}+\frac{v}{5}=8$
10. $\frac{w}{3}+\frac{2w}{3}=32$
11. $\frac{x}{4}+\frac{x}{5}=9$
12. $\frac{x}{2}+\frac{2x}{5}=9$

854. Slate Exercises.

Find the value of the unknown quantity (x).

In each of the following equations, multiply both members by the least common denominator of the fractions.

1. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=26$

Multiplying by 12, we have $6x+4x+3x=312$

2. $x+\frac{x}{2}+\frac{x}{3}=44$

Multiply by 6. $6x+3x+2x=264$

3. $\frac{x}{2}+\frac{x}{3}=35$
4. $\frac{x}{3}+\frac{x}{4}=49$
5. $\frac{x}{2}+\frac{2x}{3}=28$
6. $\frac{5}{6}x+\frac{5}{7}x=92$
7. $\frac{2x}{3}+\frac{3x}{4}=102$
8. $2\frac{7}{8}x=115$
9. $\frac{4x}{5}-\frac{2x}{3}=48$
10. $x-\frac{x}{40}=156$
11. $\frac{3x}{2}=27$
12. $1\frac{1}{2}x=27$
13. $\frac{11x}{4}=22$
14. $2\frac{3}{4}x=44$
15. $2x+\frac{3x}{4}=33$
16. $x+\frac{x}{5}=24$
17. $\frac{75x}{100}-\frac{33x}{50}=81$
18. $3\frac{1}{2}x-2\frac{3}{4}x=45$
19. $\frac{8x}{3}-\frac{2x}{5}=136$
20. $3\frac{5}{8}x=116$

21. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=39$

22. $x-\frac{x}{2}-\frac{x}{3}=37$

23. $\frac{4x}{5}-\frac{2x}{9}+\frac{3x}{4}=239$

24. $\frac{5x}{9}+\frac{2x}{3}-\frac{x}{2}=52$

25. $x-\frac{3x}{4}=80$

26. $x+2x+\frac{3x}{7}=24$

855. Slate Problems.

1. The sum of two numbers is 90, and the smaller number is one-fifth of the larger one. What are the numbers?

$$\left(x+\frac{x}{5}=90.\right)$$

2. Divide 100 into two parts, one of which shall be $2\frac{1}{2}$ times the other.

3. After losing $\frac{1}{8}$ of his money, a man has $\$714$. How many dollars had he at first?

$$\left(x-\frac{x}{8}=714.\right)$$

4. A horse was sold for $\$240$, the seller thereby gaining one-third of what he originally paid for it. How much did he pay for it?

$$\left(x+\frac{x}{3}.\right)$$

5. One-half of a number added to one-fourth of the same number equals $66\frac{3}{4}$. What is the number?

6. The difference between $\frac{3}{4}$ of a number and $\frac{2}{5}$ of the same number is 15. Find the number.

7. One number is $\frac{3}{5}$ of another. Their sum is 55. What are the numbers?

8. Find a fraction equivalent to $\frac{7}{8}$, the sum of its numerator and its denominator being 60.

(Let $7x$ = numerator and $8x$ = denominator.)

9. Find a fraction equivalent to $\frac{5}{7}$, the difference between its numerator and its denominator being 24.

10. The sum of two numbers is 480, and the quotient obtained by dividing the greater by the less is 7. What are the numbers?

11. Find two numbers whose difference is 522 and whose quotient is 30.

12. A boy buys apples at 2¢, pears at 3¢, and oranges at 4¢, the same number of each. How many of each does he buy, if he pays 81¢ for all?

13. A girl bought 70 cents' worth of peaches and plums. She paid 3¢ each for the peaches and 2¢ each for the plums, buying four times as many of the former as of the latter. How many of each did she buy?

14. $1,500 is divided among three persons, the second of whom receives three times as much as the first, and the third three and one-half times as much as the first. Find the share of each.

15. A farmer paid for a cow three-sevenths as much as he paid for a horse. How much did he pay for each, if the latter cost $80 more than the former?

16. Three times a man's money increased by two-thirds of his money is equal to $1,100. How much money has he?

17. After giving away $\frac{3}{5}$ of his marbles and losing $\frac{1}{4}$ of them, Joseph has 24 left. How many had he at first?

18. Bought a coat, a hat, and an umbrella for $15, paying for the hat $1\frac{1}{2}$ times as much as for the umbrella, and for the coat $3\frac{1}{3}$ times as much as for the hat. Find the price of each.

19. A merchant purchased two pieces of cloth for $240, paying for one piece twice as much per yard as for the other. The former contains 36 yards and the latter 48 yards. How much does he pay per yard for each?

20. A farmer sold 4 times as many cows as horses, receiving for all $840, at the rate of $40 for a cow and $120 for a horse. How many of each did he sell?

TRANSPOSING.

856. Sight Exercises.

Give values of x, y, z, etc.:

1. $x+15=21$
2. $2y+15=21$
3. $z-7=21$
4. $4w-7=21$
5. $\frac{v}{2}+3=8$
6. $\frac{x}{2}-3=12$
7. $3y+6=15$
8. $7y-13=15$
9. $9y+13=58$
10. $3y-10=56$
11. $\frac{3v}{4}+1=7$
12. $\frac{4w}{5}-1=11$

857. If $x+15=21$, $x=21-$ what?

When $x-7=21$, $x=21+$ what?

If in the equation $2x+15=21$, we take away 15 from the first member, what must we do to the second member to preserve the equality?

By *transposing* we mean bringing the unknown quantities (x, y, z, etc.) to one side of the equation, and the known quantities to the other.

NOTE. — In bringing a quantity from one side of the equation to the other the *sign* of the quantity is changed.

858. Slate Exercises.

Find values of the unknown quantities.

NOTE. — Clear of fractions when necessary; then transpose.

1. $x+37=56$
2. $4x-5=83$
3. $3x-43=98$
4. $7x+13=111$
5. $x+3x=25+11$
6. $5x=x+40$
7. $3x-20=x-8$
8. $12-3x=45-4x$

9. $3x-6=48+x$

10. $3x+6=9-2x+12$

11. $2x-2-16=x+10$

12. $\frac{x}{3}-8=24$

13. $\frac{x}{6}+4-7=21$

14. $\frac{x}{2}+\frac{x}{3}=10+5$

15. $7x-5x=20+x+4$

16. $6x-14=16+x$

17. $2x-11+6x-60=5x+25$

18. $\frac{x}{2}+\frac{x}{3}-5=10$

19. $2x-6=16+\frac{x}{2}-\frac{x}{3}$

20. $2x+\frac{3x}{5}-\frac{x}{2}=\frac{3x}{4}+27$

859. Slate Problems.

1. The sum of three numbers is 51. The second is 5 less than the first, and the third is 10 less than the first. What are the numbers?

Let x = first number,

$x-5$ = second number,

$x-10$ = third number;

$x+x-5+x-10=51.$

Transposing $x+x+x=51+5+10,$

$3x=66,$

$x=22$, first number,

$x-5=17$, second number,

$x-10=12$, third number.

2. Add 45 to four times a number, and you will have seven times that number. What is the number?

$(7x=45+4x.)$

3. Nine times a number less 27 equals six times the number. Find the number.

4. Two boys have together 48 marbles. One has 18 more than the other. How many has each?

$(x, x+18.)$

5. The length of a rectangular lot is 75 feet more than the breadth. The distance around it is 250 feet. What are its dimensions?

6. A piece of land containing 86 acres is to be divided into two fields, one of which shall be 8 acres larger than the other. How many acres in each field?

7. At a certain election 2,436 votes were cast for two candidates, the successful one receiving 318 more votes than his opponent. How many votes did each receive?

8. A man, being asked his age, replied that if he were half as old again and 7 years more he would be 100. What was his age?

9. The sum of two numbers is 96, and their difference is 72. Find the numbers.

(Let x = less, $x + 72$ = greater.)

10. After paying $\frac{1}{3}$ and $\frac{1}{4}$ of my debts, I still owe \$45. How much did I owe originally?

11. Divide 45 into two parts, one of which shall be 6 less than twice the other.

12. William has \$5 more than John, and three times William's money added to five times John's would be \$103. How many dollars has each?

13. I bought 3 cows and 4 horses for \$635, paying \$80 apiece less for the cows than for the horses. How many dollars apiece did I pay for each?

14. Mary has a dollar in dimes and five-cent pieces. She has 11 more of the latter than of the former. Find the number of pieces of each denomination.

15. Divide 100 into two parts whose difference shall be 48.

16. In a class of 54 pupils, the girls outnumber the boys by 12. How many are there of each?

17. $18,000 is divided among three children, the second of whom receives $2,400 more than the first, and the third of whom receives $2,400 more than the second. Find the share of each.

18. The greater of two numbers is 11 more than 3 times the less. Their difference is 33. What are the numbers?

19. A boy spent a dollar for postal cards, 2-cent stamps, and 5-cent stamps. He bought 15 more 2-cent stamps than 5-cent stamps, and 15 more postal cards than 2-cent stamps. How many of each did he buy?

20. A farmer has 88 head of stock — horses, cows, and sheep. He has 17 more cows than horses, and the number of sheep is 22 greater than that of the cows and horses together. How many are there of each?

CHAPTER XI.

PERCENTAGE. — INTEREST. — DISCOUNT. — SURFACES AND VOLUMES.

PERCENTAGE.

860. Preliminary Exercises.

Per cent means hundredths. Seven per cent means seven hundredths, $\frac{7}{100}$, or .07. It is written 7%.

How many hundredths of a number is its half? $\frac{1}{2}$ = how many hundredths? $\frac{1}{5}$? $\frac{1}{10}$? $\frac{1}{25}$? $\frac{1}{50}$? $\frac{3}{4}$? $\frac{2}{5}$?

What per cent of a number is the half of it? $\frac{1}{4}$? $\frac{1}{8}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$? $\frac{1}{15}$? $\frac{1}{20}$? $\frac{1}{25}$? $\frac{1}{50}$? $\frac{1}{100}$? $\frac{1}{200}$? $\frac{1}{400}$?

861. What per cent of a number is $\frac{1}{8}$ of it? $\frac{3}{4}$? $\frac{2}{5}$? $\frac{3}{5}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{3}{10}$? $\frac{7}{10}$? $\frac{9}{10}$? $\frac{3}{20}$? $\frac{7}{20}$? $\frac{9}{20}$? $\frac{2}{25}$? $\frac{3}{25}$? $\frac{4}{25}$? $\frac{6}{25}$? $\frac{7}{25}$? $\frac{3}{50}$? $\frac{7}{50}$? $\frac{3}{100}$?

862. 1 per cent of a number is equal to what fraction of it? 2%? 3%? 4%? 5%? 6%? 7%? 8%? 9%? 10%? 12%? 15%? 20%? 25%? 30%? 36%? 40%? 50%? 60%? 75%? 80%? 90%?

863. What fractions are equal to the following:

$12\frac{1}{2}$%? $16\frac{2}{3}$%? $33\frac{1}{3}$%? $37\frac{1}{2}$%? $6\frac{1}{4}$%? $6\frac{2}{3}$%? $3\frac{1}{3}$%? $62\frac{1}{2}$%? $66\frac{2}{3}$%? $87\frac{1}{2}$%? $\frac{1}{2}$%? $\frac{1}{4}$%? $2\frac{1}{2}$%? $\frac{1}{3}$%?

864. 3 times a number is what per cent of it? $2\frac{1}{2}$ times? $1\frac{1}{4}$ times? $16\frac{1}{3}$ times? $4\frac{1}{5}$ times? $9\frac{1}{10}$ times?

865. Oral Exercises.

1. Find $12\frac{1}{2}\%$ of 1 gal.
2. $37\frac{1}{2}\%$ of \$24
3. $33\frac{1}{3}\%$ of 81 cows
4. 6% of 150 lb.
5. 4% of 125 bu.
6. $62\frac{1}{2}\%$ of 1 pk.
7. $4\frac{1}{2}\%$ of \$200
8. 99% of 200 gal.
9. $\frac{1}{2}\%$ of \$640
10. $\frac{1}{4}\%$ of 800 yd.
11. $66\frac{2}{3}\%$ of 66 horses
12. $16\frac{2}{3}\%$ of 1 yd.
13. 81% of \$300
14. $2\frac{1}{2}\%$ of 80 sheep
15. 40% of \$2.50
16. 20% of 65 rd.
17. 10% of 15 lb.
18. $3\frac{1}{3}\%$ of \$60
19. $\frac{1}{8}\%$ of \$72
20. $1\frac{1}{4}\%$ of \$96

866. Slate Exercises.

1. Find $6\frac{2}{3}\%$ of \$95.10
2. 12% of \$37.50
3. $33\frac{1}{3}\%$ of \$28.80
4. $\frac{1}{8}\%$ of \$1,240
5. $4\frac{1}{2}\%$ of \$92.40
6. 450% of \$92.40
7. 20% of \$51.60
8. $1{,}400\%$ of \$89.70
9. $12\frac{1}{2}\%$ of \$73.28
10. $13\frac{1}{3}\%$ of \$27.60
11. $6\frac{1}{4}\%$ of \$25.60
12. $3\frac{1}{3}\%$ of \$47.40
13. $5\frac{1}{2}\%$ of \$29.50
14. 860% of \$38
15. $\frac{1}{8}\%$ of \$24
16. 25% of \$52.36
17. 60% of \$33.30
18. 6% of \$19.50
19. $6\frac{2}{3}\%$ of \$47.40
20. 12% of \$62.50
21. $4\frac{1}{2}\%$ of \$71.50
22. 40% of \$28.30
23. 160% of \$39.40
24. 84% of \$23.75
25. $66\frac{2}{3}\%$ of \$825
26. 75% of \$59.20
27. $82\frac{1}{2}\%$ of \$392
28. $93\frac{3}{4}\%$ of \$4.96

TO FIND THE BASE OR THE RATE.

867. Slate Problems.

29. Find 25% of 280.

280 is called the *base*, 25 is called the *rate*.

Multiplying 280 by $\frac{25}{100}$, we get 70, which is called the *percentage;* that is,

868. $Base \times \frac{Rate}{100} = Percentage.$

30. Find x per cent of 65.

31. If x per cent of 65 is 26, find the value of x.

32. Find 25% of x.

33. If 25 per cent of x is 42, what is the value of x?

34. Increase x by 20% of itself.

35. If x increased by 20 per cent of x equals 132, what is the value of x?

36. Diminish x by $33\frac{1}{3}\%$ of itself.

37. Find the value of x, when x diminished by $33\frac{1}{3}$ per cent of itself equals 78.

38. Find $x\%$ of $\frac{2}{3}$.

39. What is the value of x, if $x\%$ of $\frac{2}{3} = 3\frac{1}{3}$?

40. Find $\frac{1}{3}\%$ of x.

41. Find the value of x, when $\frac{1}{3}\%$ of $x = 23$.

42. What (x) per cent of 65 is 26?

43. 24 is 18 per cent of what number (x)?

44. 250% of what number (x) = 180?

45. What number (x) increased by 25% of itself equals 85?

46. $\frac{3}{4}$ = what per cent of $\frac{1}{4}$?

47. $\frac{3}{4}$ is what per cent of $\frac{4}{5}$?

$\frac{3}{4} = \frac{x}{100}$ of $\frac{4}{5}$; *i.e.*, $\frac{x}{125} = \frac{3}{4}$. Clear of fractions and solve.

48. Find the interest on x dollars for 1 year at $5\frac{1}{2}\%$.

49. What sum at $5\frac{1}{2}\%$ gives a yearly interest of \$44?

50. What is $66\frac{2}{3}\%$ of $\frac{3}{5}$?

51. What per cent of 88 is 33?

52. Find the difference between $\frac{1}{8}$ of 800 and $\frac{1}{8}$ per cent of 800.

53. Find the selling price of a horse that cost \$175 and was sold at 25% profit.

54. How much insurance does a man receive for \$12.50 when the rate is $2\frac{1}{2}\%$?

55. What is 16% of $6\frac{1}{4}$?

56. $3\frac{1}{3}$ is what per cent of $\frac{2}{3}$?

57. What per cent of \$389.50 is \$124.64?

58. \$174.04 is 95% of what sum of money?

59. What number increased by 16% of itself equals 1,276?

60. 984 is $133\frac{1}{3}\%$ of what number?

61. $\frac{3}{4}\%$ of a number is 81. What is the number?

$$\frac{3}{400} \text{ of } x = \frac{3x}{400} = 81.$$

62. An importer paid duties amounting to \$386.75. If the duty was 25% of the value of the goods, what was their value?

63. A collector deducts $2\frac{1}{2}\%$ commission, and returns to his employer \$745.68. How much did he collect?

$$x - \frac{2\frac{1}{2}}{100}x = 745.68; \text{ or, } x - \frac{x}{40} = 745.68; \text{ or, } \frac{39x}{40} = 745.68.$$

64. A commission merchant receives $2\frac{1}{2}\%$ commission for buying grain for a customer. The cost of the grain and his commission amount to \$4,223. How much does the grain cost?

x = cost of the grain; $\frac{x}{40}$ = commission.

65. A capitalist sends a commission merchant \$8,670 to invest in cotton and to include commission at 2%. How much does the commission amount to?

66. A commission merchant receives \$1,071 to invest in oats after deducting 2% commission. How many bushels of oats at $30¢$ per bushel does he purchase?

Should the commission merchant deduct 2% of \$1,071, or 2% of the cost of the oats?

67. A house is insured for $\frac{2}{3}$ of its value at $\frac{7}{8}\%$. The annual cost (premium) is \$8.40. What is the value of the house?

Let x = value. Then $\frac{2x}{3} \times \frac{7}{800} = \frac{7x}{1200} = 8.40$.

68. What will be the taxes on a house worth \$48,000 and assessed at $\frac{3}{4}$ of its value, the tax rate being \$18.50 per \$1,000 of assessed value?

869. Oral Exercises.

1. 3 is what part of 6?
2. 3 is what decimal of 6?
3. 3 is how many hundredths of 6?
4. 3 is what per cent of 6?
5. 6 is what per cent of 3?
6. What number is 50% of 6?
7. 3 is 50% of what number?
8. 2 is what $\%$ of 100?
9. 2 is what $\%$ of 200?

10. What number is 5% of 100?

11. What % of 20 is 1?

12. 4 is what % of 200?

13. 3 is ½% of what number?

14. What per cent of 9 is 20¼?

15. What number, increased by ¼ of itself, equals 10?

16. What number, increased by 25% of itself, equals 20?

17. 65 diminished by 5% of itself, equals what?

18. Buying price $100, selling price $112.50. Gain %?

19. Cost $80, profit 20%. Selling price?

20. What principal will give $30 yearly interest at 6%?

21. At ⅛ of 1%, how much will I pay for insurance on $10,000?

22. If insurance costs ¾%, how much can I get for $60?

23. Bought sugar at 4¢ per lb.; sold it for 5¢. Profit %?

NOTE. — Per cent of profit is based upon the cost.

24. ⅔ is what % of ⅘?

(Reduce both to a common denominator.)

25. 1½ is what % of 6⅔?

PROFIT AND LOSS.

870. Slate Problems.

Find the profit or loss, and the selling price:

1. Cost $1,876; gain 15%.
2. Cost $36.75; loss 20%.
3. Cost $1,012.50; gain 16⅔%.
4. Cost $875; loss 5%.
5. Cost $934.56; gain 12½%.

Find the profit or loss per cent.

N. B. — Profit or loss per cent is based upon the *cost*.

6. Cost \$600; selling price \$618.

Gain or loss = x% of cost. $18 = x$% of 600.

7. Cost \$1,203; selling price \$802.

8. Cost \$86.20; selling price \$73.27.

9. Cost \$908.40; selling price \$1,090.08.

10. Cost \$84; selling price \$78.75.

11. Selling price \$78.75; loss \$5.25.

Gain or loss = x% of *cost*.

12. Selling price \$150; gain \$25.

13. Selling price \$831.25; loss \$43.75.

14. Selling price \$1,051.38; gain \$116.82.

15. Selling price \$843.75; loss \$168.75.

Find the cost, and the profit or loss:

16. Selling price \$468.75; gain 25%.

Let x = cost.

17. Selling price \$73.84; loss 20%.

18. Selling price \$1,646.08; gain $33\frac{1}{3}$%.

19. Selling price \$204; loss 15%.

20. Selling price \$66.30; gain 4%.

21. A man buys a horse for \$275, and sells it at a profit of 20 per cent. How much does he gain?

22. A cow is sold for \$75, on which the profit is \$15. What is the gain per cent?

23. A lot is sold for \$960, which is 20 per cent more than it cost. Find the cost of the lot.

24. Tea that costs 32¢ per pound is sold for 48¢. What is the gain per cent?

25. A man buys a horse for $175 and sells it for $200. What per cent does he gain?

26. What per cent is lost on a horse costing $200, and sold at a loss of $25?

27. What is the selling price of dress goods costing 33⅓¢ per yard, on which a profit of 12½ per cent is made?

28. Sold a coat for $33.60, thereby losing 16 per cent. What was its cost?

29. How much did I gain on a house for which I paid $8,760, my profit being 2½ per cent?

30. A man after spending 10 per cent of his money for clothing, 25 per cent for board, and 30 per cent for incidentals, has $70 left. What did he have at first?

31. The population of a city was 16,000 in 1880. In 1890 it was 22,000. What was the gain per cent in ten years?

(Which is the base?)

32. The population of a city was 30,000 in 1890, a gain of 6,000 over the previous census. What was the increase per cent?

(Have the correct base.)

33. 16 shares of bank stock, face (par) value $100 each, were sold at 2⅞ per cent above the face value. How much was received for the stock?

34. What will be the cost of 84 shares of stock, face (par) value $50 each, at 3½ per cent below par?

35. Find the interest on $784.50, at 4 per cent, for 3 years 7 months 15 days.

MEASUREMENTS.

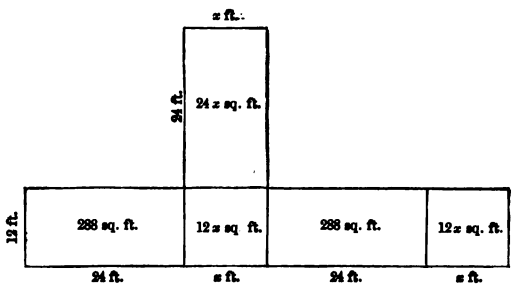
871. Slate Problems.

1. A tank 18 feet long, 15 feet wide, contains 30 cubic yards. What is its depth in feet?

$$\left(18 \times 15 \times x = 30 \times 27;\ x = \frac{30 \times 27}{18 \times 15}\right).$$

2. The surface of the walls and the ceiling of a room 24 feet long and 12 feet high is 1,440 square feet. What is the width of the room?

$$288 + 12x + 288 + 12x + 24x = 1{,}440.$$



3. A man owns a rectangular plot of ground 132 feet long, 110 feet wide. He divides it into two equal parts by a fence running from the north-east to the south-west corner. What part of an acre does each piece contain?

Indicate operations. Cancel.

4. How many yards of fence will be needed to enclose a rectangular field, 80 rods long, containing 25 acres?

5. Make a diagram of a floor 6 yards long, 4 yards wide. Show how many strips of carpet, 27 inches wide, running cross-wise, will be needed to carpet the room, no allowance being made for waste?

6. How many sheets of tin 6 inches by 4 inches will be required to cover a roof 20 feet by 30 feet, making no allowance for overlapping?

7. The owner of a lot 25 feet by 100 wishes to build around it a tight board fence 6 feet high. How many running feet of fence will there be? How many square feet?

(Draw the "development" of the fence, marking on it the dimensions.)

8. How many boards 12 feet long, 6 inches wide, will it take to build such a fence?

9. A plot of ground 300 feet by 200 feet is bought at $181.50 per acre. It is sold in lots 100 feet square at $160 each. What is the profit?

10. How many cakes of ice 4 feet long, 2 feet wide, can be cut from a rectangular pond 320 feet long, 160 feet wide, no allowance being made for waste? (Cancel.)

11. If the ice is $1\frac{1}{2}$ feet thick, how many cubic feet of ice will there be? Find its weight in tons, at 57.5 pounds per cubic foot.

12. Give the length of a shed 15 feet high, 32 feet wide, that will exactly hold the ice cut from the above pond. (Cancel.)

13. The owner of a plot of ground 640 feet long, 440 feet wide, cuts two streets, each 40 feet wide, through the middle of the plot, one running north and south, the other east and west. How many square feet of land had he originally? How many square feet has he left for building purposes? (Make diagram.)

14. What will it cost him, at 60¢ per square yard, to make the above streets?

15. If the remaining ground is divided into building lots 25 by 100, how many lots will there be?

16. How many square yards of plastering will be required for the walls and the ceiling of a room 21 feet wide, 30 feet long, 9 feet high, deducting for 2 windows, each 6 feet by 3 feet, and a door 8 feet by $4\frac{1}{2}$ feet?

PROFIT AND LOSS.

872. Slate Exercises.

1. Bought for $36; sold for $40. Gain per cent?
2. Bought for $40; sold for $36. Loss per cent?
3. Cost 36¢; selling price 40¢. Gain per cent?
4. Cost $24; gain 10%. Selling price?
5. Selling price 70¢; loss 75%. Cost?
6. Buying price 70¢; gain 75%. Selling price?
7. Cost $20; selling price $29. Gain %?
8. Cost $20; selling price $290. Gain %?
9. Cost $20; selling price $20.90. Gain %?
10. Cost $20; selling price $20.09. Gain %?
11. Selling price $300; loss $100. Loss %?
12. Cost $300; gain $100. Gain %?
13. Selling price $175; cost $150. Gain %?
14. Selling price $375; gain 25%. Profit?
15. Cost $36.50; selling price $28.50. Loss %?
16. Selling price $33.95; loss 3%. Cost?
17. Cost $75.50; loss 5½%. Selling price?
18. Selling price $20.16; gain 5%. Cost?
19. Selling price $64; profit $16. Gain %?
20. Cost $37.50; selling price $42. Gain %?
21. Selling price $26.88; loss 6⅔%. Loss?
22. Cost $24; gain 1%. Selling price?
23. Selling price $41.16; gain 5%. Cost?
24. Selling price $29.83; loss 5%. Loss?

25. Cost $19.50; loss 6%. Selling price?
26. Cost $84; selling price $184. Gain %?
27. Selling price $700; gain 250%. Cost?
28. Cost $324.80; gain 175%. Selling price?
29. Selling price $6.50; loss 13⅓%. Loss?
30. Cost $346.50; selling price $339.57. Loss %?
31. Selling price $17.64; loss 2%. Cost?
32. Cost $4,613; gain 13⅓%. Profit?
33. Selling price $26.69; gain 6¼%. Profit?
34. Cost $8,766; loss ⅞%. Loss?
35. Selling price $50.00; profit $4.20. Gain %?
36. Cost $37.50; gain $5.40. Gain %?
37. Selling price $205.20; loss $45.90. Loss %?
38. Cost $25.60; gain 6¼%. Selling price?
39. Selling price $17.35; loss ⅝%. Loss?
40. Profit $36; gain ⅓%. Cost?
41. Loss $28.17; loss $3\frac{13}{100}$%. Selling price?
42. Cost $3,864.25; loss 8%. Loss?
43. Selling price $89.37; profit $6.62. Gain %?
44. Cost $8⅔; loss $⅜. Loss %?
45. Selling price $22.35; gain 6⅔%. Cost?
46. Profit $47.25; gain 7½%. Selling price?
47. Loss $38.46; loss ½%. Cost?
48. Cost $75.52; gain 3⅛%. Profit?
49. Selling price $49.95; loss $4.05. Loss %?
50. Cost $3,879; loss ⅝%. Loss?

INTEREST.

873. In calculating interest, take 30 days to a month, 12 months to a year.

874. $Principal \times \frac{Rate}{100} \times Time\ (in\ years) = Interest.$

NOTE. — Change given time to years.

2 yr. 6 mo. $= 2\frac{1}{2}$ yr. $= \frac{5}{2}$ yr.

1 yr. 7 mo. $= 19$ mo. $= \frac{19}{12}$ yr.

4 mo. 10 da. $= 4\frac{1}{3}$ mo. $= \frac{4\frac{1}{3}}{12}$ yr. $= \frac{1}{36}$ yr.

1 yr. 5 mo. 15 da. $= 17\frac{1}{2}$ mo. $= \frac{17\frac{1}{2}}{12}$ yr. $= \frac{35}{24}$ yr.

5 mo. 17 da. $= 167$ da. $= \frac{167}{360}$ yr.

875. Slate Exercises.

Find the interest on:

1. $750, for $2\frac{1}{2}$ years, at 6%.
2. $84.75, $3\frac{1}{3}$ months, at 4%.
3. $308.25, from Oct. 1 to Oct. 21, at 5%.
4. $464.75, 8 mo. 12 da., at 6%.
5. $360, 33 da., at 5%.
6. $94.43, 2 mo. 3 da., at 7%.
7. $400, 1 yr. 1 mo. 1 da., at $4\frac{1}{2}$%.
8. $720, 21 da., 7%.
9. $1,000, 8 da., 5%.
10. $630, from April 1, 1890, to Jan. 16, 1892, at 6%.

$$\begin{array}{r} 1892-1-16 \\ 1890-4-\ 1 \\ \hline \end{array}$$

11. $394.50, 2 yr. 1 mo. 7 da., at 6%.
12. $1,560, 3 yr. 4 mo. 9 da., at $4\frac{1}{2}$%.

13. $960, 11 mo. 24 da., 2%.

14. $86.40, 1 yr. 9 mo. 20 da., 5%.

15. $108.36, 4 yr. 7 mo. 10 da., $3\frac{3}{4}$%.

876. *Amount = Principal + Interest.*

Find the amount:

16. $813, from April 19, 1889, to March 4, 1894, at 6%.

17. $960, from Jan 1, 1893, to Dec. 21, 1894, at 4%.

18. $27.84, for 3 yr. 6 mo. 9 da., at 6%.

19. $48.90, for 17 da., at 6%.

20. $144, for 2 yr. 5 da., at $3\frac{3}{4}$%.

21. $834.76, for 15 mo. 27 da., at $4\frac{1}{2}$%.

22. $5,760, for 1 yr. 5 mo. 29 da., at 5%.

23. $2,346.50, for 7 yr. 13 da., at 3%.

24. $1,892, for 3 yr. 5 mo., at 7%.

25. $150.40, for 1 yr. 2 mo. 3 da., at 6%.

877. Interest-bearing Demand Notes.

26. SAN FRANCISCO, Jan. 7, 1893.

On demand, I promise to pay William Britt, or order, Seven Hundred Sixty-five $\frac{40}{100}$ Dollars, value received, with interest at 6 per cent.

$765\frac{40}{100}$. ARTHUR TOWNSEND.

How much money will be required to pay the above note, with interest, July 15, 1894?

27. A demand note, dated Sept. 25, 1892, with interest at 8% from date, is paid Jan. 2, 1895. How much was due, the face of the note being $750?

28. Find the amount due March 4, 1894, on a note for $365.84, dated May 20, 1892, with interest from date at 7%.

29. Find the amount necessary, Oct. 16, 1896, to pay a note of $1,240, with interest at 6% from Aug. 15, 1892.

30. An interest-bearing note for $87.60 is dated April 3, 1886. How much is due on it for principal and interest Jan. 2, 1894? Rate 4½%.

878. Oral Problems.

1. Find the interest on $300, for 1 year 7 months, at 4%.

$12 per year is how much for 7 months?

2. On $60, for 33 days, at 6%.

$3.60 for 360 days is how much for 33 days?

3. On $120, from Jan. 1, 1893, to July 1, 1894, at 5%.

4. How long will it take $100 to produce $15 interest at 6%?

5. At what rate per cent will $50 produce $6 in 2 years?

6. What is the interest on $300, at 6%, from Feb. 1 to Feb. 21?

7. What part of a year is 72 days?

8. Find the interest at 4%, for 90 days, on $150.

9. On $240, for 36 days, at 5%.

10. What is the amount of $200, for 3 years 1 month, at 6%?

11. How long will it take $1 to make $1 interest at 5%?

12. How long will it take any sum to double itself at 6%?

13. How long will it take $14.90 to double itself at 4%?

14. At ½ per cent per month, find the interest on $90 for 16 months.

15. 5% per year is 1% for how many days?

16. 4½% per year is 1% for how many days?

17. Find the interest on $75, at 5%, for 72 days.

SPECIAL DRILLS.

884. Give sums:

$425 + 99$	$999 + 425$	$\$2.63 + \$6.37$	$\$5.45 + \$9.99$
$99 + 576$	$576 + 999$	$\$4.56 + \$2.84$	$\$9.99 + \$6.73$
$685 + 99$	$999 + 685$	$\$6.49 + \$3.12$	$\$12.68 + \$0.99$
$99 + 599$	$599 + 999$	$\$3.58 + \$5.67$	$\$0.99 + \$13.33$

885. Give differences:

$565 - 99$	$1{,}424 - 999$	$\$7.00 - \$2.63$	$\$15.44 - \$9.99$
$488 - 99$	$1{,}575 - 999$	$\$6.40 - \$3.56$	$\$9.44 - \$6.45$
$794 - 99$	$1{,}684 - 999$	$\$9.61 - \$4.49$	$\$7.88 - \$4.89$
$898 - 99$	$1{,}598 - 999$	$\$8.15 - \$5.58$	$\$9.53 - \$2.99$

886. Give products:

24×21	21×31	41×41	$19 \times 37\frac{1}{2}$
33×21	32×31	32×41	$18 \times 62\frac{1}{2}$
42×21	43×31	21×41	$22 \times 66\frac{2}{3}$
51×21	31×31	42×41	33×75

887. Give quotients:

$16 \div .25$	$228 \div 19$	$24 \div .33\frac{1}{3}$	$656 \div 16$
$18 \div .25$	$234 \div 18$	$36 \div .33\frac{1}{3}$	$544 \div 17$
$24 \div .75$	$238 \div 17$	$16 \div .66\frac{2}{3}$	$558 \div 18$
$36 \div .75$	$336 \div 16$	$66 \div .66\frac{2}{3}$	$418 \div 19$

888. Give answers:

$14\frac{7}{8} \times 7$	$13\frac{1}{2} \times 8$	$13 \times 13\frac{1}{2}$	$20 \times 8\frac{3}{8}$
$15\frac{3}{8} \times 6$	$21\frac{5}{6} \times 4$	$14 \times 12\frac{4}{9}$	$21 \times 7\frac{1}{4}$
$5\frac{5}{9} \div \frac{5}{9}$	$7\frac{1}{2} \div \frac{3}{4}$	$15 \div 3\frac{1}{3}$	$63 \div 3\frac{1}{2}$
$14\frac{3}{8} \div 1\frac{1}{8}$	$6\frac{2}{3} \div \frac{5}{6}$	$20 \div 3\frac{3}{4}$	$64 \div 5\frac{1}{8}$

889. Oral Problems.

1. What will be the cost of 48 yards of cloth at $87\frac{1}{2}¢$ per yard?

2. How many square yards in a piece of carpet 48 yards long, 27 inches wide?

3. How many yards of carpet 27 inches wide will be needed to cover a floor containing 48 square yards?

4. Paid $3.45 for groceries, $1.50 for dry goods, and 99¢ for sundries. What is the total?

5. From a chest containing $25\frac{1}{4}$ pounds of tea, $8\frac{1}{2}$ pounds were sold. How many pounds remain?

6. At $37\frac{1}{2}¢$ per peck, what will I receive for 4 bushels of potatoes?

7. $83\frac{1}{4}$ yards of cloth are divided into 9 pieces. How many yards are there in each piece?

8. I buy hardware to the amount of $6.37. I give the storekeeper two $5-bills. How much change should I receive?

9. What will be the cost of 24 yards of calico at $4\frac{3}{4}¢$ per yd.?

10. What will I have to pay for 19 base-balls at $1.25 each?

11. At $1.87\frac{1}{2}$ per yard, what will be the cost of 120 yards of silk?

12. For $120, how many yards of silk can I buy at $1.87\frac{1}{2}$ per yard?

13. What will be the cost of a ton of hay at $97\frac{1}{2}¢$ per cwt.?

14. A square field requires 320 rods of fence. How many square rods are there in the field?

15. How many acres are 6,400 sq. rd.?

16. At $4\frac{3}{4}¢$ per yd., how many yards of calico can I buy for 95¢?

17. If slate pencils cost 2 mills each, how many will I receive for $4?

18. At $5.00 per ton, how many pounds of coal can be bought for 1¢?

19. Find the cost of 3 T. 480 lb. coal at $5 per ton.

20. At $5 per ton, how many tons and pounds of coal can I buy for $10.80?

21. How many square yards are there in a field 41 yards long, 42 yards wide?

22. If I pay 15¢ for $3\frac{1}{3}$ yards of muslin, what is the price per yard?

23. How many acres of land are there in two farms containing, respectively, 347 and 495 acres?

24. At $87\frac{1}{2}$¢ each, how many base-balls can be bought for $56?

25. If one man can do a piece of work in 24 days, and another man can do it in 48 days, how long will it take both, working together?

APPROXIMATIONS.

890. Give approximate answers at sight (Art. 521):

1. Find the interest of $150, at 4%, from Jan. 1, 1893, to Dec. 30, 1895. (Nearly 3 years.)

2. What is the weight, at $57\frac{1}{2}$ lb. per cu. ft., of a cake of ice 4 ft. by 2 ft. by $1\frac{1}{2}$ ft.? (Nearly 60 lb. per cu. ft.)

3. Find the amount of goods sold, the commission at $2\frac{7}{8}$% being $11.75. (About 3%.)

4. What % of 497 is 249?

5. What % of $31\frac{5}{16}$ is $11\frac{21}{32}$?

6. Cost of 19,987 ft. boards at $30.05 per M.?

7. How much will be paid for 4 bbl. sugar, each containing 299 lb., @ $5\frac{1}{16}$¢ per lb.?

8. $18.0327 \div 4.5026$.

9. $83\frac{19}{20} \div 3\frac{29}{30}$.

10. 74 A. 155 sq. rd. land at $79 per A.?

SHORT METHODS.

891. Slate Exercises.

$7{,}854 \times \frac{3}{4}$	$9{,}365 \times \frac{7}{8}$
$1{,}963\frac{1}{2}$ Deduct $\frac{1}{4}$.	$1{,}170\frac{5}{8}$ Deduct $\frac{1}{8}$.
$5{,}890\frac{1}{2}$ *Ans.*	$8{,}194\frac{3}{8}$ *Ans.*

Multiply 6,578 by $9\frac{3}{8}$.

$65{,}780$ = 10 times number.
$2{,}192\frac{3}{8}$ = $\frac{1}{8}$ number (Deduct).
$63{,}587\frac{1}{8}$ *Ans.*

892. Find products:

1.	$176 \times \frac{15}{16}$	11.	$4{,}844 \times 9\frac{1}{2}$
2.	$273 \times \frac{12}{13}$	12.	$8{,}960 \times 8\frac{7}{8}$
3.	$4{,}554 \times \frac{8}{9}$	13.	$3{,}245 \times 7\frac{7}{8}$
4.	$1{,}001 \times \frac{10}{11}$	14.	$9{,}060 \times 11\frac{4}{5}$
5.	$3{,}243 \times \frac{4}{5}$	15.	$658 \times 99\frac{1}{2}$
6.	$6{,}776 \times \frac{5}{6}$	16.	$658 \times 99\frac{3}{4}$
7.	$2{,}307 \times \frac{6}{7}$	17.	$725 \times 119\frac{4}{5}$
8.	$7{,}284 \times \frac{7}{8}$	18.	$347 \times 79\frac{5}{6}$
9.	$5{,}631 \times \frac{9}{10}$	19.	$418 \times 89\frac{1}{2}$
10.	$9{,}657 \times \frac{11}{12}$	20.	$543 \times 49\frac{2}{3}$

893. Multiply:

21.	418×99	26.	$724 \times 36 \times 45$
22.	$674 \times 87\frac{1}{2}$	27.	$484 \times \frac{10}{11} \times 9\frac{3}{4}$
23.	$36 \times 999 \times 25$	28.	$576 \times 9\frac{11}{12} \times 12\frac{1}{2}$
24.	$48 \times 125 \times 71$	29.	$95 \times 36 \times 19\frac{4}{5}$
25.	$64 \times 7\frac{7}{8} \times 33\frac{1}{3}$	30.	$74 \times 31 \times 13 \times 9\frac{5}{6}$

895. Divide. Do not write products (Arts. 385, 616):

1. 611,463,874 ÷ 87,659
2. 279,864,597 ÷ 45,678
3. 387,250,005 ÷ 34,567
4. 800,700,900 ÷ 68,439
5. 453,211,687 ÷ 576,258
6. 703,205,104 ÷ 71,685
7. 923,812,701 ÷ 18,789
8. 575,646,828 ÷ 59,764
9. 1,234,567,890 ÷ 169,375
10. 3,126,045,000 ÷ 483,729

896. Write answers (Art. 385):

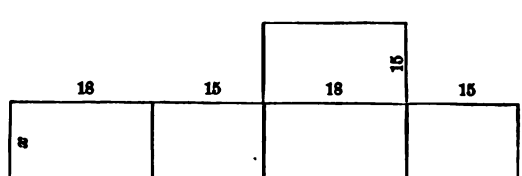
11. $\frac{876,459}{94,317}$
12. $\frac{763,154}{82,915}$
13. $\frac{654,817}{73,295}$
14. $\frac{512,000}{69,999}$
15. $\frac{453,124}{123,456}$
16. $\frac{375,005}{59,687}$
17. $\frac{208,040}{17,613}$
18. $\frac{963,018}{126,748}$
19. $\frac{862,304}{87,925}$

MEASUREMENTS.

897. Slate Problems.

1. How many boards 16 ft. long, 8 in. wide, will be required for a tight fence 8 ft. high, around a piece of ground 240 ft. long, 180 ft. wide? How many posts, 6 ft. apart, will be needed?

2. A room is 18 ft. long, 15 ft. wide. The walls and the ceiling contain 930 sq. ft. What is the height of the room?



3. What will it cost to cover a table 6 ft. long, $2\frac{1}{4}$ ft. wide, with baize $\frac{3}{4}$ yd. wide, at 75¢ per yd.?

4. 160 square rods make 1 acre. How many square yards are there in an acre? About how many yards long is a square field containing 1 acre?

5. A 40-acre field is 160 rods long. How many rods of fence are needed to enclose the field?

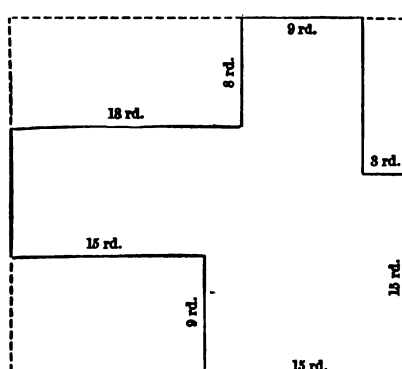
6. A room 30 ft. long, 24 ft. wide, 15 ft. high, contains 40 persons. How many square feet of floor space are there for each occupant? How many cubic feet of air space are there for each?

7. How many yards of carpet, 27 inches wide, would be needed for the floor of such a room?

8. How many bundles of laths, each bundle covering 3 square yards, would be needed for the walls and ceiling of the above room, no allowance being made for doors and windows?

9. A farmer owned a rectangular piece of ground 30 rods long, 27 rods wide. He sold three lots 18×8 rods, 12×3 rods, 15×9 rods.

Find the number of square rods in the original piece. Mark in the diagram the area of each lot sold, and the area of the part remaining.



10. How many rods of fence will be needed to enclose the part remaining?

898. 1 gallon = 231 cubic inches.
1 bushel = 2,150.4 cubic inches.
1 cord = 128 cubic feet.

11. How many gallons will fill a tank 22 feet long, 14 feet wide, 9 feet deep?

Indicate operations. Cancel where possible.

12. How many cords of wood are there in a pile 48 feet long, 16 feet wide, 12 feet high?

BANK DISCOUNT.

900. Wm. Brown and Sons receive the following note in settlement of their account with Thomas Tierney:

ST. PAUL, May 30, 1893.

Thirty days after date, I promise to pay to the order of Wm. Brown and Sons, Three Hundred Fifty-Four $\frac{75}{100}$ Dollars, value received, at the Park National Bank.

$354\frac{75}{100}$. THOMAS TIERNEY.

901. This money is payable 33 days after May 30, 3 days of grace being allowed. If Wm. Brown and Sons desire to use the money at once, they may have the note discounted at a bank. In this case, the bank deducts from the face of the note ($354.75), the interest thereon for 33 days, and pays over the difference (*proceeds*).

Face of note $354.75
Discount (Int. for 33 da.) 1.95 (at 6%)
Proceeds $352.80

902. Slate Exercises.

Find the discount at 6% on the following, allowing 3 days of grace in each case. (See Appendix, section 1305.)

1. A 30-days note for $75.
2. 15-days note for $183.60.
3. 60-days note for $275.40.
4. 20-days note for $96.
5. 4-months note for $336.

Find the proceeds, at 7%, on

6. A 6-months note for $180.

7. A 3-months note for $36.90.

8. A 24-days note for $795.60.

9. A 90-days note for $180.

10. A 72-days note for $1,000.

903. In computing the discount on a note, the banks ascertain the exact number of days.

A 3-months note, dated February 1, is payable three days after May 1, which is May 4. The discount is taken for $27+31+30+4=92$ days. A 90-days note of the same date is payable 93 days after February 1, which is May 5.

The year, however, is considered to contain 360 days; the interest in the first case being taken for $\frac{92}{360}$ of a year, and in the latter for $\frac{93}{360}$ of a year.

Find the discount, at 6%, on

11. A 1-month note for $600, dated Feb. 6, 1894. $(\frac{31}{360}.)$

12. A 2-months note for $240, dated July 17, 1894. $(\frac{65}{360}.)$

13. A 3-months note for $360, dated April 8, 1894.

14. A 4-months note for $84, dated Dec. 24, 1894.

15. A 6-months note for $172.60, dated March 4, 1895.

16. A 60-days note for $240, dated July 17, 1894.

17. A 90-days note for $360, dated April 8, 1894.

904. In each of the preceding examples, it has been assumed that the note has been presented for discount the day on which it was made.

In some of the following examples, the notes are discounted at a later date, and the *term of discount* is to be ascertained; that is, the time between the date of discount and that of maturity (including the days of grace).

The term of discount of a 30-days note dated May 1, and discounted May 19, is 15 days.

905. In the following examples, find (*a*) date of maturity; (*b*) term of discount; (*c*) discount; (*d*) proceeds:

	Dated.	Face.	Time.	Discounted.	Rate.
18.	June 16, 1894;	$87.60;	30 days;	July 1, 1894;	6%.
19.	Sept. 9, 1894;	$124.18;	4 months;	Nov. 18, 1894;	8%.
20.	Dec. 5, 1894;	$504.60;	30 days;	Dec. 12, 1894;	7%.
21.	Nov. 14, 1894;	$72.36;	3 months;	Dec. 20, 1894;	6%.
22.	Oct. 30, 1894;	$234;	90 days;	Jan. 5, 1895;	6%.
23.	Jan. 2, 1895;	$95.90;	2 months;	Feb. 13, 1895;	6%.
24.	Aug. 5, 1895;	$164;	60 days;	Aug. 31, 1895;	8%.
25.	Feb. 27, 1895;	$83.20;	100 days;	March 9, 1895;	6%.

INTEREST.

906. Find interest:

When the time is less than one year, ascertain the exact number of days. When greater than a year, find the time by compound subtraction, taking the month of 30 days.

1. $160; Jan. 2, 1893, to May 16, 1896; $4\frac{1}{2}\%$.

   3 yr. 4 mo. 14 da. $= 3\frac{134}{360}$ yr. $= 3\frac{67}{180}$ yr. $= \frac{607}{180}$ yr.

2. $342.18; April 5, 1895, to Sept. 30, 1895; 6%.

   Time, 178 days $= \frac{178}{360}$ year.

3. $59.80; Feb. 24, 1896, to Dec. 24, 1896; 5%.

   (Leap year.)

4. $1,234.56; Aug. 3, 1890, to Jan. 1, 1896; $5\frac{1}{2}\%$.

5. $387.90; March 15, 1894, to Sept. 1, 1894; 6%.

6. $96; July 6, 1894, to Feb. 4, 1895; $3\frac{3}{4}\%$.

7. $240.72; May 20, 1893, to Jan. 15, 1894; 7%.

8. $983.25; Dec. 15, 1899, to March 3, 1900; 6%.

   How many days in February, 1900? Be *sure* you are right.

SHORT METHODS.

908. When multiplying mixed numbers, many accountants prefer not to reduce them to improper fractions.

909. Multiply without reducing:

1. $43\frac{3}{4} \times 8\frac{1}{2}$

$$\begin{array}{r} 350 \\ 21\frac{7}{8} \\ \hline \end{array}$$

2. $57\frac{1}{8} \times 9\frac{1}{4}$

$$\begin{array}{r} 516 \\ 14\frac{1}{8} \\ \hline \end{array}$$

3. $17\frac{3}{5} \times 7\frac{1}{5}$

4. $24\frac{7}{8} \times 5\frac{1}{6}$

5. $42\frac{2}{3} \times 12\frac{1}{8}$

6. $35\frac{5}{16} \times 8\frac{1}{5}$

7. $56\frac{3}{7} \times 4\frac{1}{7}$

8. $83\frac{3}{8} \times 3\frac{1}{8}$

9. $72\frac{1}{4} \times 10\frac{1}{6}$

10. $95\frac{5}{6} \times 11\frac{1}{4}$

910.

$$\begin{array}{r} 76\frac{1}{2} \times 9\frac{5}{6} \\ \hline 765 \quad \text{product by } 10 \\ 12\frac{3}{4} \quad \text{less product by } \frac{1}{6} \\ \hline 752\frac{1}{4} \quad \textit{Ans.} \qquad 9\frac{5}{6} \end{array}$$

$$\begin{array}{r} 64\frac{1}{2} \times 11\frac{5}{7} \\ \hline 709\frac{1}{2} \quad \text{product by } 11 \\ 9\frac{3}{14} \quad \text{product by } \frac{1}{7} \\ 36\frac{1}{14} \quad \text{product by } \frac{4}{7} \\ \hline 755\frac{4}{7} \quad \textit{Ans.} \qquad 11\frac{5}{7} \end{array}$$

$$\begin{array}{r} 84\frac{3}{5} \times 5\frac{2}{5} \\ \hline 421\frac{1}{5} \quad \text{product by } 5 \\ 16\frac{7}{8} \quad \text{product by } \frac{1}{5} \\ 16\frac{7}{8} \quad \text{product by } \frac{1}{5} \\ \hline 455\frac{3}{5} \quad \textit{Ans.} \qquad 5\frac{2}{5} \end{array}$$

911. Multiply:

11. $19\frac{3}{5} \times 4\frac{2}{5}$ $(4 + \frac{1}{5} + \frac{1}{5})$

12. $24\frac{1}{2} \times 3\frac{2}{5}$ $(4 - \frac{1}{5})$

13. $35\frac{3}{5} \times 7\frac{3}{7}$ $(7 + \frac{1}{7} + \frac{2}{7})$

14. $40\frac{2}{3} \times 8\frac{2}{3}$ $(9 - \frac{1}{3})$

15. $51\frac{1}{3} \times 6\frac{2}{3}$

16. $63\frac{1}{2} \times 10\frac{5}{8}$

17. $29\frac{3}{8} \times 2\frac{3}{4}$

18. $31\frac{1}{4} \times 12\frac{5}{6}$

19. $42\frac{3}{5} \times 9\frac{9}{10}$

20. $56\frac{5}{8} \times 7\frac{2}{3}$

REVIEW.

915. Slate Problems.

1 pound troy = 5,760 grains.
1 pound apothecaries' = 5,760 grains.
1 pound avoirdupois = 7,000 grains.

How many grains in a troy ounce? In an avoirdupois ounce?

1. Find the value of a dozen silver spoons, each weighing 3 oz. 5 pwt., at $1.20 per oz. (See table, Art. 828.)

2. A gold chain weighs 384 grains. What is its cost at $1.15 per pwt.?

3. Add 4 lb. 6 oz. 18 gr., 5 oz. 9 pwt., 3 lb. 20 gr., and 9 lb. 11 oz. 15 pwt. 5 gr.

4. Gold coin contains 90 per cent gold, 9 per cent silver, 1 per cent copper. Find the quantity of each metal in 50 double-eagles ($20), each containing 516 grains.

5. How many spoons, each weighing 2 oz. 18 pwt., can be made from 5 lb. 9 oz. 12 pwt. silver?

6. How much money in silver dollars, 412½ grains each, will weigh 165 lb. avoirdupois (7,000 grains)?

7. What fraction of a pound avoirdupois is a pound troy? What per cent of an ounce avoirdupois is a troy ounce?

8. What is the value, at $1.60 per oz. troy, of a silver pitcher weighing 4 lb. 8 oz. avoirdupois?

9. At 75¢ per ounce, what is the value of the silver contained in a half-dollar, which weighs 192.9 grains, $\frac{9}{10}$ being pure silver?

10. What is the capacity, in gallons (231 cu. in.), of a barrel that will contain 2½ bu. (2,150.4 cu. in.)?

11. A tank 16 ft. long, 14 ft. wide, 8 ft. deep, is lined with lead on the bottom and sides. How many square feet of sheet lead will be required? (Draw the "development" of the tank.)

12. A man sold 18 bbl. sugar, each containing 306 lb.; 21 bbl., each containing 297 lb.; 5 bbl., each containing 291 lb. What is the average weight per barrel?

13. A, B, and C buy a farm. A pays $8,750, B pays $7,200, C pays $4,100. What per cent of the purchase money does each furnish?

14. If 11 weavers in 9 days weave 1,584 yards, what will 1 man do in 1 day? 6 men in 7 days?

15. The tax rate of a certain city is $1\frac{3}{4}\%$ upon the assessed value of property. If this value is 75% of the actual value, how much taxes does Mr. Smith pay upon a house and lot, the actual value of which is $24,000?

16. What per cent of a lb. avoirdupois is a troy pound?

DISCOUNT OF INTEREST-BEARING NOTES.

919. Slate Problems.

BROOKLYN, N. Y., Oct. 15, 1894.

Sixty days after date I promise to pay to the order of Harman P. Payne, Forty-eight $\frac{50}{100}$ Dollars, value received, with interest at 6%.

$48\frac{50}{100}$. GEORGE P. POST.

1. Find the amount due on the above note at maturity.

If the above note is discounted at a bank, the discount is taken on the amount due at maturity.

Find the proceeds of the above note if discounted Dec. 1, 1894, at 6%.

2. Find the proceeds of a 90-days note for $175, bearing interest at 6%, discounted 33 days after date, at 6%.

3. Find the proceeds of a 60-days note for $350, bearing interest at 6%, discounted at 6%, 10 days after date.

4. Find the proceeds of a three-months note for $840, bearing interest at 7%, discounted at bank 47 days before maturity, at 8%.

5. A four-months note for $720, dated March 17, 1894, bearing interest at 6%, is discounted at 7%, May 10. What are the proceeds?

REVIEW.

920. Slate Problems.

1. Change 7,643 inches to rods, etc.

2. Change 1,875 feet to rods, etc.

3. Change 964 yards to rods, etc.

921. English Money.

12 pence (*d.*)	1 shilling (*s.*)
20 shillings	1 pound (£)

A farthing is one-fourth of a penny, and is generally written as a fraction.

4. Reduce 4,000*d.* to £, *s.*, *d.*

5. How many pence in £87 17*s.* 6*d.*?

6. What will be the cost of 150 yd. silk at 3/6 per yd.?

3/6 = 3*s.* 6*d.*, read three and sixpence.

7. If £1 = $4.8665, what will be the cost in U. S. money of 75 books at 18 pence each?

8. A merchant sells 37 coats at £3 5*s.* each, less 10%. What is the amount of his bill in English money?

9. Find 25% of £183 14*s.* 8*d.*

10. A silver dollar weighs 412½ grains. How many ounces of pure silver are there in 1,000 silver dollars if the coin is $\frac{9}{10}$ pure silver?

11. The one-cent pieces weigh 48 grains. How many dollars would weigh 120 pounds avoirdupois (7,000 grains to pound)?

12. A coal dealer buys 150 tons of coal, 2,240 pounds each, at \$4.50 per ton. He sells it at \$4.75 per ton, giving 2,000 lb. to the ton. What is his profit?

13. Three workmen receive \$283.50 for doing a piece of work. One worked 32 days, the second worked 53 days, the third worked 41 days. What is the share of each?

14. Three men engage in a business venture. One furnishes \$3,000, another furnishes \$5,000, a third furnishes \$4,000. They gain \$1,800. What is each one's share of the profit?

15. A garrison of 1,200 men has rations for 40 days. Ten days later it receives a reinforcement of 300 men. How many more days will the rations last?

16. Two trains start at 9 o'clock from towns A and B, 120 miles apart. The train leaving A travels 20 miles per hour, the other 30 miles per hour. What time do they meet, and at what distance from A?

17. Divide \$900 among four persons, so that the second will have three times as much as the first; the third, twice as much as the second; the fourth, as much as the three others.

18. Divide \$540 among three persons, so that the first will have \$48 more than the second, and the second \$75 more than the third.

19. Four men working 7 hours a day need 15 days (105 hours) for a piece of work. How many days would it take 6 men, working 10 hours per day, to do the same work?

20. A grocer buys some eggs at 15¢ per dozen. He breaks 15, and then finds that by selling the rest at 16¢ per dozen, he will neither gain nor lose. How many eggs did he buy?

21. If a person lends me \$250 for 8 months, for how long ought I to lend him \$400 as an equivalent?

22. A fort had a garrison of 4,000 men, and provisions for 18 weeks. If 1,000 men were sent away, how long would the provisions last?

COMMERCIAL DISCOUNT.

924. Slate Problems.

1. On a bill of goods amounting to $583.40, a discount of 5% is given for cash. What is the amount paid?

2. What will be the cost of 16 gross of Roman candles at $26.75 per gross, less 60%?

3. Sept. 1, 1894, I bought tea amounting to $1,876.50. If 5% is deducted for payment within ten days, how much would I have to pay if I paid the bill Sept. 9?

4. What will be the cost of 15 cases cocoa @ $13.20 each, less 20%?

5. Bought 5 gross of essence of lemon at 50¢ per doz., less 5%. What is the amount of my bill?

6. Find the cost of 15 cases of chloride of lime, 50 lb. per case, at $9\frac{1}{2}$¢ per lb., less 15%.

7. What will be the net cost of a bill of plated ware amounting to $84.75, on which a discount of $33\frac{1}{3}$ and 10% is allowed?

	$84.75
less $\frac{1}{3}$	28.25
	56.50
less $\frac{1}{10}$	5.65
Ans. $	net.

This means $33\frac{1}{3}$% discount on the gross amount, and 10% discount on the remainder.

8. Find the difference between $390 less $43\frac{1}{3}$% discount, and $390 less $33\frac{1}{3}$ and 10% discount.

9. Thurber, Whyland Co. sold the following goods. Make out the bill, less 50 and 10 and 10 and 10% discount.

500	$\frac{1}{4}$-pound	bags at	$1.00	per M.
1,500	$\frac{1}{2}$-pound	" "	1.20	" "
3,000	1-pound	" "	1.60	" "
5,500	$1\frac{1}{2}$-pound	" "	1.70	" "
2,000	2-pound	" "	2.00	" "

10. Find the net cost of 18,500 bags at $4.40 per M. less 60 and 10 and 5%.

11. Which is the better discount for the buyer, 40 and 10% or 30 and 20%? What will be the difference on a bill of $100?

12. $100 less 33⅓ and 10% discount is equal to what? What per cent discount is 33⅓ and 10% equal to? To what per cent net is it equivalent?

926. Oral Problems.

1. A piano, marked $800, is sold at a discount of 25 and 10%. What is the selling price?

2. Bought goods amounting to $600, less 5% for cash. What is the net cost of the goods?

3. What single discount is 50 and 10% equal to?

4. What single discount is 30 and 30% equal to?

5. Paid $729 for goods, on which 10% was allowed. What was the "gross" price?

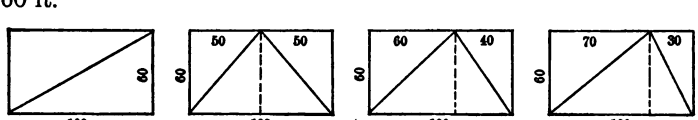
6. How much will I have to pay for 12 doz. bottles flavoring extract, at 60¢ per doz., less 10%?

7. What is the "list" price of an article for which I paid $48, after a discount of 25% was deducted?

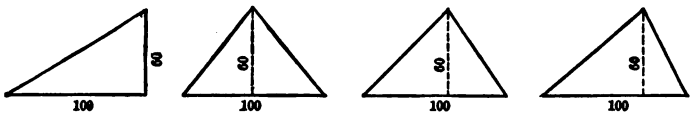
MEASUREMENTS.

929. Slate Problems.

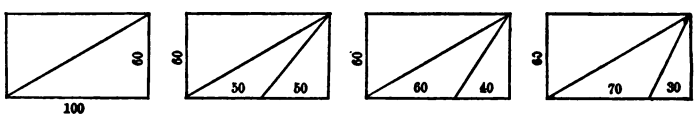
1. Mark in each division of each of the following rectangles its area in square feet. Dimensions of each rectangle 100 ft. by 60 ft.



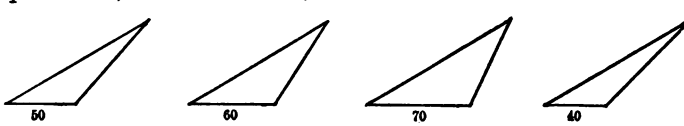
2. Find the area of each of the following triangles in square feet; base of each 100 feet, altitude 60 feet.



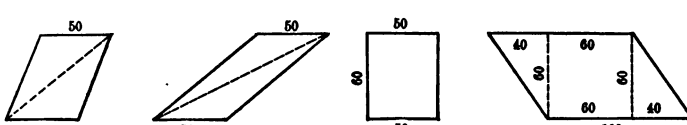
3. Mark in each of the following eleven triangles its area in square feet; altitude of each, 60 ft.



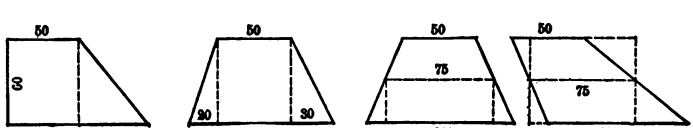
4. Find the area of each of the following four triangles in square feet; altitude of each, 60 ft.



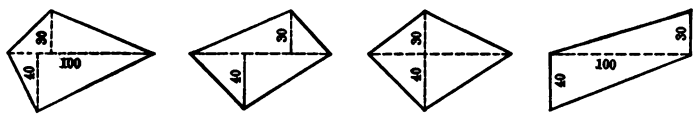
5. Find the area of each of the following four parallelograms; altitude of each, 60 ft.



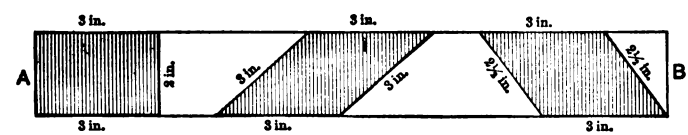
6. Find the area of each of the following trapezoids; altitude of each, 60 ft. The number of square feet in each is equal to 60 multiplied by what?



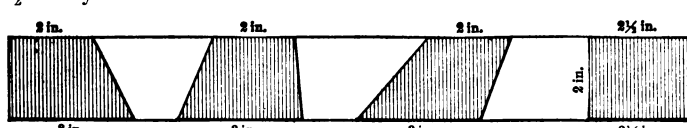
7. One diagonal of each of the following quadrilaterals measures 100 feet. The perpendiculars let fall on this diagonal from the opposite corners measure 30 ft. and 40 ft., respectively. Find the area of each in square feet.



8. Cut from a strip of paper, AB, two inches wide, a rectangle, a rhombus, and a rhomboid, as given in the accompanying diagram. Show that the three parallelograms are equal in surface.



9. Cut from a strip of paper, two inches wide, three trapezoids. Make one parallel side of each 2 inches long, and the other parallel side 3 inches long. Divide up each trapezoid in such a way as to show that its surface is equal to that of a rectangle $2\frac{1}{2}$ in. by 2 in.



SURFACES AND VOLUMES.

930. Slate Problems.

1. How many cubic feet are there in a block of granite whose base is $4\frac{1}{2}$ feet square, and whose height is 6 feet?

2. Find the value of the above block at 60¢ per cubic foot, and the cost of polishing its six faces at 60¢ per square foot.

3. A contractor charged 30¢ per cubic yard for digging a cellar 21 feet wide, 60 feet long, 9 feet deep. What was the amount of his bill?

4. What will be the cost of cementing the floor of the above cellar at 75¢ per square yard? Deduct from the given dimensions the thickness of the four walls, eighteen inches each. Make a diagram.

5. A cubic foot of water weighs 1,000 ounces. Marble is 2.8 times as heavy. Calculate the weight, in tons, of a marble shaft 4 feet square and 12 feet high. (Cancel.)

6. A carpenter is making a cubical box whose inside measurement is 1 foot. Each side consists of a single piece of wood 1 inch thick. Give the dimensions, in inches, of each of the six pieces used.

7. How many pounds avoirdupois would there be in a brick of pure gold $8 \times 4 \times 2$ inches, gold being 19.4 times as heavy as water?

8. Find the weight in pounds of a cord of pine wood, which is .66 times as heavy as water.

9. A cube of marble, 12 inches on a side, is enclosed for transportation in a tightly fitting wooden box made of material one inch thick. What are the outside dimensions of the box? How many cubic inches in the wood and the marble together? How many cubic inches are there of each?

10. An iron cube 2 feet long weighs 2 tons. How many times as heavy as water is iron? Calculate the weight of an iron cube 1 foot long. Of one 3 feet long.

CHAPTER XII.

SIMPLE AND COMPOUND INTEREST. — DISCOUNT. — CAUSE AND EFFECT. — PARTNERSHIP. — BONDS AND STOCKS. — EXCHANGE. — LONGITUDE AND TIME. — SURFACES AND VOLUMES.

TO FIND PRINCIPAL, RATE, OR TIME.

931. At what rate per cent will \$723.60 amount to \$759.78 in 1 yr. 1 mo. 10 da.?

Let x = rate.

Then $723.60 \times \frac{x}{100} \times \frac{402}{360} = 8.04x$ = interest.

$723.60 + 8.04x$ = amount = 759.78

Transposing, $8.04x = 759.78 - 723.60 = 36.18$

Clearing of decimals, $804x = 3{,}618$

$x = \frac{3618}{804} = 4\frac{1}{2}$ *Ans.* $4\frac{1}{2}$ per cent.

932. In what time will \$85.50 produce \$8.17 interest, at 4 per cent?

Let x = time in years.

Then $85.50 \times \frac{4}{100} \times x = 3.42x$ = interest.

$3.42x = 8.17$

Clearing of decimals, $342x = 817$

$x = \frac{817}{342}$ = time in years.

2 yr. 4 mo. 20 da.

342)817 yr.

133 yr. remainder.

12

1596 mo. new dividend.

228 mo. remainder.

30

6840 da. new dividend.

0 *Ans.* 2 yr. 4 mo. 20 da.

933. Slate Exercises.

Find rate, time, etc.

1. Principal, \$2,000; time, 3 yr.; interest, \$300. Rate?

2. Principal, \$1,800; rate, 4%; interest, \$144. Time?

3. Time, 8 mo.; rate, $4\frac{1}{2}\%$; interest, \$2.88. Principal?

4. Principal, \$38; time, 2 yr.; amount, \$40.28. Rate?

5. Principal, \$140; rate, $3\frac{1}{2}\%$; time, 3 mo. 15 da. Interest?

6. Amount, \$39.60; rate, 4%; time, 2 yr. 6 mo. Principal?

7. Amount, \$484.15; rate, $3\frac{1}{2}\%$; principal, \$460. Time?

8. Principal, \$39.60; rate, 4%; time, 1 yr. 7 mo. 15 da. Amount?

9. Time, 8 yr.; rate, 3%; amount, \$6,200. Principal?

10. Principal, \$7,548; time, 3 mo. 5 da.; interest, \$119.51. Rate?

11. Principal, \$9,000; rate, 4%; interest, \$632. Time?

12. Time, 2 yr. 3 mo. 20 da.; rate, 5%; amount, \$160.60. Principal?

13. Principal, \$756; rate, $3\frac{1}{4}\%$; time, 3 yr. 4 mo. 20 da. Interest?

14. Principal, \$120; time, 1 yr. 2 mo. 15 da.; interest, \$4.35. Rate?

15. Amount, \$97.57; rate, 4%; interest, \$7.57. Time?

16. Time, 3 yr. 8 mo. 19 da; rate, $4\frac{1}{2}\%$; amount, \$93.39. Principal?

17. Principal, \$1,848; rate, $3\frac{3}{4}\%$; time, 4 yr. 9 mo. 25 da. Amount?

18. Rate, 5%; time, 4 yr. 6 mo. 23 da.; interest, \$16.43 Principal?

934. Oral Exercises.

1. In what time will $100 amount to $109, at 6% interest?
2. At what rate will $200 produce $16 interest in 2 years?
3. What principal will produce $12 interest in 3 years, at 4%?
4. In what time will $300, at 4%, produce $29 interest?
5. In what time will $170 produce $1.70 interest, at 5%?
6. In what time will $360 produce $3.60 interest, at 4%?
7. In what time will $725 produce $7.25 interest, at 6%?
8. In what time will $45 produce 45¢ interest, at 4½%?
9. In what time will $72 produce $1.44 interest, at 6%?
10. Find the interest on $84 for 144 days, at 5%.
11. Find the interest on $125, at 5%, for 2 months 12 days.
12. At what rate will $64 produce 64¢ interest in 80 days?
13. At what rate will $40 produce $1.20 interest in 6 months?
14. A certain principal produces $120 interest, at 6%. What would be the interest if the rate were 4%?

INTEREST BY ALIQUOT PARTS.

936. Slate Exercises.

1. Find the interest on $387.45, for 2 yr. 8 mo. 18 da., at 7%.

$387.45 × .07.

	$27.1215	interest for 1 year.
	27.1215	interest for 1 year.
6 mo. = ½ yr.	13.5607	interest for 6 months.
2 mo. = ⅓ (of 6 mo.)	4.5202	interest for 2 months.
15 da. = ½ (of 2 mo.)	1.1301	interest for 15 days.
3 da. = ⅕ (of 15 da.)	.2260	interest for 3 days.
Ans.	$73.68	interest for 2 yr. 8 mo. 18 da.

2. Find the interest on $432.90, at 6%, for 1 yr. 7 mo. 12 da.

$432.90 × .06.

	interest for 1 yr.
6 mo. = ½ yr.	interest for 6 mo.
1 mo. = ⅙ (of 6 mo.)	interest for 1 mo.
10 da. = ⅓ (of 1 mo.)	interest for 10 da.
2 da. = ⅕ (of 10 da.)	interest for 2 da.
	interest for 1 yr. 7 mo. 12 da.

3. Find the amount of $874.16, at 5%, for 1 yr. 9 mo. 4 da.

	$874.16	principal.
5% = 1/20	43.708	interest for 1 yr.
6 mo. = ½ yr.		interest for 6 mo.
3 mo. = ½ (of 6 mo.)		interest for 3 mo.
3 da. = 1/30 (of 3 mo.)		interest for 3 da.
1 da. = ⅓ (of 3 da.)		interest for 1 da.
		amount for 1 yr. 9 mo. 4 da.

4. What is the amount of $95.72, for 3 yr. 6 mo. 20 da., at 5%?

	$95.72	principal.
(10% =) 1/10	9.572	interest for 2 yr.
1 yr. = ½ (of 2 yr.)	4.786	interest for 1 yr.
6 mo. = ½ yr.		interest for 6 mo.
20 da. = ? of 6 mo.		interest for 20 da.
		amount for 3 yr. 6 mo. 20 da.

5. Interest of $1,806.45, at 4%, for 1 yr. 7 mo. 25 da.

1 yr., 6 mo., 1 mo., 15 da., 5 da., 5 da.

6. Interest for 10 mo. 29 da., at 4%, on $380.40.

$380.40 × .04.

	$15.2160	interest for 1 year.
1 mo. = 1/12 yr.		interest for 1 month ⎫ deduct from interest
1 da. = 1/30 mo.		interest for 1 day ⎭ for 1 year.
		interest for 10 mo. 29 da.

7. Amount, at 6%, of $125.73, for 2 yr. 10 mo. 4 da.

8. Interest on $84.66, at 7%, for 1 yr. 4 mo. 12 da.

9. Interest, at 5%, for 4 yr. 2 mo. 7 da., on $250.

10. Amount of $1,000, at 6%, for 33 days.

937. When the time is less than a year, the following facts should be remembered:

6% for a year is 1 per cent for 60 days.
5% for a year is 1 per cent for 72 days.
4½% for a year is 1 per cent for ? days.
4% for a year is 1 per cent for ? days.

11. Find the interest for 81 days, at 5%, on $876.40.

72 days = 1% = $8.764
9 days = ⅛ (of 72 da.) = 1.095
$9.86 interest for 81 days.

12. Amount of $954, at 4%, for 4 mo. 10 da.

Principal $954.
3 months' interest = 1% 9.54
1 mo. = ⅓ (of 3 mo.) 3.18
10 da. = ⅓ (of 1 mo.)
amount for 4 mo. 10 da

13. Interest of $1,874, at 4½%, for 93 days.

80 days = 1%.
10 days
2 days
1 day

14. Interest of $753.20, at 5%, for 158 days.

72 da., 72 da., 12 da., 2 da.

15. Amount of $1,234.50, for 193 days, at 6%.

60 da., 120 da., 12 da., 1 da.

16. Find the proceeds of a 90-days note, for $873.60, at 6%.

Face	$873.60	
60 da.	8.736	} Deduct.
30 da.	4.368	
3 da.	.437	
	$860.06	proceeds.

17. Find the discount on a 3-months note, for $1,596, at 6%.

18. What are the proceeds of a 6-months note, for $785, discounted at 6%.

19. Find the interest on $484.40, for 1 yr. 3 mo. 17 da., at 7%.

20. Find the amount of $683, for 3 yr. 4 mo. 11 da., at 4½%.

938. N.B. — Do not use unnecessary figures.

21. Principal, $360; 5%; 3 yr. 7 mo. 18 da. Interest?

22. Principal, $613; 4½%; 157 da. Amount?

23. Principal, $1,774; 3¾%; 17 mo. 23 da. Interest?

24. Principal, $875; 6%; 2 yr. 3 mo. 1 da. Amount?

25. Principal, $976; 7%; 325 da. Interest?

939. By the *time* of a note is meant the number of days, etc., for which it is drawn. Find the discount for three additional days.

26. Face of note, $254; time, 30 days; 7%. Proceeds?

27. Face of note, $515; time, 6 months; 5%. Discount?

28. Face of note, $493; time, 60 days; 8%. Proceeds?

29. Face of note, $717; time, 15 days; 6½%. Discount?

30. Face of note, $1,000; time, 90 days; 4%. Proceeds?

940. Find the exact number of days. Take 360 days to year.

31. Principal, $1,836.50; 6%; Jan. 2 to Dec. 1. Amount?

32. Principal, $1,295.70; 7%; March 8 to April 9. Interest?

33. Principal, $1,433.11; 5%; Feb. 13 to Sept. 4. Amount?

34. Principal, $765.90; 4%; Oct. 1 to Dec. 17. Interest?

35. Principal, $275.84; 5½%; May 9 to July 3. Amount?

941. By the *term* of a note is meant the number of days it has to run after it has been discounted, including days of grace.

36. Face of note, $100; term, 63 days; 7%. Discount?

37. Face of note, $200; term, 93 days; 6½%. Proceeds?

38. Face of note, $300; term, 24 days; 5½%. Discount?

39. Face of note, $400; term, 117 days; 8%. Proceeds?

40. Face of note, $500; term, 88 days; 5%. Discount?

942. In examples 41–45, inclusive, find the time by compound subtraction.

41. Principal, $25.83; 6%; Jan. 14, 1892, to Sept. 5, 1894. Interest?

42. Principal, $47.96; 5%; Feb. 6, 1893, to Aug. 1, 1896. Amount?

43. Principal, $85.30; 7%; March 25, 1894, to Jan. 13, 1897. Interest?

44. Principal, $75.00; 4%; April 15, 1888, to Feb. 6, 1895. Amount?

COMMERCIAL DISCOUNT.

944. Slate Problems.

1. On a bill of goods amounting to $874.40, a discount of 5% is allowed. How much must be paid?

$$\begin{array}{rl} & \$874.40 \\ 5\% = \tfrac{1}{20} & \underline{\quad 43.72} \\ & \$830.68 \ \textit{Ans.} \end{array}$$

Divide by 2, placing the quotient figure one place to the right of the corresponding figure of the dividend.

2. Find the cost of a wagon, the catalogue price of which is $750, the discount being 30%.

$750 × .70 = $525. *Ans.*

The net cost is .70 of the catalogue price.

3. What will be the cost of goods amounting to $1,837.60, on which there is allowed a discount of 17½%?

	$1,837.60	
10% = 1/10	183.76	} Deduct.
5% = ½ (of 10%)	91.88	
2½% = ½ (of 5%)	45.94	
	$1,516.02 *Ans.*	

For 10% rewrite the original amount, placing first figure one place to the right. 5% is ½ of 10%. 2½% is ½ of 5%.

4. $784.68 less 75%.

¼ of $784.68 = *Ans.*

5. $937.52 less 36%.

		$937.52
Deduct {	25% = ¼	234.38
	10% = 1/10 of $937.52	
	1% =	

6. Find the net cost of 1,630 yd. silk, invoiced at $1.10 per yard, less 16% discount.

7. What is the cost, in francs, of 843.72 meters silk, at 5.75 francs per meter, less 12%?

8. What is the net cost of a lot of musical instruments amounting to $1,875.60, on which a discount of 10, 5, and 2½% is allowed? (Art. 924, Ex. 7.)

9. What would be the net cost of the same articles, if the discount were 2½, 5, and 10%?

10. Find the net cost of the same, at 17½% discount.

11. Goods catalegued at x dollars are sold at a discount of 20 and 10%. Find value of x, if net price is $360.

946. Oral Problems.

1. A can do a piece of work in 5 hours, B in 7 hours. How long will it take both working together?

2. An agent collected a bill, and sent to his employer the amount, less $2\frac{1}{2}\%$ commission. If his commission was $1.60, how much did he remit to his employer?

3. My house, worth $12,000, is insured for $\frac{3}{4}$ of its value, at $\frac{1}{4}\%$. What premium do I pay?

4. A floor 6 yards long, 4 yards wide, needs 32 yards of carpet to cover it. What is the width of the carpet?

5. What will be the interest on $87, at 5 per cent, for 144 days?

6. Find the discount, at 8 per cent, on a note for $176, which has 90 days to run.

7. An agent receives $8,200 to invest after deducting his commission of $\frac{1}{40}$ of the amount invested. What is the agent's commission?

8. By selling a house for $3,500, I lose $500. What is my loss per cent?

9. A lot is sold for $1,200, at a loss of 20 per cent. What part of $1,200 is the loss?

10. A merchant's receipts are $1,200; his gain is 20 per cent. What part of his receipts is profit?

11. If 3 men earn $72 in 8 days, how many dollars will 5 men earn in 11 days?

12. If a dealer loses 25% by selling a horse for $225, what per cent would he gain or lose by selling the horse for $325?

13. Find the cost of 4 yd. 1 ft. of ribbon, when 2 yd. 2 ft. cost 40 cents.

TO FIND FACE OF NOTE, RATE OF DISCOUNT, AND TIME.

947. I wish to obtain $1,000 from a bank. What must be the face of a 30-days note, which will give the above proceeds, if it is discounted at 6%?

Let x = face of the note.

$$x \times \frac{6}{100} \times \frac{33}{360} = \frac{11x}{2000} = \text{discount.}$$

$$x - \frac{11x}{2000} = \text{proceeds} = 1{,}000$$

Clearing of fractions, $2{,}000x - 11x = 2{,}000{,}000$

$$1{,}989x = 2{,}000{,}000$$

$$x = \frac{2000000}{1989} = 1{,}005.53$$

Ans. $1,005.53, face of note.

Proof. Face of note, $1,005.53

Deduct { 30 days' discount = 5.0276 } ½%.
{ 3 days' discount = .5027 } $\frac{1}{10}$ of 30 days.

Proceeds $1,000.00

948. A note for $1,980 was discounted at 6%. The proceeds were $1,959.21. How many days had the note to run?

Let x = *term* in days.

$$1{,}980 \times \frac{6}{100} \times \frac{x}{360} = \frac{33x}{1000} = \text{discount.}$$

$$1{,}980 - \frac{33x}{100} = \text{proceeds} = 1{,}959.21$$

Clearing of fractions, $198{,}000 - 33x = 195{,}921$

$$-33x = -2{,}079$$

$$x = 63$$

Ans. 63 days.

949. Slate Exercises.

1. Three-months note; face, $108; rate 6%. Find proceeds.

(Term of discount is 93 days.)

2. 90-days note; face, $360; discount, $6.51. Find rate.

3. Proceeds, $717.60; rate, 5%; face, $720. Find term.

4. Discount, $11.20; rate, 7%; term, 48 days. Find face.

5. 15-days note; face, $1,560; rate, 6%. Find discount.

6. Term, 20 days; face, $158.40; proceeds, $157.96. Find rate.

7. Rate, 7%; discount, $2.10; face, $150. Find term.

8. Two-months note; discount, $14.70; rate, 7%. Find face.

9. For what amount must a 60-days note be drawn, so that the proceeds will be $300 when the rate of discount is 8 per cent?

10. A note for $120 was discounted at a bank March 15, 1894. What is the date of the maturity of the note, the proceeds being $119.52 and the rate of discount 6 per cent?

11. Find the proceeds of a 6-months note for $875 drawn Jan. 2, 1894, and discounted at 6 per cent 35 days after that date.

12. A merchant bought 300 barrels of flour at $4.75 per bbl., cash, and sold it for $5 per bbl., taking in payment a 60-days note for the amount. If he has the note discounted immediately at a bank, at 7 per cent, what does he gain by the transaction?

13. What will be the face of a 30-days note (without grace), the proceeds of which when discounted at a bank at 6%, will pay for 3,000 bu. corn at $49\frac{3}{4}¢$ per bushel?

14. The proceeds of a note for $1,200, due March 15, 1896, and discounted at 6%, were $1,184.80. When was it discounted?

SPECIAL DRILLS.

951. Find sums:

$23+37+48$	$\$7.56+\$5.38$	$52+41+34$	$325+865$
$44+66+19$	$\$2.74+\$8.54$	$28+38+43$	$472+935$
$75+42+37$	$\$3.49+\$9.89$	$81+49+24$	$567+629$
$16+71+62$	$\$4.83+\$6.52$	$63+47+33$	$784+796$

952. Find remainders:

$1{,}300-654$	$90-34-39$	$\$63.20-\$48.50$	$\$98.63-\$75.21$
$1{,}295-986$	$94-27-66$	$\$27.80-\$19.90$	$\$63.44-\$50.20$
$1{,}111-777$	$85-42-37$	$\$34.10-\$17.30$	$\$86.75-\$42.50$
$1{,}463-684$	$79-16-12$	$\$56.70-\$20.70$	$\$73.24-\$31.10$

953. Find products:

91×19	51×29	27×99	67×101
82×19	42×29	38×99	78×101
73×19	32×29	49×99	89×101
64×19	23×29	56×99	98×101

954. Find quotients:

$378\div18$	$256\div16$	$468\div18$	$175\div12\frac{1}{2}$
$462\div14$	$289\div17$	$900\div75$	$75\div6\frac{1}{4}$
$475\div19$	$493\div17$	$675\div75$	$675\div37\frac{1}{2}$
$448\div16$	$465\div15$	$975\div75$	$375\div62\frac{1}{2}$

955. Find answers:

$136\times\frac{7}{8}$	$64\frac{1}{2}\div5$	$22\times19\frac{1}{2}$	$65\frac{1}{2}\div13$
$290\times\frac{9}{10}$	$73\frac{3}{8}\div8$	$24\times19\frac{3}{4}$	$109\frac{1}{8}\div12$
$315\times\frac{14}{15}$	$38\frac{5}{8}\div7$	$32\times29\frac{7}{8}$	$160\div1\frac{1}{2}$
$378\times\frac{17}{18}$	$47\frac{1}{8}\div9$	$45\times9\frac{14}{15}$	$18\frac{4}{5}\div1\frac{4}{5}$

956. Oral Problems.

1. Paid 23¢ for calico, 27¢ for ribbon, and 48¢ for collars. What was the amount of my bill?

2. A farmer had 95 sheep. He sold 39, and 17 died. How many had he left?

3. What will be the cost of 16 base balls, at 49¢ each?

4. How much paint will there be in 27 casks, each containing 75 lb.?

5. A man divided a 429 acre farm into plots of 13 acres each. How many such plots were there?

6. There are 900 men in a certain regiment. How many companies of 75 men each are in the regiment?

7. Find the cost of 136 lb. sal-soda, at $\frac{1}{8}$¢ per lb.

8. At $19\frac{1}{2}$¢ per yd., what will I have to pay for 64 yd. gingham?

9. How many square inches in a sheet of paper $10\frac{1}{2}$ inches long by $4\frac{1}{2}$ inches wide?

10. If $2\frac{2}{3}$ yards of cloth are needed for a jacket, how many jackets can be made from $18\frac{2}{3}$ yd.?

11. How many yards around a field 96 yards long, 75 yards wide?

12. What will be the area, in square rods, of a triangle 33 rods base, altitude 42 rods?

13. How many acres in 4,960 square rods?

14. How many feet in a mile?

15. I paid \$16.25 for cloth at \$1.25 per yard. How many yards did I buy?

16. What will be the cost of 3 lb. 7 oz. of tea, at 64¢ per lb.?

17. Half a number $+\frac{1}{3}$ of the same number $= 85$. What is the number?

18. I mix 4 lb. of coffee costing 20¢, with 6 lb. costing 25¢. What is the mixture worth per lb.?

19. A tailor makes up 99 yd. cloth into trousers, using $2\frac{3}{4}$ yd. per pair. How many pairs of trousers does he make?

20. How many feet in $2\frac{1}{4}$ rods?

21. At 60¢ per pound, what will be the cost of a chest of tea weighing 45 lb.?

22. A man owns a strip of land with a frontage of 576 feet. How many lots 18 feet front can he make?

23. How much will be paid for 21 lb. butter, at 28¢ per lb.?

SHORT METHODS.

Slate Exercises.

6,748
× 427
47 236 Multiply by 7. Multiply this product by 6. Why?
2 834 16
2,881,396

958. Find products:

1. $3{,}925 \times 328$
2. $12{,}345 \times 273$
3. $2{,}087 \times 287$
4. $20{,}308 \times 142$
5. $4{,}321 \times 189$
6. $31{,}265 \times 164$
7. $5{,}763 \times 426$
8. $87{,}093 \times 486$
9. $6{,}905 \times 364$
10. $64{,}271 \times 357$

3,289
832
2 631 2 Multiply by 8. Multiply this product by 4. Where is the second product placed? Why?
105 248
2,736,448

959. Find products:

11. $4{,}008 \times 214$	21. $7{,}214 \times 99$	31. $876 \times 9\frac{7}{8}$
12. $8{,}736 \times 742$	22. $3{,}281 \times 999$	32. $547 \times 19\frac{3}{8}$
13. $3{,}764 \times 327$	23. $6{,}085 \times 75$	33. $734 \times 29\frac{1}{8}$
14. $1{,}087 \times 848$	24. $6{,}984 \times 25$	34. $615 \times 39\frac{1}{2}$
15. $8{,}319 \times 416$	25. $5{,}796 \times 62\frac{1}{2}$	35. $427 \times 71 \times 16$
16. $6{,}352 \times 927$	26. $8{,}383 \times 12\frac{1}{2}$	36. $284 \times 31 \times 19$
17. $2{,}781 \times 525$	27. $3{,}428 \times 37\frac{1}{2}$	37. $876 \times 27\frac{7}{8}$
18. $9{,}060 \times 1{,}166$	28. $7{,}154 \times 87\frac{1}{2}$	38. $973 \times 24\frac{4}{5}$
19. $6{,}329 \times 618$	29. $6{,}419 \times 33\frac{1}{3}$	39. $5{,}147 \times 126\frac{6}{11}$
20. $2{,}345 \times 1{,}272$	30. $6{,}208 \times 66\frac{2}{3}$	40. $4{,}284 \times 451\frac{1}{3}$

960. Supply missing amounts:

41.	42.	43.
\$834,682.50	\$16,945.84	\$380,086.77
95.00	123,456.78	64,593.25
734.18	9,876.54	8,737.84
.69	385.89	695.27
3.75	57.40	47.16
28.14	.98	
1,059.23	7.23	96.86
22,965.89	16.84	408.08
387.42	275.30	2,766.59
1,369.78	8,888.88	32,059.87
777,777.77		165,384.26
83,008.08	64,935.27	32,564.37
699.69	148,376.95	6,999.88
.88	834.11	840.30
3.86	2,070.08	27.63
50.05	12,316.99	5.98
	7,456.83	.86
\$2,000,000.02	\$456,789.01	\$743,869.05

44. £7 16*s.* 7*d.* **45.** 8 yd. 2 ft. 6 in. **46.** 6 bu. 3 pk. 5 qt.
9 yd. 1 ft. 10 in.

4 18*s.* 11*d.* 5 bu. 2 pk. 7 qt.

£20 1*s.* 3*d.* 30 yd. 0 ft. 3 in. 25 bu. 1 pk. 1 qt.

961. Divide. 2 decimal places (Arts. 385, 616):

47. 46,893,647 ÷ 3,986,048 **50.** 76,538,061 ÷ 5,736,804

48. 26,053,862 ÷ 1,898,637 **51.** 92,647,318 ÷ 4,863,978

49. 38,627,000 ÷ 2,679,835 **52.** 57,913,246 ÷ 2,597,384

962. Write answers (Art. 385):

53. $\frac{46,893,647}{9,728,759} =$ **56.** $\frac{76,538,061}{8,736,804} =$

54. $\frac{26,053,862}{2,198,684} =$ **57.** $\frac{92,647,318}{9,863,978} =$

55. $\frac{38,627,000}{3,568,879} =$ **58.** $\frac{57,913,246}{7,384,597} =$

TABLE OF EXPORTS.

963. The following table contains the values of the goods exported in 1891 by the United States to the various countries of the world.

Find the total value of the goods exported, and the per cent of this value for each section. Carry out to two places of decimals.

Countries to which Exported.	1891.	Per cent.
1. Europe	$697,614,106	?
2. Asia and Oceanica	43,813,519	?
3. British North American Possessions	37,345,515	?
4. West Indies	33,416,178	?
5. South America	33,226,401	?
6. Mexico, Central America, and British Honduras	21,236,545	?
7. Africa	4,738,847	?
8. All other	879,172	?
	?	100.00

REVIEW.

964. Oral Problems.

1. $\frac{3}{4}$ of a number is 48. What is the number?

2. A base ball club won 17 games, and lost 13 games. What per cent of its games did it win?

3. What per cent of 4 is 64?

4. $2\frac{2}{3}$ is what per cent of $3\frac{1}{5}$?

5. How many acres in a rectangular farm 1 mile long, $\frac{1}{4}$ mile wide?

6. What per cent of the "list" price is paid by a buyer who receives a discount of 20 and 10 per cent?

7. A tank is filled by two pipes, one of which can fill it in 6 hours, and the other in 8. How long will it take both together to fill the tank?

8. Find the interest on \$80, for 72 days, at 6%.

9. A man sold a wagon for \$420, which was 16% less than it cost. How much did he lose?

10. A kilo is 2.2046 lb. How many pounds in 1,000 kilos?

965. Slate Problems.

In the first four examples, carry out to two places of decimals, as: 135.67%, 25.83%, 6.03%, 0.52%, 0.09%.

1. The population of Montana was 39,159 in 1880, 132,159 in 1890. Find the per cent of increase.

2. The population of South Dakota in 1890 was 328,808, a gain of 230,540 over the population in 1880. What was the gain per cent?

3. The enrollment in the South Dakota schools was 9,972 in 1880, 66,150 in 1890. Find the gain per cent.

4. What was the gain per cent in the population of a state that had 416,396 inhabitants in 1880, and 416,552 in 1890?

5. A man marks an article \$1.50, and sells it at a discount of 25% from the marked price. If the article cost him 90¢, what is his gain per cent?

6. Goods costing \$8 are sold at an advance of 20 per cent. The marked price is \$12. What per cent reduction is made on the marked price?

7. A rug costs \$20. It is sold at a profit of 20%. The selling price is 20% below the marked price. How much is received for the rug? What is the marked price?

8. What price must cloth, which costs \$2 per yard, be marked so that a profit of 20% will be made when the cloth is sold at 20% less than the marked price?

REVIEW FRACTIONS.

969. Slate Exercises.

NOTE. — Do not use too many figures.

1. Add $\frac{4}{5}$, $2\frac{1}{4}$, $\frac{3}{7}$, $\frac{5}{8}$.

2. Divide each of the following fractions by 6:

$$\frac{5}{7},\ \frac{12}{13},\ \frac{25}{46},\ \frac{84}{103}.$$

3. Reduce $\frac{7}{8}$ of $\frac{6}{11}$ of $\frac{5}{12}$ of $2\frac{2}{7}$ to a simple fraction.

4. $38\frac{5}{9} - 21\frac{11}{12}$.

5. What fraction of £1 18*s.* 9*d.* is 5*s.* 6*d.*?

6. Multiply $24\frac{1}{8}$ by $\frac{4}{5}$ of $\frac{3}{7}$.

7. What is the greatest common divisor of 657 and 1,168? The least common multiple of 12, 16, 20, 30?

8. What must be taken from $8\frac{5}{16}$ to leave $3\frac{7}{12}$?

9. Reduce $\frac{48}{528}$ and $\frac{404}{606}$ to their lowest terms.

10. Which is the greatest and which is the least, of $\frac{4}{9}$, $\frac{2}{3}$ of $\frac{5}{8}$, and $2\frac{1}{8}$ of $\frac{2}{11}$?

11. What must be added to $3\frac{4}{11}$ to make $5\frac{1}{3}$?

12. Add $\frac{5}{8}$ of a week, $\frac{5}{6}$ of an hour, $\frac{7}{12}$ of a minute.

13. How much is 9 times each of the following fractions?

$$\frac{5}{8},\ \frac{6}{11},\ \frac{7}{27},\ \frac{19}{65}.$$

14. $30\frac{2}{7} \div \frac{3}{5}$ of 7.

15. $\frac{7}{12} + \frac{5}{6}$ of $\frac{1}{10} + \frac{5}{8}$ of $\frac{2}{3}$.

16. What part of a ten-acre field is 4 A. 100 sq. rods?

17. What is the least number that will contain each of the numbers 6, 15, 18, and 20?

18. What must be multiplied by $4\frac{1}{3}$ to produce $16\frac{1}{4}$?

19. What is the value of $\dfrac{\frac{1}{2} + \frac{3}{7}}{4\frac{1}{4}}$?

20. What quantity must be divided by $4\frac{2}{7}$ to produce $8\frac{5}{9}$?

21. Find the value of $\dfrac{2\frac{1}{4} \div \frac{3}{5}}{4\frac{1}{12} - \frac{5}{6}}$.

22. How much is $\dfrac{\frac{3}{4} - \frac{2}{3}}{2\frac{1}{6}}$ of 3 da. 15 hr. 32 min.?

23. Reduce $\frac{4}{15}$ mile to rods.

24. Add $\frac{2}{3}$, $\frac{4}{9}$, $5\frac{1}{6}$. Subtract $4\frac{5}{12}$ from the sum.

25. Multiply $\frac{3}{4}$ of $5\frac{1}{8}$ by $7\frac{1}{5}$. Divide the result by $1\frac{1}{5}$.

MEASUREMENTS.

970. Slate Problems.

1. A tank 18 ft. long, 15 ft. wide, requires 63 sq. yd. of lead to line its sides and bottom. How many feet deep is it?

(Make diagram.)

2. A farmer has a 3-acre field in the form of a right-angled triangle. If one perpendicular side measures 242 yards, what is the length of the other?

3. One parallel side of a field in the shape of a trapezoid measures 150 yd., the other measures 200 yd. How many square yards in the field, the perpendicular distance between the sides being 50 yards? (Make diagram.)

4. The shorter parallel side of a trapezoid is x yards, the other is 100 yd., the perpendicular is 60 yd. Find the area in square yards.

How long is the shorter parallel side when the area is 5,400 sq. yd.?

5. One parallel side of a trapezoid is 80 yd., the other is 120 yd., the perpendicular is x yd. Find the area in square yards.

How long is the perpendicular when the area is 4,000 sq. yd.?

6. One parallel side of a trapezoid is x yd., the other is $x + 40$ yd., the perpendicular is 60 yd. Find the area.

Find the length of the parallel sides when the area is 6,000 sq. yd.

7. The sum of the parallel sides of a trapezoid is 200 yards, the perpendicular is 100 yards. How many square yards in the area?

8. How many flagstones $5\frac{1}{2}$ ft. long, 3 ft. wide, will be needed to lay a sidewalk 1 mile long, 6 ft. wide?

9. What would it cost, at 10¢ a square yard, to paint the walls of a room 16 ft. 6 in. long, 14 ft. 9 in. wide, 13 ft. 4 in. high?

10. A room 20 ft. long and 17 ft. 6 in. wide will require how many yards of carpet 2 ft. 6 in. wide to cover it, making no allowance for waste?

11. Find the weight of a plank 15 ft. 9 in. long, 10 in. wide, and 2 in. thick, at $41\frac{1}{4}$ lb. per cu. ft.

12. Find the entire surface of a block of marble $3\frac{3}{4}$ ft. long, $2\frac{1}{2}$ ft. wide, and $1\frac{1}{8}$ ft. thick. (Draw "development.")

13. The area of a floor is 135⅓ sq. ft., and its length is 12 ft. 8 in. What is its width?

14. How many bushels will a bin contain, its dimensions being 10 ft. 8 in. × 12 ft. 8 in. × 8 ft. 9 in.? (Cancel.)

15. Find the capacity in gallons of a tank 6 ft. 5 in., by 3 ft. 9 in., by 4 ft. 6 in.

16. A farmer wishes to construct a post and rail fence around a square field containing 40 acres. He pays 15¢ each for the posts, which are placed ½ rod apart. The rails cost 10¢ each. If the fence is 5 rails high, how much will the material cost?

17. A farm 1 mile square is divided into square fields each containing 40 acres. Make a diagram of the farm, and say how many miles of fence will be needed to enclose all the fields.

(640 A. = 1 sq. mi.)

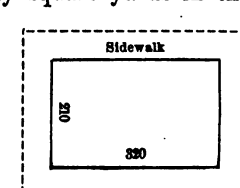
18. How many cords of wood in a pile 164 ft. long, 16 ft. wide, 30 ft. high?

19. A cubic foot of water weighs 1,000 oz. What will be the weight of a gallon of water? (Give answer in pounds and fraction.)

20. Find the weight of a quart of mercury, considering mercury 13 times as heavy as water.

21. A plot of ground 320 ft. long, 210 ft. wide, is enclosed by a tight board fence 6 ft. high. How many square yards in the surface of the fence?

22. Find the number of square yards in a sidewalk, six feet wide, on the outside of the above plot.



The inner rectangle measures 320 × 210 ft. What are the dimensions of the outer rectangle? Find the difference between the area of the outer rectangle and that of the inner one.

23. Find the area of a gravel walk, 6 ft. wide, just inside a fence surrounding a plot 320 ft. long, 210 ft. wide.

Make a diagram. Find difference of areas, as above.

CAUSE AND EFFECT.

971. Oral Problems.

1. If 4 books cost $1.25, what will a dozen cost?

2. If 3 lb. sugar cost 13½¢, what will be the cost of 50 lb.?

3. If 48 lb. tea cost $20, what will 12 lb. cost?

4. Bought 17 yards of cloth for $30. How many yards could I have bought for $90?

5. If 36 men do a piece of work in 105 days, how long will it take 72 men to do it?

6. If 7 railway trucks weigh 14 tons, how much would 29 trucks weigh?

7. How long will it take 8 horses to plow a field, if 3 horses can do it in 8 days?

8. What is the height of a steeple that casts a shadow of 300 ft., if an 8 ft. pole casts a shadow of 12 ft.?

9. If 18 men mow 90 acres of grass in 5 days, how many acres will 36 men mow in 5 days? In 10 days?

10. If 60 yd. carpet ¾ yd. wide will cover a floor, how many yards ⅝ yd. wide will be required?

972. Slate Problems.

1. A piece of cloth, measured with a yard measure that is 1 inch too short, appears to be 25 yd. long. What is its true length?

2. Exchanged 40 yd. muslin, worth 10½¢ per yd., for 15 yd. linen. What is the value of the linen per yd.?

3. If 3 men or 6 women can do a piece of work in 56 days, in what time will 1 man and 2 women working together do it?

4. If 5 men can do as much in a day as 8 boys, how long will it take 32 boys to finish a piece of work which 15 men can do in 12 days?

5. If \$100 gain \$4 in 1 year, what will \$350 gain in $3\frac{1}{2}$ years?

6. If 48 horses in 10 days consume 180 bu. oats, how many bushels will 32 horses consume in 10 days? In 12 days? In 15 days?

7. If 5 men mow 45 acres of grass in 6 days, in how many days will 12 men mow 90 acres?

973. If 5 men mow 45 acres in 6 days,

1 man will mow 45 acres in 6×5 days.

1 man will mow 1 acre in $\frac{6 \times 5}{45}$ days.

12 men will mow 1 acre in $\frac{6 \times 5}{45 \times 12}$ days.

12 men will mow 90 acres in $\frac{6 \times 5 \times 90}{45 \times 12}$ days.

Canceling, $\frac{\overset{2}{\cancel{6}} \times 5 \times \overset{2}{\cancel{90}}}{\underset{2}{\cancel{45}} \times \underset{2}{\cancel{12}}} = 5$. *Ans.* 5 days.

974. In practice, the work is somewhat shortened. Since the number of days is required, we write the given number of days last, with a line underneath.

$$\left.\begin{array}{l} 5 \text{ men mow } 45 \text{ acres} \\ 1 \text{ man mows } 1 \text{ acre} \\ 12 \text{ men mow } 90 \text{ acres} \end{array}\right\} \begin{array}{c} \text{days} \\ \dfrac{6 \times 5 \times 90}{45 \times 12}. \end{array}$$

If 5 men do the work in a certain time, 1 man will require 5 times as many days. We place 5 in the numerator (as a multiplier). To cut 1 acre, he will take $\frac{1}{45}$ of the time required to cut 45 acres. Place 45 in the denominator (as a divisor).

12 men will take $\frac{1}{12}$ of the time 1 man requires. Place 12 in the denominator. To cut 90 acres will require 90 times as long. Place 90 in the numerator.

8. If 12 horses eat 60 bushels of oats in 6 days, how many bushels will 24 horses eat in 3 days?

Make bushels the last term.

12 horses in 6 days eat
1 horse in 1 day eats
24 horses in 3 days eat
} bu. 60

975. This example can be solved more easily. 6 days' food for 12 horses will supply how many horses for 1 day? 3 days' food for 24 horses will supply how many horses for 1 day?

9. If 24 men use 240 lb. of beef in 2 weeks, how many pounds will 18 men use in 8 weeks?

24 men in 2 weeks use 240 lb.

10. If 6 printers can print 1,656 sheets in 9 days, how many sheets will 15 printers print in 10 days?

11. How much will it cost to feed 520 sheep for 36 days, if it costs $128 to feed 160 sheep 48 days?

12. In what time will 8 masons build a wall 84 ft. long, working 10 hours a day, if 12 masons build a wall 96 ft. long in 8 days, working 8 hours a day?

13. How much money must I lend for 1 year and 3 months, when the rate of interest is 5 per cent, in return for $60 lent me for 9 months, which I borrowed at 4 per cent?

14. If 27 men build 54 rods of wall in 6 days, how many rods will 32 men build in 9 days?

15. If 50 men can do a piece of work in 90 days, working 8 hours a day, in how many days will 72 men do it, working 10 hours a day?

16. If $350 earns $42 interest in 3 years, how much will $225 earn in 5 years?

17. If a wall 34 feet high could be built by 68 men in 15 days, how many men could build a wall 32 feet high in 8 days?

18. If a ship's crew of 500 men have provisions to serve for 48 days, at the rate of 27 ounces a day for each man, how many men will the same provisions serve for 60 days, allowing each man 30 ounces a day?

19. How many hours a day must 9 men work so that they may do as much in 16 days as 12 men can do in 15 days of 8 hours each?

20. If 30¢ is paid for 6 lb. 14 oz. of bread, when wheat is $1.14 per bu., what should be paid for 23 lb. 12 oz., when wheat is $1.32 per bu.?

NOTE. — Reduce weights to ounces, or to pounds and fractions.

21. If 3 men can do as much work as 7 boys, how long will it take 28 boys to do as much work as 16 men can do in 24 days?

22. A crew of 16 men have provisions for 36 days, allowing 20 ounces to each man per day. After sailing 10 days they pick up 10 shipwrecked sailors. How long will the provisions then last at the daily rate of 16 ounces per man?

PARTNERSHIP.

977. Slate Problems.

1. B and C gain by trade $182. What is the gain of each, B having put in $300, and C $400?

The gain of $700 is $182. What should $300 gain? What should $400 gain?

2. A, B, and C invest $720, $340, and $960, respectively. The profits are $101. What is each one's share?

How many dollars of capital produce $101 profits?

3. Two men hire a pasture for $45. One puts in 15 cows, the other puts in 12 cows. What should each pay?

4. A and B hire a boat for 50 days, paying $30. A uses it 27 days, B uses it 23 days. How much should each pay?

5. Our standard gold coin consists of 900 parts gold, 90 parts silver, 10 parts copper. What is the quantity of each metal in 50 pounds of coin?

6. Gunpowder is composed of 15 parts of saltpeter, 2 of sulphur, and 3 of charcoal, mixed together. How many pounds of each are there in 72 pounds of powder?

7. Three farmers hired a threshing-machine for $54. A used it to thresh his crop of 900 bu., B to thresh his crop of 828 bu.; C, 672 bu. How much should each pay?

8. A, B, and C rented a warehouse. A stored in it 2,400 bales cotton; B, 1,500; C, 1,100. A fire destroyed 1,800 bales. How much of the loss should each sustain?

9. X and Y rent a field for $32. X puts in 8 horses for 6 months, and Y 10 horses for 8 months. How many dollars should each pay?

8 horses for 6 months = how many for one month?
10 horses for 8 months = how many for one month?

10. M and N entered into partnership. M puts $200 into the business for 5 months, and N, $300 for 4 months. They gained $132. Find the share of each.

REVIEW DECIMALS.

978. Slate Exercises.

1. Express as decimals $\frac{57}{100}$, $\frac{7}{1000}$, and $\frac{27}{820}$.

2. .395 + 86.7 + 209.0043 + .81 + 3.075 + 27.

3. Divide 34,020.072 by 5.309.

570 ÷ .005 = ?

4. Multiply 80.037 by 10.

Seventy-three one hundred-thousandths by one hundred.

.2054 × 1,000 = ?

5. Subtract 48.8067 from 53.07.

.0539 × 26.08 = ?

6. The smaller of two numbers is 8.5307, and their sum is 25.07. Find the larger number.

7. Express .39, 6.175, .00036, and 74.0005 as common fractions (or mixed numbers).

8. Divide .826 by 100; 543.71 by 10,000; and fifty-nine ten-thousandths by one thousand.

9. Find the difference between 9.84 and 38.005, and the continued product of 83.09, .734, and 5.007.

10. Reduce 6 shillings 9 pence to the decimal of a pound sterling.

11. Express as decimals seven hundredths, forty-three ten-thousandths, and ninety-one millionths.

12. Change $\frac{1}{16}$, $8\frac{1}{80}$, $1\frac{1}{125}$, and $\frac{1}{256}$, into decimals. Find their sum.

13. Express .42796 as a common fraction, and the sum of $\frac{7}{10}$, $\frac{3}{100}$, and $\frac{873}{10000}$ as a decimal.

14. $3.009 \times .07 \times .0907$.

15. Divide .0075 by .15, and .00044408 by .0112.

16. Divisor, 403.6; quotient, 2.709. Dividend?

17. What is the value of $\frac{.035 \times .0056}{.00007}$?

18. Change 69 rods to the decimal of a mile.

19. Change .4285 month (30 days) to days, hours, etc.

20. How many meters, each 39.37 inches, in 3 miles 220 rods?

21. Change .1875 bu. to quarts.

22. What decimal of a pound is 13 oz.?

23. Reduce 4 ft. $1\frac{1}{2}$ in. to the decimal of a rod.

24. How many links of 7.92 in. each in a 4-rod chain?

25. A chain is 66 ft. What decimal of an acre is 1 sq. chain?

APPROXIMATIONS.

981. Give approximate answers at sight (Art. 890):

1. $487\frac{3}{4}$ is what per cent of 960?

2. If 17 bu. 37 lb. of corn cost \$8.75, what will 52 bu. cost?

3. About how many cords of wood in a pile 25 ft. long, 4 ft. wide, 5 ft. high?

4. How many bushels ($1\frac{1}{4}$ cu. ft.) can be placed in a bin 6 ft. long, 5 ft. wide, 4 ft. high?

5. How many acres in a field 50 rods long, 30 rods wide?

6. About how many yards are there in the side of a square field containing 1 acre (4,840 sq. yd.)?

7. At $7\frac{1}{2}$ gal. to cu. ft., about how many gallons will a tank hold 6 ft. long, 4 ft. wide, 3 ft. high?

8. $64.3 \div .0987 = ?$

9. About how many dollars are equal to £199 17*s.* 6*d.*?

10. A mark = 23.8¢. How many marks in \$100?

BONDS AND STOCKS.

982. Slate Problems.

1. The people of a certain town wish to build a street railroad. A company is formed. Five hundred shares of stock, of the par value of \$100 each, are sold.

At the end of 6 months it is found that the profits are \$2,000. How much should the owner of 10 shares receive?

2. Profits thus distributed are termed *dividends*. What % semi-annual dividend is declared on the stock of the above railroad? To what per cent interest per year is it equal?

3. Mr. H. has \$4,500 in the savings bank, on which he receives a low rate of interest. Hearing of the success of the

new road, he gives that amount for 30 shares of the stock. What price does he pay per share? What per cent of the par value?

4. If the semi-annual dividend is again 4%, how much more income does Mr. H. receive from the railroad stock than he would obtain from the savings bank in six months, interest 4 per cent per annum?

5. What per cent, for six months, does the stock pay on his investment of $4,500? What % per year?

6. If he sells the stock (30 shares) at 164½ (per cent), how much more does he receive for it than it cost him?

7. Which investment will pay better, one in a gas company paying 6 per cent dividends annually, their stock selling at 150, the other in a bank paying 7 per cent dividends annually, stock selling at 175?

8. What annual dividend should be declared on railroad stock bought at 125, so that the buyer will receive 4% per annum on his investment? What semi-annual dividend?

9. What will be the cost of 17 shares of canal stock, par value $50, at 93¾, and 143 shares gas stock, par value $10, at 102¾?

10. If the above stock is purchased through a broker, what commission does the latter receive at ⅛% on the par value?

11. A railroad company needing more money to extend its road, issues bonds, promising to pay the holder the face value in twenty years, with interest at 4%.

If these bonds are sold at 95, what rate of interest on the money invested does the owner of a bond receive?

12. Government 4 per cent bonds sell for 116½. What per cent interest is received on the amount invested?

How is it that these bonds bring higher prices than railroad bonds?

13. Can you state a difference between stocks and bonds as to the rate of income received from each?

Bonds are redeemed at maturity. How about stocks?

If a railroad prove unsuccessful, which claims are first met, those of the stockholders or those of the bondholders?

14. Why is it necessary sometimes to employ a broker to purchase stocks or bonds? What is his fee called?

15. Mention some other persons, not owners, through whom buyers regularly make purchases.

16. What is the base in the following?

(*a*) Insurance; (*b*) taxes; (*c*) brokerage; (*d*) commission; (*e*) interest; (*f*) discount; (*g*) stocks; (*h*) bonds.

17. At $24.50 per thousand, what will have to be paid in taxes by the owner of property assessed at $18,750?

18. Mr. Cartwright owns a house and lot worth $36,000. The tax rate is $2\frac{1}{4}\%$, and his tax bill is $540. What is the assessed value of the property? What per cent of the actual value is the assessed value?

19. If the property in the last problem were assessed at its real value, what should be the rate to make Mr. Cartwright's tax bill the same?

20. For insuring his property, Mr. Cartwright pays a yearly premium of $135. If the rate is $\frac{3}{4}\%$, for how much is his property insured?

21. Reduce 1,674 feet to rods, etc.

22. A man paid $8,575 for bank stock at 245. How many shares, par value $100, did he buy? If a quarterly dividend of $2\frac{1}{2}\%$ is declared, how much should he receive?

23. Reduce 7,481 inches to rods, etc.

24. A woman deposited $100 in a savings bank Jan. 1, 1892. On the first of July, interest at the rate of 4% per annum was

calculated, and entered on the depositor's bank book. Jan. 1, 1893, interest on the new principal was placed to the credit of the depositor. The same was done July 1, 1893. How much was there to the woman's credit at the date last mentioned?

25. Reduce 3,793 feet to rods, etc.

COMPOUND INTEREST.

983. Find the amount of $375, for 1 year, at 6%. Considering this as a new principal, find the amount for a year, same rate. Find the amount of this last principal for 3 months.

26. What is the amount of $375, for 2 years 3 months, at 6%, compound interest?

27. What is the amount of $375, for 2 years 3 months, at 6%, the interest compounded semi-annually?

Principal, $375.

3%	11.25	6 months' interest.
	386.25	Amount 6 months.
3%	11.5875	6 months' interest.
		Amount 1 year.
3%	_____	6 months' interest.
		Amount 1½ years.
	etc., etc., etc.	

Find the "compound interest" on $375, for 2 years 3 months, at 6 per cent, compounded semi-annually.

28. What is the amount of $100, at compound interest, for 3 years, interest at 6%, compounded annually?

29. Find the compound interest of $1,800, at 4%, for 2 years, interest compounded quarterly.

	$1,800.00
1%	18.00
	1,818.00
1%	18.18
	etc., etc., etc.

30. Find the difference between the simple interest of $100, for 2 yr. 3 mo., at 5%, and the compound interest for the same time, interest compounded semi-annually.

	$100.00	
$2\frac{1}{2}\% = \frac{1}{40}$	2.50	Divide by 4, and put first quotient figure
	102.50	one place to the right.
$2\frac{1}{2}\%$	2.5625	
	105.0625	
$2\frac{1}{2}\%$	2.6266	
	$107.6891	(four places of decimals are sufficient.)

984. Compound interest is allowed by savings banks. It is not collectible on notes or other debts.

REVIEW.

985. Oral Problems.

1. A capitalist wishes to realize 5% on money invested in stock. What must be the annual dividend on stock costing 300, in order to produce this rate?

2. What will be the taxes on property assessed at $25,000, the rate being $16 per $1,000?

3. Find the compound interest on $1,000, for two years, at five per cent, interest compounded annually.

4. What will be the net cost of an article marked $8, on which a discount of 50, 25, and 10% is allowed?

5. Find the "list" price of an article sold for $10 after a discount of 50 and 50 per cent had been deducted.

6. Paid 90¢ for an article. The discount is 25 and 25 per cent. What is the list price?

7. One boy can do a certain piece of work in 2 hours, a second boy requires 3 hours, a third needs 6 hours. How long will it take the three working together?

8. Sold a cow for $60, losing 25%. What was the loss?

9. Sold a cow for $60, gaining 25%. What was the gain?

10. Sold two horses at $240 apiece. On one I gained 20%, on the other I lost 20%. Did I gain or lose on both, and how much?

SUGGESTION. — $240 in the first case represents 120% of cost of horse.

The gain is 20%, which is $\frac{1}{6}$ of selling price, or $40.

The loss in the other case is 20%, which is what part of the selling price?

Do not find the cost.

11. John has $60, James has $80. James has what per cent more money than John? John has what per cent less money than James?

12. $\frac{2}{3}$ is what per cent of $\frac{1}{2}$? $\frac{1}{2}$ is what per cent of $\frac{2}{3}$?

13. Two men working together can finish a piece of work in 8 days; one can do it in 12 days. How long would the other take to do the work?

14. How many yards of cloth at $3.75 per yard can be bought for $90?

EXCHANGE.

992. If I wish to pay a bill in a distant city, ought I to enclose the money in a letter? Why?

Can money be sent by express? Can the telegraph be used in paying money at a distance?

What is a money-order?

Can I buy from the postal authorities a money-order payable in Europe?

What will be the cost of a money-order for $85, payable in San Francisco?

What is the largest money-order that can be purchased?

What is a check? Can you tell why a draft rather than a check is used in paying a bill at a distance?

Pupils should be encouraged to look up answers to the foregoing.

Bills of exchange are either foreign or domestic. A domestic bill of exchange is called a *draft*, the term *bill of exchange* being generally applied only to foreign bills.

DOMESTIC EXCHANGE.

993. Slate Problems.

William F. Smith, of Memphis, Tenn., owes John M. Thomson, of New York, $3,475.86. He purchases from a Memphis banker, Joseph E. Washington, a sight draft for the above amount on the Chemical Bank of New York. The following is the form of the draft:

$3,475$\frac{86}{100}$. MEMPHIS, TENN., Aug. 9, 1893.

At sight, pay to the order of John M. Thomson Three Thousand Four Hundred Seventy-five and $\frac{86}{100}$ Dollars, value received, and charge to the account of

To CHEMICAL BANK, New York. JOSEPH E. WASHINGTON.

1. What must William F. Smith pay for the above draft, the rate being $1.50 premium per $1,000?

(A draft for $1,000 costs $1,001.50.)

2. Find the cost of a Boston draft on New York for $1,875, at 12¢ discount per $1,000.

(A draft for $1,000 costs $999.88.)

3. What will a St. Louis merchant have to pay for a draft on New York for $2,460.53, at 50¢ premium per $1,000?

4. If the rate of exchange is 50¢ discount per $1,000, what is the face of the sight draft on Boston, that can be bought in New York for $1,000?

5. When the premium is $1.25 per $1,000, Mr. Brown pays $1,634.04 for a draft on Louisville. What is the face of the draft?

6. At $\frac{1}{8}\%$ premium, find the cost of a sight draft for $\$1{,}843.60$.

$$\frac{1}{8}\% = \begin{array}{r} \$1{,}843.60 \\ 2.30 \\ \hline \$ \end{array} \text{ Add.}$$

7. At $75¢$ discount per $\$1{,}000$, how much will cost a sight draft on Milwaukee for $\$946.75$?

$$\text{less} \begin{cases} 50 \text{ per M.} & \$946.75 \\ 25 \text{ per M.} & .473 \\ & .237 \end{cases}$$

8. Paid $\$632.18$ for a sight draft on Milwaukee. What was the face of the draft, the discount being $\frac{3}{40}\%$?

9. I sent a commission merchant $\$1{,}000$ to buy grain. How much will he spend for grain, if his commission at $1\frac{1}{2}\%$ is included in the amount sent?

(Let x = amount spent for grain. ? = commission.)

10. A farmer ships produce to a commission merchant, which the latter sells for $\$339.66$, charging 2 per cent commission. For the remainder of the money he buys groceries and dry-goods, charging 2 per cent commission on the amount spent. What is the cost of the goods purchased?

REVIEW.

994. Slate Problems.

1. A joiner worked on Monday 9 hr. 45 min., on Tuesday and Wednesday 10 hr. 45 min. each day, on Thursday and Friday 10 hr. 15 min. each day, and on Saturday 6 hr. 45 min. What was the average length of his day's work?

2. A watch that loses 35 seconds in an hour was set right at noon on Monday. What time did it show at 6 P.M. the following Thursday?

3. There are 5 boys whose heights are 4 ft. 9 in., 5 ft. 1 in., 4 ft. 5 in., 3 ft. 11 in., and 4 ft. 4 in., respectively. What is their average height?

4. A man had a plot of ground 20 yards long and 12 yards wide, which he planted in cabbage. How many plants did he require, if the rows, which ran lengthwise, were 2 feet apart and 2 feet from the fence surrounding the plot, and the plants in the rows 16 inches from each other and from the fence?

Get the correct number of rows, and the correct number of plants in a row. How many plants would have been needed if the rows ran crosswise?

5. How long would it take a person to count a million silver dollars, at the rate of 100 a minute, and working 8 hours a day?

6. The front wheel of a wagon is 13 ft. 4 in. in circumference. How many revolutions will it make in a journey of 14 miles? How many more revolutions will it make than the hind wheel, the circumference of the latter being 17 ft. 6 in.?

7. The wheels of an engine being 16 ft. 8 in. in circumference, and the number of revolutions 150 per minute, how far does it go in an hour? Give answer in miles and rods.

995. Circular Measure.

60 seconds (″)	1 minute (′)
60 minutes	1 degree (°)
360 degrees	1 circle.

8. If the equatorial circumference of the earth is 25,000 miles, how many miles apart are two places on the equator, the distance between them being 20°?

9. What is the length of a degree on a circle whose diameter is 18 feet?

The circumference = diameter × 3.1416.

10. The 60th parallel of latitude is a circle about one-half as long as the equator. How far due east of Christiania is St. Petersburg, both situated on this parallel, the former being 10° east of Greenwich, and the latter 30° east?

11. How many miles north of the equator is a place in latitude 46° 22′ 30″? Take 69½ miles to a degree.

12. Two places in latitude 45° are 22° 30′ apart, measured on that parallel. Find the distance in miles, assuming the 45th parallel to be a circle .7071 times the length of the equator, and considering the length of the latter to be 25,000 miles.

996. Time Drafts.

987\frac{65}{100}$. NEW ORLEANS, June 15, 1893.

At three days' sight, pay to the order of John D. Hallock, Nine Hundred Eighty-seven $\frac{65}{100}$ Dollars, value received, and charge to account of

FRANK PHILLIPS.

To NATIONAL PARK BANK,
New York.

When Mr. Hallock receives the above, he presents it to the National Park Bank for *acceptance*. The proper bank official writes across the face of the draft in red ink "Accepted," with the date, say "June 18, 1893," and signs his name. Three days thereafter, plus three days of grace, or June 24, the draft will be payable.

997. Sight drafts are usually not allowed days of grace. Time drafts are generally allowed three days of grace. (See Appendix.)

998. The premium on the above draft at $1.50 per $1,000 is calculated on the face of the draft, and amounts to $1.48.

999. Since it is not payable until six days after acceptance, the interest (or bank discount) for that time is deducted.

Interest on 987\frac{65}{100}$ for 6 days at 6% = $.99.
Cost of draft = $987.65 + $1.48 − $.99 = $988.14.

N.B. Take 6% as the interest rate, unless a different rate be expressed.

1000. Slate Exercises.

1. What will I have to pay for a 90-days draft on San Francisco for $840, at $1.75 premium per $1,000?

2. Face $400; 30 days' sight; discount $\frac{1}{8}$%. Cost?

3. Face $560; 60 days' sight; premium 50¢ per $1,000. Find cost.

4. What will be the cost of a sight draft for $625.38 at 7½¢ discount per $1,000?

5. Find the cost of a 60-days draft for x dollars, premium 25¢ per $1,000.

6. Find the cost of an x-day draft for $1,200, discount ⅛%.

7. Find the cost of a 30-days draft for $1,600, premium x dollars per $1,000.

8. Paid $1,188.90 for a 60-days draft, at ⅛% premium. What was the face of the draft?

9. A time draft for $1,800 at $1 premium per $1,000, cost $1791.90. At how many days' sight was it drawn?

10. At what rate did I purchase a 90-days draft for $900, its cost being $884.70?

LONGITUDE AND TIME.

NOTE. — This topic should be taught in connection with the study of Mathematical Geography. The globe should be used to show the pupils that all places on the same meridian have the same time, that a difference in longitude of 15 degrees produces a difference in time of 1 hour, and that the more easterly of two places has the later time.

1001. Oral Problems.

1. The difference in time being 1 hour for each 15 degrees, find the difference in longitude between two cities differing in time 3⅓ hours.

2. Two places differ in longitude 61 degrees. What is their difference in time?

3. London is 75° east of Philadelphia. When it is 1 o'clock at Philadelphia, what is the time at London?

4. When it is 2 P.M. at London, what is the time at Philadelphia?

5. When it is noon at a city 25 degrees west of Vienna, what is the time at the latter place?

6. How many degrees of longitude correspond to a time difference of 3 hours 40 minutes?

7. What is the difference in longitude between Philadelphia, 75° west longitude, and St. Petersburg, 30° east longitude?

8. When it is 3 P.M. at St. Petersburg, what is the time at Philadelphia?

9. Washington is in 77° west longitude, and uses "standard time," that is, the time of 75° west longitude. What is the difference between the correct time at Washington and its clock time?

10. A town in 84° west longitude uses standard time (of 90°). What is the correct time when the clocks are striking 12, noon?

1002. Slate Problems.

1. Find the difference in longitude between two places differing in time 3 hr. 44 min.

2. Two places differ in longitude 37° 18′. What is their difference in time?

3. Chicago is 87° 35′ west of Greenwich. What is the difference in time between the two places?

Is it earlier or later than noon at Chicago when it is noon at Greenwich? Why?

What is the standard time at Chicago when it is 1 P.M. at Greenwich?

4. When a captain's observation of the sun shows that it is exactly noon, the ship's chronometer, keeping Greenwich time, reads 30 minutes past 2 P.M. How many degrees west of Greenwich is the vessel?

5. Find the difference in time between two places in longitude 74° 31′ and 93° 14′ west of Greenwich, respectively.

6. When it is noon at a place 11° east of Greenwich, it is 1.30 P.M. at another place. Find the longitude of the latter place.

7. A train ran from New York to San Francisco, 3,313.5 miles, in 3 da. 12 hr. 17 min. How many miles per hour did it average?

8. If for $6 I can have 1,200 pounds carried 36 miles, how many pounds can I have carried 24 miles for the same money?

9. At 80¢ per ounce, what is the value of 36 ingots of silver, each weighing 2 lb. 10 oz. 15 pwt.?

10. Find 30 per cent of 27 yards 8 inches.

11. The solid contents of a block 12 feet 6 inches wide and 3 feet 9 inches thick are 27 cubic yards 1 cubic foot 810 cubic inches. Required its length.

12. A farmer sold 237 bushels 3 pecks of wheat, which was 48 per cent of his crop. How many bushels, pecks, etc., did he have left?

(48% is given; you have to find what %? What part of 48% added to itself will give the required per cent? Do not find the whole crop.)

13. How many spoons, each weighing 2 ounces 12 penny-weights, can be made from 4 pounds 4 ounces of silver?

14. A man travels due west, on the 45th parallel of latitude, 34 miles per hour for 24 hours. How many degrees has he traveled, the length of a degree being 48.96 miles?

REVIEW.

1003. Oral Problems.

1. A puts $600 into business; B, $400; the profits are $500. What is the share of each?

2. Two boys hire a camera for 26 weeks, paying $5.20. How much should be paid by the boy that uses it 12 weeks?

3. New Orleans is 90° west of Greenwich. When it is 2 P.M. at the latter place, what is the time at New Orleans?

4. Find the discount, at 6%, on a note for $300, that has 48 days to run.

5. What will be the cost of 84 yards of muslin at 49¢ a yard?

6. Two men hire a pasture for $84. One puts in twice as many head of cattle as the other. What should each pay?

BILLS OF EXCHANGE.

Exchange for £180 17*s.* 6*d.* NEW YORK, Dec. 14, 1895.

Sixty days after sight of this First of Exchange (Second unpaid), pay to the order of John W. Moran & Bro., One Hundred Eighty pounds sterling, seventeen shillings, six pence,

Value received, and charge the same to account of

To JAMES LENNON & Co.,
London.
No. 39.

PETER COMERFORD & SON.

1005. Slate Exercises.

1. Find the cost of the above bill at $4.87 per £.

£200 = $974.00
20 = _____
£180 = $
10*s.* = 2.435 £½
5*s.* =
2*s.* 6*d.* = _____
$

2. What would be the cost of a cable transfer of £251 11*s.* 9*d.*, at $4.88½ per £?

£250 = $1,221.25 ¼ of £1,000
1 =
10*s.* =
1*s.* =
6*d.* =
3*s.* = _____

The newspapers give quotations of foreign exchange for sight and 60-day bills, also for cable transfers.

1006. The New York quotations for French exchange give the number of francs for $1.

Paris cable transfers	5.16¼ @ 5.15⅞.
Paris bankers' 60 days	5.18⅜ @ 5.18⅛.
Paris bankers' sight	5.16⅛ @ 5.16¼.

1007. The quotations for German exchange give the value in U. S. money of 4 Reichmarks (or marks).

Reichmarks (4) 60 days	95⅛ @ 95¼.
Reichmarks (4) sight	95⅜ @ 95½.

3. Find the cost of a sight bill on Paris for 1,000 francs, at 5.16¼ francs for $1.

4. Find the cost of a 60-days bill of exchange on Berlin for 1,874.35 marks, at 95¼¢ for 4 marks.

5. What will be the face in marks of a sight bill of exchange on Berlin that can be bought for $1,000, at 95½¢ for 4 marks?

6. A New York merchant pays $1,637.50 for a 60-days bill on Paris. What is the face of the bill, the rate of exchange being 5.18½ francs for $1?

7. At $4.88 per £, what will be the face of the sight bill on London that can be bought for $1,500?

$$\frac{\cancel{1500.00}^{18750}}{\cancel{4.88}_{61}} = \frac{18750}{61}$$

£307 7s., etc.
61)18750
450
£23 remainder
20
460s., new dividend.

8. Bought goods in London amounting to £437 5s. 10d. less 4%. How much will I have to pay in Boston for a sight bill of exchange at $4.88½, to settle the account?

9. What will be the cost in Chicago for a 60-day bill on Paris, that will pay for the following articles? Rate, 1 franc = 19½¢.

18 pieces silk, 44 meters each, at 25 francs per meter, less 7½%.
3 pieces of cloth, 50 meters each, at 20 francs per meter, less 5%.
Packing charges, 60.50 francs.

10. I wish to send a sight bill of exchange on Berlin in payment of the following invoice:

4 cases musical instruments amounting to 3,598.60 marks, less 10, 5, and $2\frac{1}{2}\%$.

Freight to Hamburg, 165 kilos, at 4.80 marks per kilo.

At $95\frac{7}{8}¢$ for 4 marks, what will be the cost of the bill of exchange?

CHAPTER XIII.

PARTIAL PAYMENTS. — RATIO AND PROPORTION. — SQUARE ROOT. — SURFACES AND VOLUMES.

PARTIAL PAYMENTS.

1008. U. S. Rule.

DULUTH, MINN., Jan. 5, 1889.

On demand, I promise to pay to the order of Owen McGee Three Hundred Dollars, value received, with interest at 7 per cent.

$\$300\frac{00}{100}$. J. RANDOLPH PAGE.

Endorsements: May 20, 1889, $100; Oct. 30, 1889, $100; March 6, 1890, $50.

How much was due Jan. 5, 1891?

Find amount of $300 Jan. 5, 1889, to first payment May 20, 1889, 4 mo. 15 da. (by compound subtraction).	$307.88
Deduct first payment,	100.00
Balance May 20, 1889,	$207.88
Interest on $207.88 to Oct. 30, 5 mo. 10 da.,	6.47
Amount,	$214.35
Less second payment,	100.00
Balance Oct. 30, 1889,	$114.35
Interest on $114.35 Oct. 30 to March 6, 4 mo. 6 da.,	2.80
Amount,	$117.15
Less third payment,	50.00
Balance March 6, 1890,	$67.15
Interest on $67.15 March 6 to Jan. 5, 9 mo. 29 da.,	3.90
Due Jan. 5, 1891,	$71.05

1009. Slate Exercises.

NOTE. — Find time by compound subtraction.

1. How much is due June 3, 1896, on a demand note for $1,200, with interest at 6%, dated June 3, 1893, bearing endorsements of payment of $500, Sept. 18, 1894; $600, Jan. 3, 1895?

2. A demand note for $600, bearing interest at 5%, was given Feb. 18, 1892. A payment of $250 was made May 28, 1893; one of $150 was made Oct. 8, 1893. How much is due Jan. 23, 1895?

3. Note for $2,000; interest, 7%; dated April 15, 1891. Endorsements: $50, Sept. 20, 1891; $100, May 26, 1892; $1,000, June 20, 1893. How much is due Dec. 27, 1894?

1010.

Face of note,	$2,000.00
Interest from April 15 to Sept. 20, 1891, 5 mo. 5 da.,	60.28
Amount due Sept. 20, 1891,	$2,060.28
If the $50 payment were deducted, and interest computed on the balance, $2,010.27, the maker would be charged interest on $10.27 more than the face of the note, and this the law does not allow. Interest is taken on $2,000 until next payment, May 26, 1892, 8 mo. 6 da.,	95.67
Amount due May 26, 1892,	$2,155.95
As the two payments are not large enough to meet the interest now due, the interest is again calculated on the original $2,000 from May 26, 1892, to June 20, 1893, 1 yr. 24 da.,	149.33
Amount,	$2,305.28
Less $50 + $100 + $1,000 (three payments),	1,150.00
Balance due June 20, 1893,	$1,155.28
Interest on $1,155.26 to Dec. 27, 1894, 1 yr. 6 mo. 7 da.,	122.87
Due Dec. 27, 1894,	$1,278.15

1011. By the United States rule for partial payments, the amount of the principal is found to the time when the payment, or the sum of two or more payments, equals or exceeds the interest.

From this amount deduct the payment or sum of payments.

Use the balance then due as a new principal, and proceed as before.

4. ALBANY, N.Y., March 5, 1893.

One year after date, I promise to pay John Harrigan, or order, Nine Hundred Dollars, value received, with interest at six per cent.

$\$900\frac{00}{100}$ ANDREW T. SULLIVAN.

Endorsed as follows: June 5, 1893, \$10; Sept. 5, 1893, \$50; Jan. 5, 1894, \$120.

What was due March 8, 1894?

1012. In the United States courts, and in those of some of the states, interest for a portion of a year is taken by days, upon the basis of 365 days to the year.

To make the work easier for the pupils, however, the year of 360 days should be used in the examples given, and the time between dates should be found by compound subtraction.

PRESENT WORTH AND TRUE DISCOUNT.

1016. Problems are frequently met with in books, in which the "present worth" is asked of a sum of money payable at a future date.

1017. 1. What is the present worth of \$150 payable in 1 year 6 months, interest 6%?

By this is meant what sum at 6% interest will amount to \$150 in 1 year 6 months? Or,

Given the amount (\$150), rate 6%, time 1 yr. 6 mo., to find principal.

$$x + (x \times \tfrac{6}{100} \times 1\tfrac{1}{2}) = 150.$$

1018. By "true discount" is meant the difference between the sum payable at a future time and its "present worth."

2. What is the "true discount" of \$150, payable in 1 year 6 months, interest 6%?

The amount \$150, rate 6%, time 1 yr. 6 mo., are given. Find the interest.

Let x = principal

Amount $= x + (x \times \tfrac{6}{100} \times 1\tfrac{1}{2}) = 150$

Interest = amount $- x$

SURFACES AND VOLUMES.

1024. Slate Problems.

1. If a piece of cloth is 20 yards long and $\frac{3}{4}$ yd. broad, how broad is another piece of cloth 12 yards long that contains as many square yards as the former?

2. An iron beam 16 ft. long, $2\frac{1}{4}$ ft. broad, and 8 in. thick, weighs 1,280 lb. What is the length of a similar beam whose breadth is $3\frac{1}{4}$ ft., thickness $7\frac{1}{2}$ in., and weight 2,028 lb.?

3. What will it cost to carpet a room $22\frac{1}{2}$ ft. long by $15\frac{3}{4}$ ft. wide with carpet $2\frac{1}{4}$ ft. wide, costing \$1.50 per yd.?

4. What is the length of a box $6\frac{2}{3}$ ft. wide and $7\frac{1}{2}$ ft. high, that will exactly contain 12 boxes $4\frac{1}{3}$ ft. long, $3\frac{1}{3}$ ft. wide, and $2\frac{1}{2}$ ft. deep?

5. What is the value, at \$120 per acre, of a square field whose side is 35.25 chains?

10 sq. chains = 1 acre.

6. What is the area in square feet of a triangle whose base is 18 ft. 4 in., and whose altitude is 11 ft. 10 in.?

7. What is the area of a circle whose diameter is 7.5 feet, the area of the circle being .7854 times the area of the square that will just enclose it?

8. Find the capacity, in bushels, of a bin 22 ft. long, 14 ft. wide, 12 ft. high?

9. How many gallons will a tank hold, its dimensions being 4 ft. 1 in. by 3 ft. 8 in. by 2 ft. 3 in.?

10. How many square yards are there in the walls and the ceiling of a room 21 ft. long, 18 ft. wide, 12 ft. high? Make a diagram.

11. A tank $5\frac{1}{2}$ ft. by 6 ft. by 7 ft. can be emptied by two pipes, one of which discharges 9 gallons per minute and the

other 7 gallons per minute. How long will it take each to empty the tank? How long will it take both together?

12. A parlor is 18 feet long, 15 feet wide. Make a diagram showing how carpet 27 inches wide can be laid without cutting the carpet lengthwise. Which would be the better way to lay carpet 30 inches wide in the above room?

13. Calculate the number of running yards of carpet 30 in. wide needed for the floor of the above room, including $4\frac{1}{2}$ yards wasted in matching the pattern.

Find the cost of carpeting the room at 95 cents per running yard for carpet, 5 cents per square yard for lining, and 10 cents per running yard for sewing and laying.

14. A rug 18 feet long, 15 feet wide, is placed in the centre of the floor of a room 21 feet long, 18 feet wide. What is the width of the strip left uncovered? Find the area of the uncovered space?

15. A room is 18 feet wide, 24 feet long, 9 feet high. There are two doors 4 feet wide, $7\frac{1}{2}$ feet high; two windows 4 feet wide, 6 feet high; and a fire-place 5 feet square. How many square feet of plastering will there be on the walls and ceiling, deducting for a baseboard 12 inches wide? How many running feet of baseboard will be needed?

Draw "development" of the above room, showing the four walls and the ceiling, and locating the doors, the windows, and the baseboard.

Do not use baseboard where it is not required.

16. At the rate of $1,400 for a pile of lumber 25 ft. long, 20 ft. wide, 10 ft. high, what is the value of a pile 50 ft. long, 40 ft. wide, 20 ft. high?

17. If it costs $14 to paint the walls and the ceiling of a room 25 ft. long, 20 ft. wide, and 10 ft. high, what will it cost to paint the walls and the ceiling of a room 50 ft. long, 40 ft. wide. and 20 ft. high?

SQUARE ROOT.

1029. Squaring a number is multiplying the number by itself. The square of $8 = 8 \times 8 = 64$.

1030. The square of a number is indicated by writing a small 2 a little to the right of the upper part of the number.

$$5^2 = 25, \quad 12^2 = 144.$$

What is the square of 4? Of 6? Of 7? Of 9? Of 10? Of 11?

$$2^2 = ? \quad 3^2 = ?$$

Square 13. 15. 21. 16. 19. $14^2 = ?$ $17^2 = ?$ $24^2 = ?$ $33^2 = ?$

1031. The square of $25 = (20 + 5) \times (20 + 5)$.

$$\begin{array}{ll} & 20 + 5 \\ & 20 + 5 \\ \text{Multiplying by 20} & 20^2 + \quad 20 \times 5 \\ \text{Multiplying by 5} & \qquad\quad 20 \times 5 + 5^2 \\ & 20^2 + 2(20 \times 5) + 5^2 = 400 + 200 + 25 = 625. \end{array}$$

1032. The square of the sum of two numbers is equal to the square of the first + twice the product of the first by the second + the square of the second.

$$13^2 = (10 + 3)^2 = \quad 10^2 + 2(10 \times 3) + 3^2 = ?$$
$$18^2 = (10 + 8)^2 = 100 + 160 + 64 = ?$$
$$27^2 = (20 + 7)^2 = 400 + 280 + 49 = ?$$

1033. Oral Exercises.

Square:

1. 14	4. 22	7. 51	10. 32	13. 24
2. 15	5. 31	8. 61	11. 42	14. 33
3. 21	6. 41	9. 23	12. 52	15. 43

1034. The square root of 4 is 2; of 9 is 3; of 16 is 4; of 25 is 5.

1035. Give the square root of 36. Of 64. Of 81. Of 121. Of 49. Of 100. Of 144.

1036. The sign of square root is $\sqrt{}$.

$\sqrt{81} = 9.$ $\sqrt{121} = ?$ $\sqrt{25} = ?$ $\sqrt{49} = ?$

1037. Find the square root of 169.

$10^2 = 100$. $20^2 = 400$. The square root is between 10 and 20; it is, therefore, 10 + a second number.

$$169 = 10^2 + 2(10 \times \text{second}) + \text{second}^2.$$
$$169 = 100 + 20 \times \text{second} + \text{second}^2.$$
$$20 \times \text{second} + \text{second}^2 = 69.$$

From this it appears that the second number is 3, since

$$20 \times 3 + 3^2 = 69.$$

1038. It may be shown in this way:

$$\begin{array}{rr} & 10 \text{ (first number)} \\ & \overline{169} \\ 10^2 = & 100 \\ \hline \text{Trial divisor — twice 10} \quad 20) & 69 (3 \text{ second number}) \\ & 60 \\ & \overline{9} \\ 3^2 = & 9 \end{array}$$

Ans. 10 + 3 = 13.

1039. Find the square root of 2,116.

$$\begin{array}{rr} & 40 \text{ (first number)} \\ & \overline{2{,}116} \\ 40^2 & 1{,}600 \\ \hline 40 \times 2 = 80, \text{ trial divisor} \quad) & 516 (6 \text{ second number}) \\ & 480 \\ & \overline{36 = 6^2} \end{array}$$

Ans. 46.

1040. Instead of multiplying the trial divisor by the second number, and then ascertaining whether the remainder is the square of the second number, the second number is added to the trial divisor and this sum is multiplied by the second number.

$$\begin{array}{r} \underline{40} \text{ (first number)} \\ 2{,}116 \\ \underline{1{,}600} \\ (2 \times 40) + 6 = 86) \quad 516 (6 \text{ second number}) \\ 516 \end{array}$$

Ans. 46.

1041. In practice, the work is shortened by omitting the ciphers.

First, point off in periods of two figures, commencing at the right. Find the greatest square in the first period, and place the root in the quotient. Subtract the square from the first period. Bring down the next period. Multiply the first quotient figure by 2, and use it as a trial divisor. Place the second figure in the quotient. Affix it also to the trial divisor. Multiply the two figures in the trial divisor by the second quotient figure.

$$\begin{array}{r} 4\ 6 \\ \hline 21'16 \\ 16 \\ \hline 86)\ 516 \\ 516 \\ \hline \end{array}$$ *Ans.*

1042. Slate Exercises.

Extract the square root:

1. 196	**6.** 1,296	**11.** 2,809	**16.** 5,625
2. 256	**7.** 1,225	**12.** 2,916	**17.** 6,889
3. 324	**8.** 1,764	**13.** 3,721	**18.** 7,056
4. 576	**9.** 1,936	**14.** 3,969	**19.** 8,281
5. 676	**10.** 2,601	**15.** 5,184	**20.** 9,025

REVIEW.

1043. Slate Exercises.

Divide (Arts. 385, 616):

1. 4,270,978,096 ÷ 564,347	**6.** 2,171,008,895 ÷ 721,985
2. 4,375,621,423 ÷ 856,789	**7.** 86,409,429,120 ÷ 876,008
3. 4,518,821,072 ÷ 752,134	**8.** 57,681,954,968 ÷ 768,437
4. 3,817,832,184 ÷ 607,432	**9.** 40,333,410,989 ÷ 568,709
5. 3,462,706,614 ÷ 567,843	**10.** 53,531,676,960 ÷ 678,432

Write answers (Art. 385):

1. $\frac{450,000}{86,432}$	**4.** $\frac{700,000}{59,084}$	**7.** $\frac{901,020}{98,642}$
2. $\frac{500,000}{72,356}$	**5.** $\frac{683,427}{67,805}$	**8.** $\frac{385,093}{76,057}$
3. $\frac{583,217}{64,587}$	**6.** $\frac{701,380}{58,437}$	**9.** $\frac{673,217}{85,607}$

1044. An Invoice (English).

Invoice of 3 bales Linen Goods forwarded by rail to Glasgow, for shipment thence per S.S. Anchoria to New York, to order, and for account and risk of Messrs. Robinson & Co.

			yd.				
[R] Co.	#2	30 pcs. Bord. Crash	1500	$1\frac{7}{8}$ *d.*	£11	14	$4\frac{1}{2}$
		30 " " "	1500	2 "			
	#3	60 " Checked G. C.	3000	$1\frac{7}{8}$ "			
	#4	60 " " "	2889	$2\frac{5}{12}$ "			
					£		
		Less $2\frac{1}{2}$% disct.					
					£		

1. Find the duty in U. S. money at 50% ad valorem.

£ = $4.8665.

2. What is the cost in English money of crockery amounting to £166 13*s.* 4*d.* less a discount of 5 and 5%?

RATIO.

1045. Ratio is the relation which one number has to another of the same kind.

1046. The first term of the ratio is called the antecedent; the second, the consequent.

The ratio of 3 to 6, $9 to $18, 15 cows to 30 cows may be expressed $\frac{3}{6}$, $\frac{9}{18}$, $\frac{15}{30}$. They are each equal to $\frac{1}{2}$.

1047. Oral Exercises.

Express the ratio in lowest terms:

1. 175 to 700

$\frac{175}{700} = \frac{1}{4}$

2. $19 to $95

3. $36.50 to $18.25

4. 3 quarts to 4 gallons

NOTE. — The denominations must be the same.

3 quarts to 16 quarts = $\frac{3}{16}$

5. 6 pecks to 5 bushels
6. 20 mills to 1 dollar
7. 7 tenths to 3 fifths
8. 1 gallon to 500 cu. in.
9. 1 mark (23.8¢) to 1 franc (19.3¢)
10. 1 shilling (24.33¢) to 1 dollar

1048. Sight Exercises.

1. $\frac{3}{16}=\frac{?}{64}$
2. $\frac{18}{37}=\frac{36}{?}$
3. $\frac{15}{13}=\frac{?}{65}$
4. $\frac{1 \text{ pk.}}{3 \text{ bu.}}=\frac{\$?}{\$24}$
5. $\frac{3 \text{ qt.}}{1 \text{ gal.}}=\frac{30¢}{?¢}$
6. $\frac{17}{21}=\frac{51}{?}$
7. $\frac{18}{?}=\frac{36}{70}$
8. $\frac{?}{24}=\frac{57}{72}$
9. $\frac{\$16}{?}=\frac{7 \text{ marks}}{21 \text{ marks}}$
10. $5 \div 22 = ? \div 88$
11. 6 horses $\div$? horses $= \$600 \div \$900$
12. 1 ft. $\div$? yd. $= 15¢ \div 90¢$
13. 1 qt. 1 pt. $\div$ 1 pt. $= ?¢ \div 4¢$
14. $1\frac{1}{4} \div \frac{3}{4} = \frac{?}{8} \div \frac{6}{8}$
15. $2.8 \div .4 = .14 \div x$

1049. Oral Problems.

1. One line is a rod long, another is $5\frac{1}{2}$ ft. long. What is the ratio of the first to the second?

2. What is the ratio of 7 hours to 1 day?

3. A pound of coffee costs 30¢, of sugar 6¢. What is the ratio of their respective prices?

4. A walks in 4 hours as far as B in 5. What is the ratio of A's speed to B's?

5. E earns in 6 days as much as D earns in 8 days. Find the ratio of E's daily earnings to D's.

6. One wheel makes 300 revolutions in 2 minutes, the second requires only $1\frac{1}{2}$ minutes to make the same number. Find the ratio of the number of revolutions made by the first wheel in 1 minute to the number made by the second wheel in the same time.

7. A circle whose diameter is 1 ft. has a circumference of $3\frac{1}{7}$ ft. What is the ratio of the diameter to the circumference?

8. One train goes 40 miles an hour, a second goes 45 miles an hour. What is the ratio of the speed of the first to that of the second?

9. A window is 6 ft. 4 in. high by 4 ft. 2 in. wide. What is the ratio of the height to the width?

10. A father is 36 years old, his son is 9. What was the ratio 6 years ago of the father's age to that of the son?

1050. Slate Problems.

(Be sure your answer is correct.)

1. One line is 3 rods 4 yards long, the length of another is 5 rods 1 ft. Find the ratio of the first to the second.

2. One candle lasts 4 hours 20 minutes, another lasts 3 hours 15 minutes. Find the ratio of the first to the second.

3. A pound of coffee costs $25\frac{7}{8}$¢; 1 lb. of sugar costs $5\frac{3}{16}$¢. What is the ratio of price of sugar to that of coffee?

4. M walks in 1 hour 47 min. as far as N walks in 2 hours 3 minutes. What is the ratio of M's speed to N's?

5. P earns in $19\frac{4}{7}$ days as much as Q in $18\frac{3}{4}$ days. What is the ratio of Q's daily earnings to P's? Of P's to Q's?

6. One wheel makes 600 revolutions in $8\frac{4}{7}$ seconds; a second makes 300 revolutions in $3\frac{1}{3}$ seconds. What is the ratio of the speed of the first wheel to that of the second?

7. The circumference of a circle is 12.5664 ft., and its radius is 2 ft. What is the ratio of the diameter to the circumference?

8. One train goes 40 miles in 50 minutes, another goes 24 miles in a half hour. What is the ratio of the speed of the second to that of the first?

9. One window is 6 ft. 8 in. × 4 ft. 2 in.; a second is 4 ft. 8 in. × 2 ft. 1 in. What is the ratio of the area of the second to that of the first?

10. A mother is now 35 years old, and her son is 3 years and 6 months old. Fourteen months ago, what was the ratio of the mother's age to that of her son?

11. A farm costing $4,750 was sold for $5,750. What is the ratio between the profit and the cost?

12. A man can do a piece of work in $4\frac{2}{7}$ days. What part of it can he do in a day and a half? What decimal? What per cent?

13. What is the ratio between a ton of 2,000 pounds and one of 2,240 pounds?

INTEREST AND DISCOUNT.

1052. Slate Exercises.

(Solve the first ten by aliquot parts.)

Find the amount:

1. $1,875.25, 3 yr. 5 mo. 15 da., $4\frac{1}{2}\%$.
2. $487.50, 1 yr. 10 mo. 25 da., 6%.
3. $1,206.84, 2 yr. 1 mo. 16 da., 5%.
4. $595.00, 7 yr. 7 mo. 7 da., 7%.

7 mo. = $\frac{7}{12}$ of 7 yr. 7 da. = what part of 7 mo.

5. $763.25, 8 mo. 11 da., 4%.

6. $685.70, 19 mo. 5 da., 3½%.

7. $1,563.00, 3 mo. 20 da., 5%.

8. $998.45, 87 da., 4%.

9. $2,575.50, 149 da., 3%.

10. $693.27, 214 da., 6%.

1053. Find the value of x:

11. Principal, $240; rate, x; interest, $32.04; time, 2 yr. 11 mo. 18 da.

12. Principal, x; rate, 6%; amount, $717.40; time, 3 yr. 3 mo. 4 da.

13. Principal, $360; rate, 3%; interest, $48.87; time, x.

14. Principal, $288; rate, 2½%; amount, $307.22; time, x.

15. Principal, x; rate, 6%; interest, $13.10; time, 4 mo. 11 da.

16. Principal, $270; rate, x; amount, $273.27; time, 3 mo. 19 da.

1054. Distinguish between "term" and "time." Term of a 90-day note is 93 days. (See Arts. 939 and 941.)

17. Term, x; face, $600; discount, $6.30; rate, 6%.

18. Term, 33 days; face, x; proceeds, $397.80; rate, 6%.

19. Time, 90 days; face, $300; proceeds, x; rate, 6%.

20. Term, 21 days; face, $600; discount, $2.45; rate, x.

21. Time, 4 mo.; face, $200; discount, x; rate, 6%.

22. Term, 132 days; face, x; proceeds, $2,689.50; rate, 6%.

23. Term, x; face, $150; proceeds, $147.75; rate, 6%.

24. Term, x; face, $1,650; discount, $4.95; rate, 6%.

25. Time, 69 days; face, x; proceeds, $469.30; rate, 6%.

REVIEW.

1055. Find products:

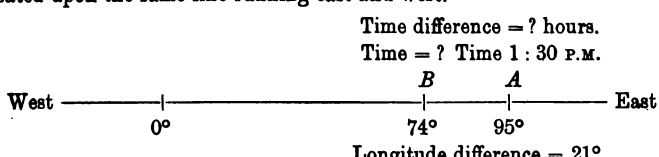
1.	$648 \times \frac{7}{8}$	11.	$1{,}864 \times 250$
2.	$976 \times 1\frac{5}{16}$	12.	983×51
3.	$1{,}648 \times 87\frac{1}{2}$	13.	$1{,}576 \times 62\frac{1}{2}$
4.	$2{,}592 \times 9\frac{5}{16}$	14.	$176 \times 23\frac{3}{4}$
5.	$2{,}416 \times 875$	15.	$1{,}128 \times 375$
6.	874×99	16.	$895 \times 44\frac{4}{5}$
7.	848×125	17.	864×486
8.	375×999	18.	975×318
9.	792×25	19.	$37\frac{2}{3} \times 6\frac{1}{9}$
10.	457×16	20.	$48\frac{3}{4} \times 4\frac{2}{3}$

LONGITUDE AND TIME.

1056. NOTE. — Making diagrams, as shown below, may assist the pupil to solve the problems.

1. Given the longitude of A as 95° east, and that of B as 74° east, and the time at A as 1 : 30 P.M., to find the time at B.

Since the latitude of B has no bearing upon its time, both places may be located upon the same line running east and west.

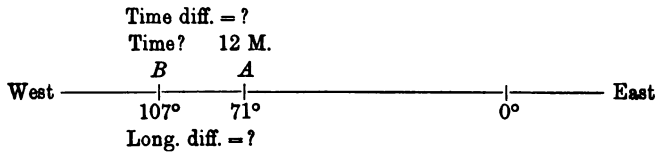


Locate the prime meridian (that of 0°), then the meridians of 74° and 95° east. Mark above the last two the names of the places, B and A. Write above A its given time, 1 : 30 P.M.

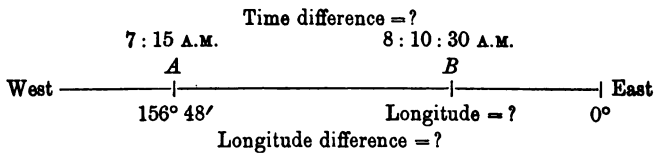
To find the time at B, we must find the difference of time between B and A. The difference in longitude is 95° − 74° = 21°. The difference in time is (21 ÷ 15) hours.

NOTE. — Remember that the more easterly of the two places has the later time.

2. A is situated in 71° west longitude, B in 107° west longitude. What time is it at B, when it is noon at A?

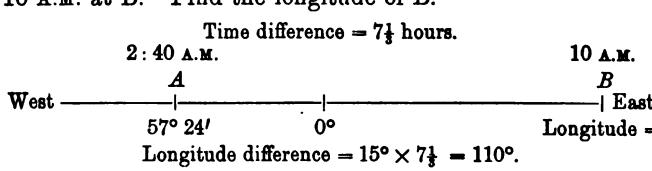


3. Find the longitude of B, whose time is 8 : 10 : 30 A.M., when it is 7 : 15 A.M. at A, whose longitude is 156° 48′ west.



Since B has the later time, its location is east of A. The difference in time, being nearly an hour, shows the difference in longitude to be nearly 15°. Find the exact difference. Is it to be added to 156° 48′ or subtracted from it, to give the longitude of B?

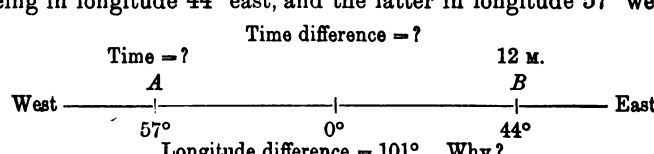
4. When it is 2 : 40 A.M. at A, in 57° 24′ west longitude, it is 10 A.M. at B. Find the longitude of B.



Longitude difference = 15° × 7⅓ = 110°.

If we go 110° eastward from A, we shall reach the prime meridian after going how many degrees and minutes? How many more degrees and minutes must we travel to reach B? Is B in east or in west longitude?

5. When it is noon at B, what is the time at A, the former being in longitude 44° east, and the latter in longitude 57° west?



Longitude difference = 101°. Why?

1057. Find the longitude or the time:

	Longitude of *A*.	Time at *A*.	Longitude of *B*.	Time at *B*.
6.	63° east	9 A.M.	54° east	?
7.	57° 25′ east	?	83° 20′ east	1 : 45 P.M.
8.	156° 48′ west	3 : 15 P.M.	?	4 : 10 P.M.
9.	?	11 : 42 A.M.	56° 25′ west	1 : 27 P.M.
10.	2° 15′ west	6 : 53 A.M.	67° 48′ east	?
11.	27° 10′ east	?	27° 10′ west	12 M.
12.	?	4 : 10 P.M.	18° 4′ east	11 : 30 A.M.
13.	74° 56′ west	3 : 50 A.M.	?	11 A.M.
14.	4′ 30″ east	8 : 47 A.M.	90° 15′ west	?
15.	?	10 : 30 P.M.	32° 30′ east	6 : 48 P.M.

SQUARE ROOT.

1058. Find the square root:

1.	$\frac{4}{25}$	**5.**	$\frac{49}{144}$	**9.**	$\frac{5929}{9025}$
2.	$\frac{9}{64}$	**6.**	$\frac{225}{1024}$	**10.**	$\frac{7921}{8649}$
3.	$\frac{169}{441}$	**7.**	$\frac{324}{2209}$	**11.**	$\frac{2304}{4225}$
4.	$\frac{36}{121}$	**8.**	$\frac{529}{3844}$	**12.**	$\frac{3025}{5041}$

NOTE. — Before extracting the square root of the following, reduce the mixed numbers to improper fractions.

13.	$12\frac{1}{4}$	**16.**	$4\frac{124}{225}$	**19.**	$156\frac{1}{4}$
14.	$11\frac{1}{9}$	**17.**	$6\frac{265}{324}$	**20.**	$264\frac{1}{16}$
15.	$2\frac{46}{49}$	**18.**	$10\frac{311}{625}$	**21.**	$102\frac{33}{64}$

SPECIAL DRILLS.

1060. Find sums:

$11+55+99$	$92+18+48$	$25+84+58$	$45+56+68$
$46+22+88$	$36+71+57$	$66+15+96$	$75+34+86$
$33+76+37$	$89+28+64$	$27+19+87$	$41+65+59$
$66+42+54$	$53+47+35$	$69+73+38$	$52+39+78$

1061. Find answers:

$150-23-48$	$162-(26+61)$	$174-41-35$	$165-(28+47)$
$172-19-66$	$154-(36+44)$	$163-38-43$	$171-(32+33)$
$183-37-42$	$184-(39+35)$	$155-49-24$	$180-(18+28)$
$161-12-71$	$173-(57+17)$	$181-47-33$	$153-(45+31)$

1062. Find products:

36×31	54×51	72×71	90×89
36×29	54×49	72×69	90×91
45×41	63×61	81×81	99×99
45×39	63×59	81×79	101×99

1063. Find quotients:

$576\div18$	$448\div14$	$600\div24$	$225\div12\frac{1}{2}$
$693\div21$	$533\div13$	$675\div75$	$150\div6\frac{1}{4}$
$608\div19$	$924\div22$	$825\div75$	$825\div37\frac{1}{2}$
$848\div16$	$943\div23$	$525\div75$	$750\div62\frac{1}{2}$

1064. Find answers:

$315\times1\frac{14}{15}$	$32\times39\frac{7}{8}$	$7\frac{1}{2}\times7\frac{1}{2}$	$18\frac{2}{5}\times5\frac{1}{3}$
$378\times1\frac{17}{18}$	$45\times19\frac{14}{15}$	$8\frac{1}{2}\times8\frac{1}{2}$	$12\frac{1}{2}\times6\frac{1}{5}$
$97\frac{1}{2}\div3$	$105\frac{1}{5}\div12$	$155\frac{1}{8}\div7$	$19\frac{1}{5}\div2\frac{2}{5}$
$85\frac{1}{4}\div11$	$109\frac{1}{4}\div4$	$200\frac{2}{3}\div18$	$25\frac{2}{3}\div3\frac{1}{3}$

1065. Oral Problems.

1. Paid 92¢ for coffee, 48¢ for butter, and 18¢ for lard. How much was my bill?

2. I had $150. Spent $23 for a suit of clothes and $48 for tools. How much was left?

3. What is the area of a field 36 yd. by 31 yd.?

4. 600 hours = how many days?

5. What is the cost of a cow if I pay $630 for 15?

6. How many ounces in $29\frac{1}{2}$ lb.?

7. $109\frac{1}{4}$ lb. sugar are divided among 4 people. What is the share of each?

8. At $1\frac{9}{10}$¢ per lb., how many lb. iron can I get for $5.70?

9. What will be the cost of 51 tons iron at $17 per ton?

10. What will be the average age of 9 boys, each 12 years old, and 6 boys, each 10 years old?

11. At 42 miles per hour, how long will it take a train to go 882 miles?

12. At 25¢ per hour, what will a man earn in 18 days of 10 hours?

13. What will be the net price of an article whose catalogue price is $20.00, the discount being 90 and 10%?

14. A man had $181 in bank. What will be his balance after taking out $47 and $33?

15. How many feet in 14 rods?

16. 77 yards = how many rods?

17. How many sq. yd. are there in a floor $10\frac{2}{3}$ yd. long and $6\frac{1}{2}$ yd. wide?

18. Cost of 372 eggs at 15¢ per doz.

19. A man owns 3 farms containing 65 acres, 86 acres, and 98 acres, respectively. How many acres does he own?

20. What is the area of a piece of glass measuring $8\frac{1}{2}$ by $6\frac{1}{4}$ inches?

21. What is the value in U. S. money of 50 marks at $23\frac{8}{10}$ cents?

22. How many francs will a calf cost, if 18 are worth 630 francs?

23. A man spends \$1,740 per year. What is the average amount spent per month?

PROPORTION.

1068. A ratio is generally expressed by the sign (:). This is another form of the division sign (÷).

1069. Two equal ratios form a *proportion.*

$3 \div 9 = 13 \div 39;$

or, $3 : 9 = 13 : 39;$

or, $3 : 9 :: 13 : 39.$

1070. Supply missing term:

1. $\frac{1\frac{2}{4}}{6\frac{2}{3}} = \frac{21}{?}$
2. $3\frac{1}{2} \div 16 = \frac{7}{8} \div x$
3. $5 : 7 :: 12\frac{1}{2} : x$
4. $\frac{14}{45} = \frac{x}{90}$
5. $3 \div x = 12 \div 20$
6. $? : 19 :: 28 : 76$
7. 1 lb. 1 oz. : 2 lb. 4 oz. : : 17¢ : x¢.
8. 3 qt. 1 pt. ÷ 1 gal. = x¢ ÷ 80¢.
9. 4 bottles : x bottles = 6 pints : 15 pints.
10. x men : 9 men = 16 acres : 36 acres.

1071. The first and the last term of a proportion constitute the *extremes;* the second and the third, the *means.*

$$5 : 15 :: 9 : 27$$

5 and 27 are the extremes. 15 and 9 are the means. The proportion is read: 5 is to 15 as 9 is to 27.

1072. The proportion $3 : 4 :: x : y$ may be written $\frac{3}{4} = \frac{x}{y}$.

Clearing of fractions, we have $3y = 4x$; *i.e.*, the product of the extremes is equal to the product of the means.

Solve the following:

1073. Make the product of the extremes equal to the product of the means, after canceling any factor common to an extreme and a mean.

1. $x : 5 :: \overset{3}{\cancel{12}} : \cancel{4}$ $\quad x = 15$, *Ans.*
2. $3 : x :: \cancel{5} : \overset{6}{\cancel{30}}$ $\quad x = 18$, *Ans.*
3. $3 : \cancel{4\frac{1}{2}} :: x : \overset{4}{\cancel{18}}$
4. $3 : 4\frac{4}{5} :: 5 : x$
5. $x : 15 :: 4 : \frac{8}{7}$
6. $22\frac{1}{9} : x :: 18 : 13$
7. $\frac{5}{9} : \frac{11}{21} :: x : \frac{11}{24}$
8. $x : \frac{1}{2} :: 2 : 7$
9. $\frac{2}{3} : x :: \frac{1}{4} : \frac{3}{5}$
10. $\frac{1}{5} : \frac{3}{5} :: x : 2\frac{1}{4}$
11. $1 : \frac{5}{8} :: 1\frac{3}{5} : x$
12. $x : \frac{5}{9} :: 11 : 3\frac{1}{3}$
13. $x : 9 :: 4 : x$

1074. Oral Problems.

1. If 9 eggs cost 25¢, what will 3 dozen cost?

2. If 7 lb. flour cost 23¢, what will be paid for 49 lb.?

3. For $5 I can get 12 straw hats. How many can I get for $20?

4. A wheel makes 75 revolutions in 5 minutes. How many does it make in an hour?

5. $100 principal gives $6 interest. How much will be the interest of $450 principal?

6. A merchant pays 75¢ freight for 125 lb. of merchandise. How much will be the freight on 1,000 lb. at the same rate?

7. A locomotive goes 3 miles in 4 minutes. How far does it go in an hour?

8. 4 horses can eat a certain quantity of hay in 10 months. How long will it last 20 horses?

9. 12 men can do a piece of work in 15 days. How long will 36 men require?

10. 15 kilos cost 270 francs. What will be the cost of 5 kilos?

1075. Slate Problems.

NOTE. — Solve by proportion or in any other way.

1. If 9 cows cost $267, what will be the cost of 36 at the same rate?

In solving such examples by proportion, we say

$$9 \text{ cows} = \$267,$$

$$36 \text{ cows} = \$x.$$

The ratio of the cost, $267 : x$, must be the same as the ratio of the number of cows, $9 : 36$. Making the proportion, we have

$$9 : 36 :: 267 : x.$$

Canceling, $$x = \$1{,}068.$$

2. 7 bbl. sugar cost $104.32. Find the cost of 42 bbl.

3. A wheel makes 248 revolutions in 5 minutes. How many does it make in 1 hour 20 minutes?

4. A locomotive goes 1.8 kilometers in 4 minutes. How far does it go in an hour?

5. From 9 kilos (kilogrammes) of yarn are made 42 meters of cloth. How many meters of cloth can be made from 165 kilos of yarn?

How many kilos of yarn are needed for 196 meters of cloth?

6. If 17 men receive $357 for a week's work, how much should 24 men receive?

7. If 17 men take 27 days to finish some work, how long would it take 54 men?

17 men take 27 days

54 men take x days

17 or 54 men ?: 17 or 54 men ? :: 27 days : x days.

8. When a sum of money is divided among 48 persons, each receives $27.50. How much would each receive if the same sum were divided among 16 persons?

9. For $85 I can purchase 238 yards of dress goods. How many yards can I purchase for $5?

10. A can do a piece of work in 6 days, B can do it in 7 days. If B's wages are $2.10 per day, how much should A receive per day?

COMMERCIAL DISCOUNT.

1076. Oral.

When the list price is $1, what is the net price after the deduction of each of the following discounts?

1. 30 and 20%	6. 10 and 5%
2. 40 and 10%	7. 20 and 20%
3. 25 and 40%	8. $33\frac{1}{3}$ and 10%
4. 50 and 10%	9. 20 and 15%
5. 40 and 20%	10. 30 and 15%

1077. What single discount is equal to each of the following double discounts?

11. 30 and 30%
12. 20 and 25%
13. 25 and 20%
14. 15 and 30%
15. 40 and 30%
16. 30 and 10%
17. 40 and 5%
18. 50 and 20%
19. 40 and 15%
20. 50 and 15%

1078. Slate Exercises.

Which is the better discount for the buyer?

21. 30 and 20%, or 40 and 10%.
22. 50 and 10%, or 40 and 20%.
23. 20 and 20%, or 30 and 10%.
24. 20 and 15%, or 30 and 5%.
25. 30 and 15%, or 25 and 20%.
26. 30 and 30%, or 50 and 10%.
27. 40 and 30%, or 20 and 50%.
28. 40 and 5%, or 30 and 15%.
29. 20 and 50%, or 60 and 10%.
30. 40 and 15%, or 30 and 25%.

1079. Find the value of x:

1. List price, \$250; selling price, \$$x$; rate, 40 and 10%.
2. List price, \$800; selling price, \$684; rate, x and 5%.
3. List price, \$$x$; selling price, \$90; rate, $33\frac{1}{3}$ and 10%.
4. List price, \$600; selling price, \$378; rate, 30 and x%.
5. List price, \$16; selling price, \$$x$; rate, 30 and 20%.

6. List price, x¢; selling price, 27¢; rate, 50 and 10%.
7. List price, \$5; selling price, \$3.20; rate, x and 20%.
8. List price, \$$x$; selling price, \$73.50; rate, 30 and 30%.
9. List price, \$200; selling price, \$$x$; rate, 25 and 20%.
10. List price, \$1.50; selling price, 60¢; rate, 50 and x%.

SQUARE ROOT.

1080. Find the square root of 425,104.

	6 5 2	
	42'51'04	
12|5	6 51	
130|2	26 04	*Ans.* 652.

1081. In finding any figure of the root after the first, we multiply the other figure or figures by 2 for a trial divisor.

1082. Find the square root of 20,857,489.

	4 5 6 7	
	20'85'74'89	
8|5	4 85	
90|6	60 74	
912|7	6 38 89	*Ans.* 4,567.

1083. Find the square root of

1. 64,516
2. 734.41
3. 1.8769
4. 718.24
5. 14.1376
6. .702244
7. 264.7129
8. .23775376
9. .093636
10. .004761

1084. Slate Problems.

1. What is the profit on 9 boxes of oranges, each containing 20 dozen, bought at $1.10 per hundred and sold at the rate of 18 for 25¢?

2. How long will it take a train to go 176 miles at the rate of 3,520 feet per minute?

3. If .0375 of an acre of land is worth $9, what is $\frac{3}{8\frac{1}{2}}$ acre worth?

4. At £1 1*s.* 7*d.* per barrel, how many barrels of flour can be bought for £161 17*s.* 6*d.*?

5. A, B, and C buy a house for $7,500. A furnishes $2,000; B, $2,500; C, the remainder. The yearly rent, less expenses, is $576. To what amount is each entitled?

6. If 580 tiles, each 6 inches square, will cover a certain area, how many tiles, each 4 inches long and 3 inches wide, will be needed to cover the same area?

7. A man receives $1,500 commission on his yearly sales. What is the amount of his sales, if he is allowed $\frac{1}{4}$ per cent commission?

8. At what rate per cent will $360 produce $3.06 interest in 2 months 12 days?

9. Find the square root of 25.00400016.

10. What will be the capacity, in gallons, of a tank 9 feet long, 6 feet 8 inches wide, and 6 feet 5 inches deep?

11. What decimal multiplied by 312.5 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, .09375, and 2.46?

12. A dealer bought a lot of coal at $4.95 per ton. What was the total cost if he gained $142.50 by selling it at $5.25 per ton?

13. Find the value of $\frac{2\frac{1}{9}+4\frac{5}{12}}{1\frac{3}{4}\times 3}-\frac{1}{6}$ of $6\frac{5}{7}$.

14. The front wheel of a wagon measures 13 feet in circumference. What is the distance traveled in miles, rods, yards, etc., when the wheel has made 527 revolutions?

15. Write in words .349, 300.049, $\frac{340}{1000}$, $300\frac{40}{1000}$.

16. If a bar of silver weighing 4 lb. 6 oz. 12 pwt. is worth £13 8*s*. 4*d*., what is the value (in English money) of a similar bar weighing 7 lb. 9 oz. 12 pwt.?

17. A and B form a partnership. A furnishes $5,000; B, $10,000. During the year A draws $1,500 of the profits and B draws $1,000. At the end of the year the entire business is disposed of for $20,000. What amount should each receive?

18. What per cent is gained on an article bought for 20 per cent less than its value and sold for 20 per cent more than its value?

19. A person loans $750 to M and $1,200 to N at the same rate. From the latter he receives half-yearly $9 more interest than from the former. What is the annual rate of interest?

20. A 4-months note for $375, drawn March 19, was discounted at a bank June 4. Find the proceeds. Rate, 6%.

21. M can do a piece of work in 4 days, N can do it in 5 days, O in 6 days. How long will it take the three together to do the work?

REVIEW FRACTIONS.

1085. Slate Exercises.

1. Divide the sum of $6\frac{3}{7}$ and $1\frac{7}{8}$ by the difference between $2\frac{1}{5}$ and $3\frac{1}{3}$.

2. What is the difference between the sum of $\frac{3}{7}$ and $\frac{5}{9}$ and the product of $\frac{4}{5}$ and $\frac{7}{12}$?

3. What is the product of the sum and the difference of $4\frac{1}{4}$ and $6\frac{1}{8}$?

4. Subtract $\frac{3}{8}$ of $\frac{1}{4}$ from $\frac{11}{16}$; and find the value of $\frac{8}{11}$ of 16*s.* 6*d.*

5. Add $7\frac{5}{9}$, $\frac{3}{4}$ of $\frac{7}{12}$, $\frac{5}{8}$ of $7\frac{2}{3}$, and $\frac{1}{24}$.

6. Reduce $\frac{3}{5}$ of a sq. rod to the fraction of an acre, and find the value of $\frac{7}{64}$ of a ton in pounds and ounces.

7. Reduce $\frac{6966}{18805}$ to its lowest terms, and $\frac{3\frac{1}{3}-2\frac{1}{5}}{3\frac{1}{3}+2\frac{1}{5}}$ to its simplest form.

8. Add $\frac{1}{2}$, $\frac{5}{9}$, $\frac{3}{4}$, and $\frac{1}{3}$; multiply the sum by $\frac{9}{22}$; and subtract the product from 1.

9. Find the value of $9\frac{4}{11}$ meters at $4\frac{5}{9}$ francs per meter.

10. Divide $2\frac{1}{4}$ by $3\frac{1}{2}$, and add the quotient to $\frac{5}{14}$.

11. Multiply $2\frac{2}{15}$ by $16\frac{5}{7}$, and divide the result by $1\frac{1}{2}$ of $2\frac{2}{5}$.

12. Reduce 7*s.* 6*d.* to the fraction of a £, and 7 hr. 12 min. to the fraction of a day.

13. Reduce to its simplest form $\dfrac{2+\frac{1}{6}\text{ of }5\frac{1}{7}}{\frac{5}{7}\text{ of }\frac{1}{2}}$.

14. Add together £$\frac{5}{8}$ and $\frac{1}{7}$ of $5\frac{5}{8}$ shillings.

15. What fractional part of 7 A. 127 sq. rd. is 5 A. 81 sq. rd.?

16. What must be added to $\frac{3}{7}$ of $\frac{5}{8}$ to make it equal to $\frac{9}{10}$ of $3\frac{3}{4}$?

17. $\frac{3}{5}$ of a number is 148. What is the number?

18. If $\frac{5}{8}$ of a field is worth $325, what is the field worth?

19. If $\frac{3}{8}$ of a house is worth $4,900, what is the value of $\frac{1}{8}$?

20. If $\frac{3}{16}$ of a ship is worth £1,273 2*s.* 6*d.*, what is $\frac{5}{12}$ worth?

$\frac{3}{16} - \frac{5}{12} =$ £1,273 2*s.* 6*d.*

Deduct $\frac{1}{12} =$ _____

$\frac{5}{12} =$

DOMESTIC EXCHANGE.

1087. Slate Exercises.

Find the value of x.

(Do not allow days of grace in the case of "sight" drafts.)

	Face of Draft.	Rate of Exchange.	Term.	Rate of Interest.	Cost of Draft.
1.	\$100	20¢ per M. premium	sight		x
2.	x	par	6 days	6%	\$499.50
3.	\$1,800	x	60 days	6%	\$1,778.85
4.	x	$\frac{1}{4}$% premium	sight		\$701.75
5.	\$200	\$1 per M. discount	30 days	6%	x
6.	\$600	$\frac{3}{8}$% premium	x	6%	\$598.95
7.	\$1,000	75¢ per M. discount	x	6%	\$999.25
8.	\$1,200	par	93 days	x	\$1,178.30
9.	\$800	x	24 days	6%	\$796.80
10.	\$400	\$2 per M. premium	9 days	x	\$400.30

APPLICATIONS OF SQUARE ROOT.

1088. Slate Problems.

1. How many inches in the side of a square table top containing 529 square inches?

2. The surface of a square piece of board contains 3 sq. ft. 97 sq. in. What is the length of one side in feet and inches?

(Reduce area to square inches.)

3. How many rods long is a square field containing 90 acres? How many yards of fence would be needed to enclose it?

4. Land surveyors use a measure called a chain. What is its length in feet, 10 square chains being equal to an acre?

It is subdivided into 100 "links." Find the length of a link in inches and decimal.

5. The surface of the six equal faces of a cube is 1,350 sq. inches. What is the length of the cube?

6. Carefully construct a right-angled triangle, base 4 inches, perpendicular 3 inches. Measure the hypotenuse.

Take the square of the length of each side, and endeavor to show the relation between the square of the hypotenuse and the squares of the other two sides.

7. Construct a right-angled triangle, base 3 in., perpendicular $1\frac{1}{2}$ in. Measure the hypotenuse, and see if the relation between this hypotenuse and the other two sides of this triangle is the same as that found in the other triangle.

8. A right-angled triangle has a base 12 inches long; its perpendicular is $3\frac{1}{2}$ inches. What is the length of the hypotenuse?

9. The hypotenuse of a right-angled triangle is 25 inches; its perpendicular is 7 inches. What is the base?

10. The base of a right-angled triangle is 12 feet; the hypotenuse is 13 ft. Find the perpendicular.

1089. Draw a right-angled triangle (Fig. 1). Upon each side construct a square (Fig. 2). From the upper portion of the

Fig. 1.

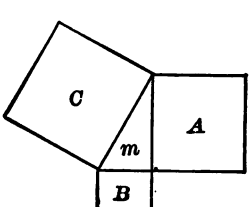


Fig. 2.

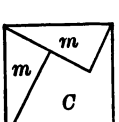


Fig. 3.

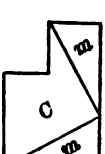


Fig. 4.

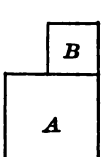


Fig. 5.

largest square C, cut a right-angle triangle of the same dimensions as those of the original triangle m. Cut another triangle of the same dimensions from the left-hand portion (Fig. 3). Place one of these triangles below the remainder of the square C, and the other at the right, as in Fig. 4, and the resulting polygon will be exactly equal in surface to the two squares A and B (Fig. 5).

COMPOUND INTEREST.

1091. Slate Exercises.

1. Find compound interest of \$2,048, for 3 years, at 5%, interest compounded semi-annually.

	\$2,048	Do not use unnecessary figures. $\frac{1}{40}$ each 6 mo.
first ½ year	51.20	
	\$2,099.20	
second ½ year	52.48	Carry to only 4 places of decimals.
	\$2,151.68	
etc.	etc.	

2. Compound interest of \$1,864, at 4%, for 2 years, interest compounded quarterly.

3. Compound interest of \$1,500, at 6%, for 3 years, interest compounded semi-annually.

4. Amount of \$800 at compound interest, for 3 years, at 3%, interest compounded semi-annually.

		\$800.00
first ½ year	1%	8.00
	½%	4.00
		\$812.00
second ½ year	1%	8.12
	½%	4.06
		\$824.18
third ½ year	1%	8.2418
	½%	4.1209
		\$836.5427
	etc.	etc.

STOCKS AND BONDS.

1092. Slate Problems.

Brokerage is calculated on the *par* value. The dividends are based on the *par* value.

1. Find the cost of 240 shares mining stock, par value $10, at 87¾, brokerage ⅛%.

2. Paid $11,460 for 120 shares R. R. stock, par value $100, brokerage ¼%. What was the value of the stock per share?

3. Bought 150 shares canal stock at 87½, brokerage ¼%, paying for it $5,265. What is the par value per share?

4. How much brokerage is paid by the buyer of 275 shares bank stock, par value $100, brokerage ⅛%?

5. A broker sells for a customer 200 shares stock, par value $25, at 102¼. If he retains ⅛% brokerage, how much does he pay over to the former owner of the stock?

6. A man buys 60 shares bank stock, par value $100, at 450, no brokerage. If the annual dividend is 18%, what is his income therefrom? What per cent does he receive on his investment?

7. A manufacturing corporation makes $20,000 a year over all expenses. The stock consists of 4,000 shares, par value $50. What rate of dividend can be declared?

What per cent on his investment does a man receive who has bought his stock at 175, no brokerage?

8. A railroad company's stock consists of 10,000 shares, par value $100. Its profits for the year are $47,500, out of which must be paid the interest for the year on $200,000 worth of bonds, at 5%. What rate of dividend can be paid the stockholders?

9. A capitalist bought 360 shares stock, par value $25, at 168½. He paid therefor, including brokerage, $15,176.25. What was the rate of brokerage?

10. A broker sold 250 shares, par value $100, at 107¾. He deducted brokerage, and paid over the proceeds, amounting to $26,875. Find the amount of the brokerage and the rate per cent.

11. A woman invests $35,050 in stock at 175, brokerage ¼%. If the annual dividends are 7½%, what is her income from the investment?

12. Mr. Tower pays $104 for a $100 five per cent bond. At the end of six years, the bond is redeemed at par. What rate of simple interest does he receive on his investment of $104?

SQUARE ROOT.

1094. Slate Exercises.

Find square roots to three decimal places:

1. 7	3. 38	5. 350	7. 807	9. 1,874
2. 14	4. 74	6. 758	8. 1,384	10. 4,000

1095. Oral Exercises.

What is the square of .1? Of .3? Of .11? Of .12?

1096. How many decimal places in the first two answers? In the last two?

$\sqrt{.01} = ?$ $\sqrt{.09} = ?$ $\sqrt{.0121} = ?$ $\sqrt{.0144} = ?$

1097. Slate Exercises.

Find square roots to three decimal places:

1. .10	5. .4	9. 3.6	13. 4.9	17. .121
2. .40	6. .9	10. 1.60	14. 6.4	18. .144
3. .90	7. 1.6	11. 2.50	15. 8.1	19. .169
4. .1	8. 2.5	12. 3.60	16. 10.0	20. .196

MEASUREMENTS.

1099. Slate Exercises.

Find the missing side of each of the following ten right-angled triangles:

1. Base, 15; perpendicular, 8; hypotenuse, x.
2. Base, 35; perpendicular, x; hypotenuse, 37.
3. Base, x; perpendicular, 15; hypotenuse, 39.
4. Base, 20; perpendicular, 21; hypotenuse, x.
5. Base, x; perpendicular, 45; hypotenuse, 53.
6. Base, 56; perpendicular, x; hypotenuse, 65.
7. Base, 55; perpendicular, 48; hypotenuse, x.
8. Base, x; perpendicular, 14; hypotenuse, 50.
9. Base, 63; perpendicular, x; hypotenuse, 65.
10. Base, 112; perpendicular, 15; hypotenuse, x.

1100. Slate Problems.

11. One parallel side of a trapezoid measures 160 yd., the other measures 200 yd., the area is 32,400 sq. yd. Find the perpendicular.

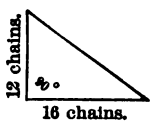
12. One parallel side of a trapezoid is 20 rods, the perpendicular is 15 rods, the area is 225 sq. rods. Find the length of the other parallel side.

13. One parallel side of a trapezoid measures x rods, the other measures $x + 6$ rods, the perpendicular is 10 rods, the area is 150 sq. rods. Find the length of the parallel sides.

14. Find the area in acres of a right-angled triangle, the length of the sides being 24 rods, 7 rods, 25 rods.

15. A court yard 84 ft. by 36 ft. is to be paved with flagstones measuring 6 ft. by 3 ft. How many stones will be needed? What will be the cost of the work at $1.25 per sq. yd.?

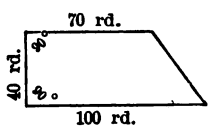
16. How many rods of fence will be needed to enclose the field shown in the diagram?



1 chain = 66 feet.

NOTE. — A right angle contains 90 degrees.

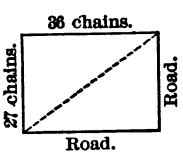
17. Find the length of the fourth side of the following piece of ground.



How many yards of fence are needed to enclose it? How many acres does it contain?

18. What is the length of the diagonal of a rectangular field 90 yd. wide, 120 yd. long?

19. The dotted line in the accompanying diagram indicates a path through the field. How many yards are saved by taking the path instead of following the road?



20. Find the length (in rods and a decimal) of the diagonal of a square 40-acre field.

REVIEW.

1101. Oral Problems.

To the following ten problems the wrong answers are very frequently given.

1. Sold a horse for $250, losing $50. What is the loss per cent?

2. If 3 boys solve 3 problems in 3 minutes, how long will it take 6 boys to solve 6 problems?

3. Two boys go fishing; one brings 7 cakes for lunch, the other brings 5 cakes. A third boy joins them at noon, and pays 12¢ for his share of the dinner. How should the first two divide the money received?

4. If 100 per cent is gained by selling an article for $1, how much would be gained by selling it for $2?

5. A boy had a slate 5 inches by 7 inches. He buys one twice as large. Give the dimensions of the new slate.

6. A man wishes to put up on the front of his lot a fence 30 feet long. If the posts are 6 feet apart, how much will they cost at 25¢ each?

7. One-half the money received by a newsboy is profit. What per cent does he make?

8. 50 per cent of a number multiplied by 30 per cent of the same number equals 60. What is the number?

9. Three-fourths per cent of a number is 90. What is the number?

10. An importer receives some cases of goods numbered consecutively. How many cases are there, if the number of the first is 28, and of the last 75?

1102. Slate Problems.

1. If a bar of silver weighing 2 lb. 3 oz. 6 pwt. is worth £6 13*s.* 7*d.*, what is the value in English money of a similar bar weighing 15 lb. 7 oz. 4 pwt.?

2. A quantity of provisions would last a ship's crew 20 days, allowing each man 2 lb. 4 oz. daily. What should each man be allowed so as to make the provisions last 4 days longer?

3. If 40 men are able to do a piece of work in 10 hours, how many extra men must be employed to finish it in 8 hours?

4. If it requires 40 yd. carpet 2 ft. 9 in. wide to cover a floor, how many yards of carpet 2 ft. 6 in. wide would be needed?

5. How long will it take a train to go 112 miles, at the rate of 46 miles in 1 hour 20 min. 30 sec.?

6. Change 1,759 yards to rods, yards, etc.

7. If a beam 5 ft. 6 in. long, 10 in. wide, and 8 in. thick, weighs 924 lb., find the length of another beam of the same material which weighs 3,024 lb., and whose end is a square foot.

8. A field 110 yd. long and 44 yd. wide contains an acre. What is the area of a field 220 yd. long and 88 yd. wide? Of one 440 yd. long and 176 yd. wide?

9. A ship with a crew of 32 men has provisions that will serve for 45 days, at a daily allowance of 3 lb. for each man. If it then picks up another vessel's crew consisting of 16 men, what must be the daily allowance, to make the provisions last for 40 days?

10. If a steel bar 12 ft. long, 4 in. broad, and $2\frac{1}{2}$ in. thick, weighs 480 lb., what is the weight of another steel bar 18 ft. long, 3 in. broad, and 2 in. thick?

11. If 8 horses eat 13.5 bushels of oats in 9 days, how many days will 15.75 bushels last 14 horses?

12. A person deposits in two banks \$750 and \$1,200, respectively, at the same rate. The latter sum draws \$18 more interest per year than the former. What is the rate per cent?

13. Two men have saved \$2,000 each. One has loaned \$1,400, at 4%, and the remainder at 5%. What rate must the other man receive for his money in order to get the same interest?

14. I owe \$8,625, payable in 3 years 4 months. I have at present \$7,500. What rate of interest must I receive to pay my debt at maturity?

15. A person loans $\frac{1}{3}$ of his capital at 5%, and the other half at 4%. He receives annually \$40 more interest from the former than from the latter. What is his capital?

16. A certain sum loaned at 4% produces \$30 less interest than a sum \$400 greater, loaned at 5%. How much is loaned at each rate?

17. A capitalist has placed $\frac{4}{5}$ of his money at 4%, and his remainder at 5%. His income is $2,940 per year. What is his capital?

18. Change 13,576 inches to rods, etc.

19. Three men buy a lot for $600. After selling it A receives $220 as his share of the proceeds, B receives $280, and C $300. How much did each invest originally?

20. D receives $\frac{1}{3}$ of a sum of money, E $\frac{1}{8}$, and F the remainder. E's share is $90 more than D's. What is the share of F?

21. A man receives $593.70 as the proceeds of his note. 63 days thereafter he pays the bank $600. What rate of interest has the bank charged on the $593.70 loaned?

What is the rate of bank discount on the $600 note?

22. A tank is fed by two pipes, one of which can fill it in 2 hours, and the other in 3 hours. A third pipe can empty it in 1 hour. If, when the tank is full, the supply pipes and the exhaust pipe are all set to work, in what time will it be emptied?

23. What per cent is gained on oranges bought at 20 cents per dozen and sold at the rate of 10 for 25 cents?

EXACT INTEREST.

Exact interest is used by the Government in its calculations. 365 days are taken to the year.

1104. Slate Exercises.

1. Find the exact interest of $280 from April 14 to Sept. 6 at 4%.

Time 145 days. *Ans.* $280 \times \frac{4}{100} \times \frac{145}{365}$.

2. Find the exact interest on $76.65 from March 4 to Dec. 15 at 6 per cent.

3. On $384 at $7\frac{3}{10}$ per cent for 75 days.

4. On $438 at 5% from Jan. 1 to March 15.

5. On $109.50 at $4\frac{1}{2}$% for 87 days.

6. On $847.60 at 5% from April 29 to Sept. 20.

7. On $584 at $3\frac{3}{4}$% from May 16 to Dec. 1.

1105. Unless "exact" or "accurate" interest is specified, use 360 days to the year.

MEASUREMENTS.

1. What is the area of a triangle whose sides measure 15, 16, and 17 inches, respectively?

15
16
17
2)48
24 − 15 = 9
24 − 16 = 8
24 − 17 = 7

From the half sum of the three sides subtract each side separately. The square root of the continued product of the half sum and the three remainders will be the area.

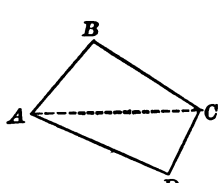
$\sqrt{24 \times 9 \times 8 \times 7} =$

$\sqrt{12{,}096} = 109.98$ sq. in. *Ans.*

2. Find the area in square feet of a triangle whose sides measure 35 ft., 84 ft., 91 ft.

3. Find the area of a triangle whose sides measure 21, 28, and 35 rods, respectively.

4. In the following field, AB measures 39 rods; BC, 52 rods; CD, 25 rods; AD, 60 rods; and the diagonal, AC, 65 rods. Find the area of the field in square rods.



5. Find the area of an isosceles triangle whose base is 30 yards, its equal sides measuring 25 yards.

6. What is the altitude of an isosceles triangle, base 96 ft., equal sides 64 ft.? Find its area.

7. Find the area of an equilateral triangle, each side being 6 ft.

8. Find the area of a right-angled triangle, base 42 ft., hypotenuse 70 ft.

9. Find the area of an isosceles triangle, altitude 48 ft., equal sides 50 ft.

10. Place two equilateral triangles, sides 2 inches, base to base, making a rhombus. Find its area, also the length of each diagonal.

11. Find the radius of a circle whose circumference is 132 ft. (3.1416 × diam. = circum.)

12. Find the area of a circle whose radius is 4 inches. (Area = $\frac{1}{2}$ circumference × $\frac{1}{2}$ diameter.)

13. Find the area of a circle whose radius is x inches.

14. Find the radius of a circle whose area is 314.16 sq. in.

15. Find the area of a circle whose circumference is 15.708 ft.

PARTIAL PAYMENTS.

1108. Merchants' Rule.

BROOKLYN, N.Y., June 19, 1894.

On demand, I promise to pay William R. Budd, or order, Two Thousand Four Hundred Fifty-four $\frac{75}{100}$ Dollars, value received, with interest at 6 per cent.

$2,454\frac{75}{100}$. ARTHUR TOWNSEND.

The following payments are endorsed on the note:

July 5, 1894, $200.
July 29, 1894, $450.
Sept. 18, 1894, $700.
Oct. 25, 1894, $300.

Find the amount due Jan. 2, 1895.

If no payments had been made, there would be due	$2,454.75
And interest from June 19 to Jan. 2, 197 days,	80.60
Total due,	$2,535.35
The credits are: Payment July 5, 1894,	200.00
Interest on $200, July 5 to Jan. 2, 181 days,	6.03
Payment July 29, 1894,	450.00
Interest on $450, July 29 to Jan. 2, 157 days,	11.78
Payment Sept. 18, 1894,	700.00
Interest on $700, Sept. 18 to Jan. 2, 106 days,	12.37
Payment Oct. 25, 1894,	300.00
Interest on $300, Oct. 25 to Jan. 2, 69 days,	3.45
Balance due,	$851.72

1109. By the merchants' rule, interest is calculated on the face of an interest-bearing note from its date until settlement, and interest is allowed on all credits from their payment until settlement.

1110. Slate Exercises.

1. A note for $500, with interest at 6%, is dated July 25, 1893. Payments are made: $100, Sept. 18; $200, Feb. 5, 1894. How much is due April 1, 1894?

2. Find amount due Sept. 15, 1894, on a demand note for $1,875, with interest at 6%, dated Jan. 18, 1894. Payments of $1,000 and $500 were made March 30 and June 17, respectively.

3. June 12, 1892, Robert Colgate bought goods amounting to $600. Dec. 31, 1892, he paid $300; April 5, 1893, $200; June 1, 1893, he settled the account. How much did he pay on that date, if he is charged 6% on the purchase from its date, and is allowed 6% interest on his payments?

4. T. J. Minturn loaned Chas. A. Dorsey $500, Sept. 1, at 6%. Payments of $200 each were made Oct. 1 and Nov. 1. How much is due Dec. 1?

Dr. WILSON T. JONES. *Cr.*

1893.					1893.				
Feb.	5	To merchandise,	840	00	Mar.	9	By cash,	500	00
Dec.	31	To interest to date,			Sept.	13	By cash,	200	00
					Dec.	31	By interest to date,		
					"	"	By cash,		

5. Find the amount paid in settlement of the foregoing account, Dec. 31, 1893. Interest 6%.

6. A merchant's books show the following debits: Feb. 13, merchandise, $725.00; April 14, merchandise, $603.00. The credits are: April 5, cash, $600; Aug. 29, cash, $300. How much is due Oct. 5, interest 6%?

1111. The merchants' rule is frequently used where the transactions all take place within a year. The exact number of days is taken, and the interest is calculated on the basis of 360 days to the year.

CHAPTER XIV.

EQUATION OF PAYMENTS. — MENSURATION OF SURFACES AND VOLUMES. — BOARD MEASURE. — ANNUAL INTEREST. — GOVERNMENT LANDS. — METRIC SYSTEM.

EQUATION OF PAYMENTS.

1114. Oral Problems.

1. A friend loans me $800 for 6 months without interest. How long ought I to loan him $400 to cancel the obligation?

2. In what time would the interest on $450 be the same as the interest for 3 months on $600?

3. W borrows from X $200 for 5 months and $400 for 2 months. How much money should W loan X for one month in return for the accommodation?

4. A man offers a lot for $600, payable $300 in 2 months, and $300 in 4 months. How much credit should be given to a buyer who wishes to pay the $600 at one time?

5. Mr. Jones has bought $600 worth of goods on 6 months' credit, and $300 worth on 3 months' credit. For what time should he give a note (without days of grace) for the whole amount, $900?

1115. Slate Problems.

1. In what time would the interest on $1,000 be the same as the total interest on the following amounts:

$100 for 1 month, $200 for 2 months, $300 for 3 months, $400 for 4 months?

Interest on	$100	for	1	month	=	Interest on	$100	for	1	month
"	"	200	"	2	months	=	"	"	400	"	1	"
"	"	300	"	3	"	=	"	"	900	"	1	"
"	"	400	"	4	"	=	"	"	1600	"	1	"
"	"	1000	"	?	"	=	"	"	3000	"	1	"

2. A person owes $400 payable in 4 months, and $500 payable in 13 months. What would be the average time for the payment of the whole indebtedness of $900?

The debtor is entitled to the use of $400 for 4 months, which is equal to the use of $1,600 for 1 month. He is also entitled to the use of $500 for 13 months, which is the same as $6,500 for 1 month. He is entitled, in all, to the use of $8,100 for 1 month, which is equal to the use of $900 for how many months?

1116. By equation of payments is meant a method of ascertaining at what time several debts due at different times may be settled by a single payment. The time thus found is called the *average* time, or the *equated* time.

3. Find the average, or equated, time for the payment of the following:

$600 due in 2 years
$500 due in 1½ years
$300 due in 1 year
$400 due in 9 months

4. $250 due in 8 months
$450 due in 6 months
$500 due in 3 months
$600 payable in cash

$250 \times 8 =$
$450 \times 6 =$
$500 \times 3 =$
$600 \times 0 = 0$
$1800 \times ? =$

5. $200 due in 15 days
$300 due in 30 days
$400 due in 45 days

6. $840 to be paid in four equal installments in 1, 2, 3, and 4 months, respectively.

7. $960 to be paid $\frac{1}{8}$ in 2 months, $\frac{1}{6}$ in 4 months, $\frac{1}{4}$ in 5 months, and the remainder in 6 months.

8. A debt to be paid $\frac{1}{10}$ in 2 months, $\frac{1}{5}$ in 3 months, $\frac{1}{6}$ in 4 months, and the balance in 12 months.

9. $6,000; $\frac{1}{3}$ to be paid in cash, $\frac{1}{4}$ of the remainder in 3 months, another fourth in 6 months, and the balance in 9 months.

10. On what date should the following account be paid in full?

Bought, July 1, goods to the amount of $300 payable in cash, to the amount of $800 payable in 30 days, and to the amount of $1,000 payable in 60 days.

1117. Miscellaneous.

11. A farmer sold 300 bu. wheat at 92½¢ per bushel, 100 bu. at 90¢, 400 bu. at 95¢, 200 bu. at $1. What was the average price?

12. Three men hire a pasture for $84. One puts in 15 cows for 12 weeks, the second puts in 20 cows for 6 weeks, the third puts in 18 cows for 10 weeks. What amount should each pay?

13. A and B form a partnership. A furnishes $2,000, B $3,000. After a year A furnishes an additional $1,000. At the end of 2 years the business is disposed of for $7,100. How much should each receive?

SUGGESTION: A receives his $3,000 and how much of the profits? Should he receive as much as B, who had $3,000 in the business the whole time?

14. How many bushels of bran worth 40 cents per bushel should be mixed with bran worth 30 cents per bushel to make 100 bushels worth 36 cents a bushel?

x = number of bushels at 40¢

$100 - x$ = number of bushels at 30¢

$40x$ = value (in cents) of one kind

$30(100 - x)$ = value of other kind

Total value = how many cents?

15. How many bushels of corn worth 60¢ per bushel should be mixed with 80 bushels of corn worth 50¢ per bushel to make a mixture worth 52¢ per bushel?

16. A can do a piece of work in 20 days, B can do it in 30 days. They work together and receive $5 per day as the wages of both. What should be A's share of the total amount received?

How long does it take both together to do the work? What would A receive per day if he did the work alone?

17. A partnership is formed between A with a capital of $1,500 and B with a capital of $2,500. Six months thereafter, they take in C with a capital of $4,000. How should a profit of $3,500 be divided at the end of the year?

18. Three merchants shipped a cargo of iron by sea. A sent 180 tons, B sent 105 tons, C sent 315 tons. During a storm the sailors were obliged to throw overboard 180 tons to save the vessel. What portion of the loss should each merchant sustain?

19. If pure milk is reduced in value from 24¢ per gallon to 20¢ per gallon by the addition of water, how many quarts of water have been placed in a can that contains 40 quarts of the adulterated article?

20. Find the entire surface of a cube whose edge measures 7 inches.

21. What is the edge of a cube whose entire surface contains 726 square inches?

MENSURATION OF PLANE SURFACES.

1124. Slate Exercises.

1. Find the circumference of a circle whose radius is x.

(Diameter $\times$ 3.1416.)

2. Find the area of a circle whose radius is x.

($\frac{1}{2}$ circumference $\times$ $\frac{1}{2}$ diameter.)

3. Find the area of a circle whose diameter is x.

4. Find the area of a circle whose circumference is x.

5. What is the area of a circle whose diameter is 36 feet?

6. What is the radius of a circle whose area is 153.9384 sq. yd.?

7. What is the circumference of a circle whose area is 198.95 sq. rods?

8. Find the area of a square whose diagonal is x.

9. Find the area of a square whose diagonal is 150 rods.

10. Find the area of an isosceles triangle, its base being 56 meters, equal sides 100 meters.

11. Find the area of an equilateral triangle whose side is 12 feet.

12. Find the area of a triangle whose sides are 50 yd., 60 yd., 70 yd.

13. What is the area of a circle whose circumference is 10 feet?

(The square of the circumference $\times$ what = area?)

14. Find the area of the rhomboid, Fig. 1.

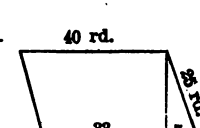


FIG. 1.

15. Of the rectangle, Fig. 2. 16. Of the rhombus, Fig. 3.

17. Of the trapezoid, Fig. 4.

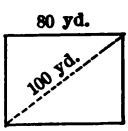


Fig. 2.

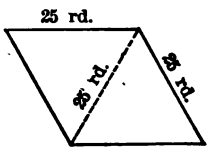


Fig. 3.

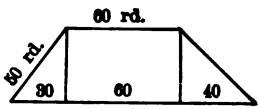


Fig. 4.

18. Of the trapezium, Fig. 5. 19. Of the rhombus, Fig. 6.

20. Find the altitude, AB, of the following triangle, Fig. 7:

(First find the area.)

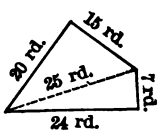


Fig. 5.

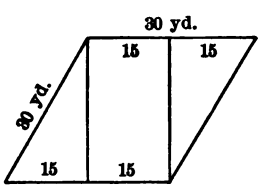


Fig. 6.

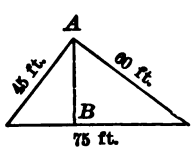


Fig 7.

21. Find the diagonal (in rods) of the square whose area is 5 acres.

22. Find the area of a hexagon, composed of six equilateral triangles, each side being 6 inches. Fig. 8.

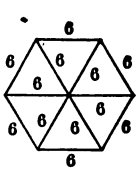


Fig. 8.

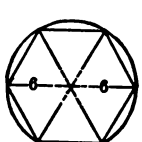


Fig. 9.

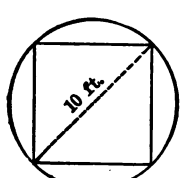


Fig. 10.

23. What is the area of the circle circumscribed about the above hexagon, Fig. 9?

24. What is the area of the square inscribed in a circle whose diameter is 10 feet, Fig. 10?

SPECIAL DRILLS.

1129. Give sums:

112+91+85	43+131+61	95+144+79	68+56+174
129+62+98	26+172+81	63+117+97	63+34+186
182+13+67	75+193+23	91+126+32	91+59+165
114+21+49	132+94+77	63+143+24	78+39+183

1130. Give answers:

150−23+48	172+19−66	183−(72−37)	161+79−12
154−36+44	184+39−35	173−(57−18)	174+41−36
155−49+24	181+47−33	165−(47−28)	171+32−34
153−45+31	151+46−24	182−(48−33)	175+33−46

1131. Give products:

44×20	63×32	33×52	17×72
44×22	54×38	26×58	71×78
44×18	54×42	26×62	71×82
63×28	33×48	17×68	45×88

1132. Give quotients:

$676 \div 13$	$527 \div 17$	$704 \div 22$	$837 \div 27$
$602 \div 14$	$738 \div 18$	$966 \div 23$	$961 \div 31$
$690 \div 15$	$950 \div 19$	$768 \div 24$	$992 \div 32$
$672 \div 16$	$924 \div 21$	$975 \div 25$	$759 \div 33$

1133. Give results:

$84 \times 1\frac{5}{6}$	$211\frac{1}{4} \div 13$	$36 \times 49\frac{5}{6}$	$162\frac{1}{2} \div 25$
$48 \times 2\frac{7}{8}$	$214\frac{3}{8} \div 14$	$32 \times 59\frac{7}{8}$	$158\frac{7}{8} \div 31$
$36 \times 9\frac{1}{2}$	$130\frac{1}{8} \div 21$	49×49	$18\frac{2}{3} \div 3\frac{3}{8}$
$48 \times 19\frac{3}{8}$	$155\frac{1}{2} \div 22$	58×58	$17\frac{3}{4} \div 5\frac{3}{4}$

1134. Oral Problems.

1. A has 96 sheep; B has 28 sheep more than A. How many sheep have both?

2. There are 56 pupils in one class, 48 in a second class, and 52 in a third class. How many pupils are there in the three classes?

3. March 29 is what day of the year 1894?

4. How far is a man from his starting-point, if he travels due east 150 miles, due west 23 miles, due east again 48 miles?

5. A body falls 16 ft. in the first second, three times as far in the second second, five times as far in the third second. How far does it fall in three seconds?

6. The base of a right-angled triangle is 12 ft., the perpendicular is 16 ft. What is the hypotenuse?

7. At $35 per month, what will be the rent of a house for 16 months?

8. A field containing 169 square rods is 13 rods long. How many rods of fence will be needed to enclose it?

9. 25 packages of sugar weigh together 87½ lb. How many pounds are there in each?

10. At 45 miles per hour, how many hours, minutes, and seconds will it take a train to go 230 miles?

11. How many years have elapsed since the invention of gunpowder, 1356?

12. What profit is made on an article bought for $175, less 12%, and sold for $200?

13. How many square rods in a field 71 rods long, 81 rods wide?

14. Assuming a kilo to be 2⅕ lb., how many kilos will be equal to 143 lb.?

15. A degree of longitude in latitude 45° is about 70% of the length of a degree on the equator. Calling the latter length 69 miles, how long is a degree of longitude in latitude 45°?

16. At $44 per acre, how much land can be bought for $968?

17. A number of marbles divided among 29 boys gives each 16 marbles, and leaves a remainder of 26. How many marbles are there?

18. What is the monthly salary of a clerk who receives $1,500 per year?

19. How many revolutions in a mile, 5,280 ft., are made by a locomotive wheel 16 ft. in circumference?

20. How many feet of fence are there around a lot 49 ft. wide, 87 ft. long?

21. How many bricks 8 in. by 4 in. by 2 in. would make a cubic foot?

22. 13 is one factor of 1,001. Find the other two prime factors.

23. What are the three equal factors of 343?

24. What is the square root of 1,225?

25. At $4\frac{1}{2}$ miles per hour, how long will it take a man to walk $37\frac{1}{2}$ miles?

26. What will be the cost of 9 dozen hats at 1.33\frac{1}{3}$ each?

MEASUREMENTS.

1137. Find the area of each of the following triangles and its altitude.

When the area of a triangle is known and the length of the base, how can its altitude be calculated?

1. Base, 51 ft.; other sides, 20 ft. and 37 ft.

2. Base, 21 yd.; other sides, 13 yd. and 20 yd.

3. Base, 148 rods; other sides, 39 rods and 113 rods.

4. Base, 28 chains; other sides, 17 chains and 25 chains.

5. Base, 75 inches; other sides, 20 inches and 65 inches.

1138. Find the areas of the following quadrilaterals:

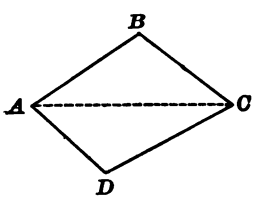
6. Given AB, 17; BC, 10; CD, 20; DA, 13. $AC=$ 21.

7. Given AB, 25; BC, 39; CD, 34; DA, 50. $AC=$ 56.

8. Given AB, 37; BC, 15; CD, 39; DA, 17. $AC=$ 44.

9. Given AB, 111; BC, 45; CD, 25; DA, 113. $AC=132$.

10. Given AB, 113; BC, 17; CD, 39; DA, 111. $AC=120$.



1139. Slate Problems.

1. A and B rented a field for a year for $175. A put in 6 horses for the whole time, B put in 5 horses for 11 months and 3 horses for 5 months. How much of the rent had each to pay?

2. A bankrupt surrenders property worth $1,287 for the benefit of three creditors to whom he owes $750, $1,125, and $1,245, respectively. How much should each creditor receive?

3. Four persons rented a pasture for 26 weeks. K put in 50 sheep and L 60 sheep for the whole time, M put in 70 sheep for 20 weeks, and N 90 sheep for 22 weeks. How much of the rent, $130, had each to pay?

4. A employs a capital of $2,500 in business, and at the end of 3 years takes into partnership B, who furnishes $4,000. Four years later they are joined by C, with a capital of $5,000. At the end of 12 years from the commencement of the business, the profits, amounting to $15,000, are divided. What amount should each receive?

A's money is in the business how many years? B's, how many years? C's, how many?

5. Four butchers rent a field, and pay for 6 months' rent $152.50. The first puts in 20 oxen for 10 weeks and 50 sheep for 8 weeks; the second, 25 oxen for 8 weeks and 30 sheep for 7 weeks; the third, 18 oxen for 10 weeks and 10 sheep for 12 weeks; the fourth, 30 oxen for 12 weeks. What share will each have to pay, counting 3 sheep equal to 1 ox?

6. A wall 700 yards long was to be built in 29 days. At the end of 11 days, 18 men had built 220 yards of it. How many extra men had then to be put to work, so that the wall might be completed in the given time?

7. If 5 needlewomen can do a piece of work in 11 days of 9 hours each, how long will it take 3 needlewomen to do two such pieces, supposing them to work $10\frac{1}{2}$ hours each day?

8. If 14 men can mow 168 acres in 12 days of 8 hours 15 minutes each, how many acres can 20 men mow in 11 days of 7 hours 48 minutes each?

9. If 12 men can do a piece of work in 20 days, what number of men will be required to do four times as much work in a fifth part of the time?

10. A ship sailed with a crew of 60 men, and provisions for 34 days, and 10 days afterwards, 12 persons were received on board from a sinking vessel. How long would the provisions last the 72 persons then on board?

How long would the provisions last the 60 persons at the time the sinking vessel was met?

11. If 76 boards, each 14 feet long and 10 inches wide, are worth $19.76, how much would 50 such boards be worth?

12. If 7 men receive $126 for 5 weeks' work, how much should they receive for 9 weeks' work?

13. If for 7*s.* 6*d.* I can buy 9 lb. of raisins, how many pounds can I buy for £56 16*s.*?

14. A field of grain was to be cut down by 40 men in 10 days. Eight of the men, however, failed to come. How long did it take the others to do the work?

TABLE.

1140. Brooklyn Assessments and Taxes for 10 years.

	Assessments.	Tax Levies.				Average Tax Rate per $1,000 of Valuation.
	Real and Personal.	For State Purposes.	For County Purposes.	For City Purposes.	Total Levy.	
1883	$298,936,506	$ 874,088	$ 1,242,476	$ 5,632,795		
1884	317,853,850	733,669	1,323,861	6,287,462		
1885	330,683,762	889,559	1,307,090	7,383,911		
1886	362,009,202	929,273	1,412,623	7,180,990		
1887	383,851,674	907,663	1,398,310	8,266,643		
1888	407,454,028	940,517	1,682,120	8,503,581		
1889	428,483,681	1,344,023	1,997,414	9,298,236		
1890	452,758,601	949,253	2,009,518	8,709,541		
1891	466,914,249	589,178	2,159,879	9,241,130		
1892	483,738,129	888,297	2,240,613	10,324,617		
Av. of 10 yr.						

Find for each year the total tax levy, and the tax rate in dollars, cents, and mills per $1,000 of assessed value.

Find the average assessment per year; the average tax levy for state, county, and city purposes; and the average tax rate.

SURFACES OF PRISMS AND CYLINDERS.

1141. Slate Exercises.

NOTE. — The pupils should be encouraged to make cardboard models of the forms studied.

1. Find the convex surface of a square prism, one side of its base being 4 inches and its height 6 inches. Draw the development.

NOTE. — The convex surface is the surface exclusive of the bases.

2. Find the convex surface of a triangular prism, each side of whose base measures 4 inches and whose altitude is 6 inches. Draw the development.

3. Find the convex surface of an hexagonal prism, each side of its base being 4 inches and its altitude 6 inches. Draw the development.

4. Can you show that the convex surface of a prism is found by multiplying the perimeter of the base by its altitude (height)?

5. Find the convex surface of a cylinder, the diameter of its base being 4 inches and its height 6 inches.

6. How do you find the entire surface of a prism or cylinder?

7. What is the entire surface of a cube whose side is 7 inches? Of a cube whose side is x inches?

8. The entire surface of a cube is 216 sq. in. What is the length of one side?

9. The convex surface of a cube is 144 sq. in. Find the entire surface.

10. Find the entire surface of a square prism, one side of whose base measures 4 inches, and whose altitude is 6 inches.

11. The convex surface of a square prism is 600 sq. ft., the altitude is 15 ft. What is the length of one side of the base?

12. The entire surface of a square prism is 1,650 sq. in. One side of the base measures 15 inches. What is its convex surface? What is its altitude?

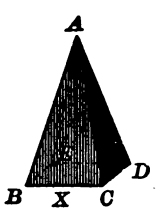
13. Find the entire surface of a square prism whose convex surface is 540 sq. in., and whose altitude is 15 inches.

14. What is the entire surface of a cylinder whose base has a diameter of 1 foot, and whose altitude is 1 foot?

SURFACES OF PYRAMIDS AND CONES.

15. The convex surface of a square pyramid consists of how many equal triangles? Find the convex surface when one side of its base measures 4 inches and its slant height (AX) 6 inches.

Draw the development.



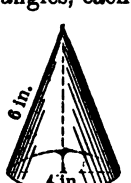
16. The convex surface of a pyramid is equal to the perimeter of the base multiplied by what?

17. Find the entire surface of the above pyramid.

18. Calculate the entire surface of a square pyramid whose slant height is 18 inches, the area of its base being 144 sq. in.

19. Find the entire surface of a triangular pyramid whose three convex faces and the base are equilateral triangles, each side measuring 2 inches.

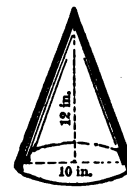
20. Draw the developed convex surface of a cone, the diameter of whose base is 4 inches, and whose slant height is 6 inches.



Calculate the convex surface.

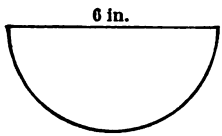
21. How many square inches of paper would be required to cover the side and the base of a cone 6 inches in diameter at the base, and having a slant height of 10 inches?

22. Calculate the slant height of a cone whose altitude is 12 inches, the diameter of its base being 10 inches. What is its convex surface?

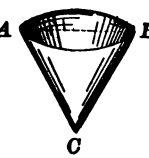


23. What is the entire surface of a cone, the diameter of whose base is 6 inches, and its slant height 10 inches?

Draw the development.



24. A semi-circular piece of paper 6 inches in diameter is folded into a hollow cone (without overlapping). What will be the diameter AB of the mouth of the cone (the base)? What will be the slant height BC?



VOLUMES OF PRISMS AND PYRAMIDS. OF CYLINDERS AND CONES.

1145. Slate Exercises.

SUGGESTION. — Have the pupils construct of cardboard a hollow square prism of convenient size, and a pyramid having base and altitude respectively equal to those of the prism. Let them use sand or water to ascertain how many times the contents of the pyramid must be taken to exactly fill the prism.

Volume of prism or cylinder = area of base × altitude.

Volume of pyramid or cone = area of base × $\frac{1}{3}$ altitude.

1. Find the volume of a square pyramid, the area of the base being 9 square feet and the altitude 6 feet.

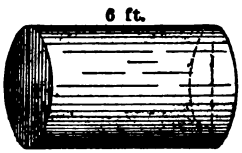
2. What is the volume of a square pyramid whose altitude is 12 inches, one side of the base being 10 inches?

3. The base of a prism is a triangle whose sides measure 3, 4, and 5 inches respectively. Find the solidity, its altitude being 10 inches.

4. The base of a prism 19 feet high is a rectangle whose sides are 9 feet and 13 feet. How many cubic yards does it contain?

5. Find the volume of a prism whose bases are equilateral triangles, each side being 4 ft., and the height of the prism being 12 ft.

6. How many cubic feet are there in a stone roller 6 ft. long, 8 ft. in circumference?



7. Find the volume of a cone whose altitude is 18 meters, diameter of base 6 meters.

8. How many gallons of oil (231 cu. in.) will fill a cylindrical tank $5\frac{1}{2}$ ft. high, radius of base 3 ft.?

9. Measure accurately the interior dimensions of a quart or a pint cup, and calculate its volume.

NOTE. — How many cubic inches in a quart, liquid measure?

10. Measure the interior dimensions of a peck or a bushel, and calculate its volume.

11. Pour a quart or a pint of water into a paper box having a rectangular base, and calculate the number of cubic inches of water in the box.

What would be the depth of a quart of water in a box whose base measures $5\frac{1}{2}$ by 3 inches?

LUMBER MEASURE.

1147. Lumber is measured in *board feet.* A board foot is 1 foot long, 1 foot wide, 1 inch thick.

A board 16 feet long, 1 foot wide, 1 inch thick, contains 16 board feet.

A board 16 feet long, 9 inches wide, 1 inch thick, contains $(16 \times \frac{3}{4})$ board feet, or 12 board feet. A board of the same length and width, 2 inches thick, contains (12×2) board feet, or 24 board feet.

In practice, the term *board foot* is seldom used, the word *foot* alone being generally employed.

1148. Find the number of feet (board feet) in each of the following boards and planks:

1. 16 feet long, 12 inches wide, 1 inch thick.
2. 14 " " 6 " " 1 " "
3. 12 " " 8 " " 1 " "
4. 14 " " 12 " " 1 " "
5. 16 " " 3 " " 1 " "
6. 12 " " 9 " " 2 " "
7. 14 " " 3 " " 2 " "
8. 16 " " 9 " " 2 " "
9. 14 " " 6 " " 2 " "
10. 12 " " 8 " " 2 " "
11. 18 " " 9 " " 3 " "
12. 9 " " 12 " " 3 " "
13. 20 " " 6 " " 3 " "
14. 15 " " 4 " " 3 " "
15. 12 " " 3 " " 3 " "

16. What is the cost, at $30 per thousand feet, of 15 planks, each 16 feet long, 9 inches wide, 3 inches thick?

17. Find the number of (board) feet of lumber required to floor a dock 36 feet long, 17 feet 6 inches wide, the planks being $2\frac{1}{2}$ inches thick.

18. Find the duty, at $1 per thousand feet, on the following lumber imported from Canada:

150 boards, 13 feet long, 8 inches wide, 1 inch thick;

60 planks, 14 feet long, 9 inches wide, 2 inches thick;

40 scantlings, 15 feet long, 5 inches wide, 4 inches thick.

19. At $18 per thousand, what will be the cost of the boards necessary to enclose a field 160 yards long, 120 yards wide, with an open fence 4 boards high, each board 6 inches wide, and 1 inch thick?

MENSURATION.

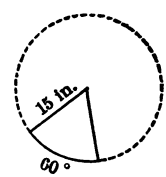
1150. Slate Problems.

Area of circle = ½ circumference × ½ diameter.

Area of sector = ½ arc × ½ diameter.

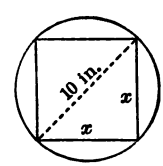
1. Find the area of a semicircle whose radius is 20 feet.

2. How many square inches are contained in a sector of 60°, the radius of the circle being 15 inches?



3. A square is inscribed in a circle 10 inches in diameter. Find its area.

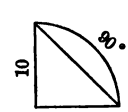
x = side of square, x^2 = area. Find x^2 from the right-angled triangle, without finding the value of x.



4. What is the difference between the area of a circle of 10 inches diameter and that of the inscribed square?

5. The sides of the above inscribed square are chords of arcs of 90°. Find the length of an arc of 90°, and of its chord.

6. A *segment* of a circle is that portion of the surface included between an arc and its chord. Find the area of a sector of 90° and the area of the segment, the radius of the circle being 10 inches.

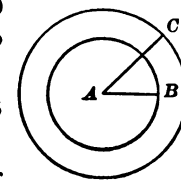


7. Calculate the area of a circle whose radius is 1 inch. Of a circle whose radius is 2 inches. What is the ratio of the two areas?

8. What is the ratio between the area of a circle whose radius is 1 inch and that of a circle whose radius is 3 inches?

The area of a circle = square of radius × ?

9. How many square yards are there in a circular walk, the radius, AB, of the inner edge of walk being 10 feet, and that of the outer edge, AC, being 15 feet?

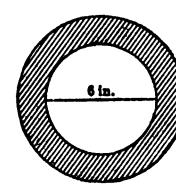


(Find the difference between the area of a circle of 15 ft. radius, and that of a circle of 10 ft. radius.)

10. A circular flower-bed 20 feet in diameter is surrounded by a walk 5 feet wide. How many square feet of surface does the walk contain?

(If you have to subtract 100 times 3.1416 from 225 times 3.1416, how can you shorten the work?)

11. How many square inches are there in the surface of a frame 3 inches wide, around a looking-glass 6 inches in diameter?

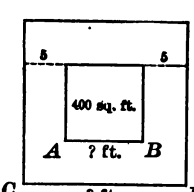


(Area = ? × 3.1416.)

12. What is the ratio between the surface of the above frame and that of the looking-glass?

(Indicate operations and cancel.)

13. What is the area of a walk 5 feet wide around the outside of a square plot containing 400 sq. ft.?



(What is the area of the large square, including the walk?)

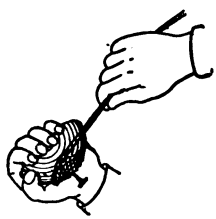
14. The outer edge of a walk 5 feet wide, surrounding a plot of ground, measures 120 feet, the inner edge measures 80 feet. How many square feet does the walk contain?

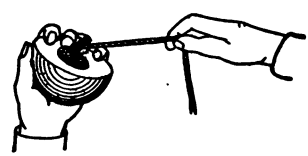
(The "average" length of the walk is $\frac{120 + 80}{2}$ = 100 ft.; that is, its length measured on a line along the center of the walk.)

15. Find the ratio between the area of a triangle whose sides measure 16, 30, and 34 feet, respectively, and the area of another whose sides are 32, 60, and 68 feet.

SURFACE OF SPHERE.

1151. Take a wooden hemisphere and drive a tack into the center of its curved surface. Commencing at the tack, carefully wind a waxed cord about the curved surface, in the way a boy winds a top. When this surface is exactly covered, cut the cord.





Wind the same cord around a tack driven into the plane surface of the base of the hemisphere, pressing it closely to the surface. When the latter is entirely covered, just one-half of the cord will be used.

If a sphere is cut through in any direction, the section made will be a circle. The section formed when the sphere is cut through the center is called a *great circle.*

The above experiment shows that the surface of the hemisphere is equal to that of two great circles of the same sphere.

1152. The surface of a sphere is equal to that of four great circles.

Since the surface of a great circle of the sphere is $\frac{1}{2}$ diameter $\times$ $\frac{1}{2}$ circumference, the surface of the sphere is $\frac{1}{2}$ diameter $\times$ $\frac{1}{2}$ circumference $\times$ 4 = diameter of sphere $\times$ the circumference.

Calling the radius of a circle R, and using the Greek letter π instead of 3.1416, we have

Diameter of circle $= 2R$.

Circumference of circle $= 2\pi R$.

Area of circle $= \pi R^2$. ($\frac{1}{2}$ of $2R \times \frac{1}{2}$ of $2\pi R$.)

Surface of sphere $= 4\pi R^2$.

1153. Slate Exercises.

16. Find the surface of a sphere whose radius is 1 inch.

Of a sphere whose diameter is 2 inches.

Of a sphere whose circumference is 6.2832 inches.

17. At 10 cents a square foot, what will be the cost of gilding a sphere 12 inches in diameter?

18. Find the ratio between the surface of a sphere 1 foot in diameter, and the convex surface of a cylinder 1 foot high, the diameter of the base 1 foot.

19. What is the ratio between the surface of the above sphere and the entire surface of the cylinder?

20. Find the surface of a sphere whose circumference is 20 inches.

CUBE ROOT.

1155. To cube a number is to employ it three times as a factor.

The cube of 4, written 4^3, is $4 \times 4 \times 4$, or 64.

Find the cube of 1, 9, 6, 3, 5, 8, 2, 7.

To find the cube root of a number is to find one of the three equal factors of the number.

The cube root of 343, written $\sqrt[3]{343}$, is 7.

The cube of 25, $20 + 5$, is equal to the following:

We have seen (Art. 1031) that

$$(20+5)^2 = 20^2 + 2 \times 20 \times 5 + 5^2$$

Multiplying by $20 + 5$ we have

$$\text{Product by } 20 = 20^3 + 2 \times 20^2 \times 5 + 20 \times 5^2$$

$$\text{Product by } 5 = 20^2 \times 5 + 2 \times 20 \times 5^2 + 5^3$$

$$(20+5)^3 = 20^3 + 3 \times 20^2 \times 5 + 3 \times 20 \times 5^2 + 5^3$$

which may be written in this way,

$$20^3 + [(3 \times 20^2) + (3 \times 20 \times 5) + 5^2] \times 5.$$

1156. Extract the cube root of 15,625.

We see by inspection that the cube root is between 20 and 30; that is, $20 + x$. Subtract from 15,625 the cube of 20, 8,000. The remainder, 7,625, is equal to the second number multiplied by the sum of three times the square of the first (1,200), etc. Using 1,200 as a trial divisor, the second number is seen to be 6 or less.

$$\begin{array}{rrrr} & & & 20+5 \\ & & & \overline{15{,}625} \\ & (20)^3 = & & 8{,}000 \\ & 3\times 20^2 = & 1{,}200 & 7{,}625 \text{ remainder} \\ 3\times 20\times 5 = & & 300 & \\ & 5^2 = & 25 & \\ & & \overline{1{,}525} & \overline{7{,}625} \end{array}$$

Taking 5 as the second number, we add to the 1,200 three times the product of the first and second (300), and the square of the second (25), making a total of 1,525. Multiplying this sum by the second number, we get 7,625, which is equal to the difference between 15,625 and 8,000. The second number is, therefore, 5, and the cube root of 15,625 is 25.

$\sqrt[3]{110{,}592}$

$$\begin{array}{rrrr} & & & 40+8 \\ & & & \overline{110{,}592} \\ \overline{40^3} & - & & 64{,}000 \\ 3\times\overline{40^2} & = & 4{,}800 & 46{,}592 \\ 3\times 40\times 8 & = & 960 & \\ & 8^2 = & 64 & \\ & & 5{,}824 & \overline{46{,}592} \end{array}$$

Ans. 48.

$\sqrt[3]{658{,}503}$

$$\begin{array}{rrrr} & & & 8\quad 7 \\ & & & \overline{658'503} \\ & 8^3 = & & 512 \\ 3\times 80^2 & = & 19\,200 & 146{,}503 \\ 3\times 80\times 7 & = & 1{,}680 & \\ & 7^2 = & 49 & \\ & & 20{,}929 & \overline{146{,}503} \end{array}$$

Ans. 87.

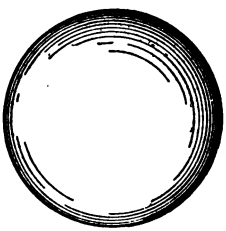
In the last example we point off three places, beginning at the right, and find the greatest cube in the first period, placing its cube root as the first figure of the answer.

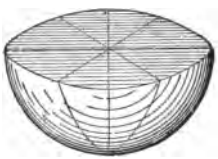
1157. Find the cube root of the following:

1. 2,197	**6.** 238,328	**11.** $\frac{8000}{12167}$
2. 9,261	**7.** 421,875	**12.** 3.375
3. 32,768	**8.** 551,368	**13.** $1\frac{217}{512}$
4. 68,921	**9.** $\frac{512}{729}$	**14.** $1\frac{397}{1331}$
5. 148,877	**10.** $\frac{1331}{2744}$	**15.** $54\frac{51}{729}$

VOLUME OF SPHERE.

1158. Cut up a sphere (a round potato, for instance) into a number of small pieces, passing the knife in each case through the center of the sphere.





Each piece is a solid, having for its base a portion of the surface of the sphere, and for its altitude the radius of the sphere.

When the pieces become very numerous, the base of each may be considered a plane, and the solid a pyramid. The volume of each pyramid is

equal to the base $\times \frac{1}{3}$ altitude; and the total volume of all, which is the volume of the sphere, is equal to the total surface of all the bases, which is the surface of the sphere, multiplied by $\frac{1}{3}$ altitude, that is, $\frac{1}{3}$ radius.

$$\text{Surface of sphere} = 4\pi R^2,$$

therefore, $$\text{volume of sphere} = 4\pi R^2 \times \tfrac{1}{3} R = \tfrac{4}{3}\pi R^3.$$

1159. Slate Exercises.

1. Find the volume of a sphere whose radius is 3 inches.

2. If the diameter of a sphere is 3 inches, what is its volume?

3. What is the ratio between the volumes of two spheres whose diameters are 1 foot and 2 feet, respectively?

4. Find the ratio between the volume of a sphere 1 foot in diameter, and that of a cube whose side is 1 foot.

5. The radius of a sphere is 18 inches. What is the circumference of a great circle? The surface? The volume?

6. What is the weight of an iron cannon-ball 12 inches in diameter, considering the weight of a cubic foot of water as 1,000 ounces, and considering iron 7.5 times as heavy as water?

7. Find the ratio between the volume of a sphere 4 inches in diameter, and that of a cylinder 4 inches in altitude, radius of base 4 inches.

NOTE. — *Indicate* the volume of each, and cancel.

8. A man has a cubical block of hard wood, its side measuring one foot, which he wishes made into a sphere one foot in diameter. What decimal part of the block is cut away?

The volume of the sphere is *about* what fraction of the volume of the cube?

CUBE ROOT.

1162. Find the cube root of 9,938,375.

When the root contains more than two figures, continue, as shown in the accompanying example, taking for divisor three times the square of the first two figures considered as tens, plus three times the product of the first two figures considered as tens by the third figure, plus the square of the third figure.

		2 1 5
		9'938'375
		8
$3 \times 20^2 =$	1 200	1 938
$3 \times 20 \times 1 =$	60	
$1^2 =$	1	1 261
$3 \times 210^2 =$	132 300	677 375
$3 \times 210 \times 5 =$	3 150	
$5^2 =$	25	
	135 475	677 375

1163. Find the value of the following:

1. $\sqrt[3]{1{,}442{,}897}$
2. $\sqrt[3]{1{,}906{,}624}$
3. $\sqrt[3]{3{,}723{,}875}$
4. $\sqrt[3]{39{,}651{,}821}$
5. $\sqrt[3]{12.977875}$
6. $\sqrt[3]{66.923416}$

ANNUAL INTEREST.

1171. Slate Problems.

DETROIT, MICH., June 1, 1890.

Four years after date, without days of grace, I promise to pay to the order of Daniel W. Lawler, Six Hundred Dollars, value received, with annual interest at six per cent.

$\$600\frac{00}{100}$. GEORGE OXNARD.

1. Find the amount due June 1, 1894, no payments of principal or interest having been made.

1172. When the maker of a note fails to keep his contract to pay interest annually, the laws of some states, including Michigan, permit the collection of *simple interest* on the deferred payments of interest.

Principal,	$600.00
Interest, 4 years, at 6%,	144.00
3 years' interest, at 6%, on the 1st year's interest, $36,	6.48
2 " " " " " " 2d " " "	
1 " " " " " " 3d " " "	
Amount due June 1, 1894,	$

2. Find the amount due, at 5%, for 5 years, on a note for $1,200, annual interest being unpaid.

3. What is the amount of a note for $720, at 4 years, at $4\frac{1}{2}\%$, annual interest unpaid after the first year?

4. The maker of a note for $900, with annual interest at 7%, makes the first and the second interest payments when due. How much will he owe at settlement, 6 years after the date of the note?

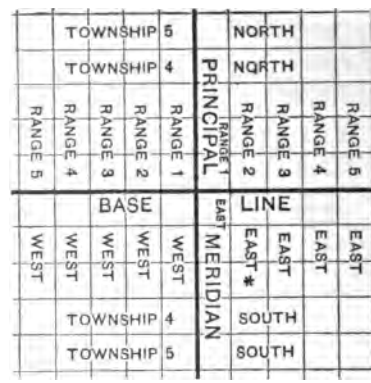
5. Find the difference between the amount due at 6% for 3 years on a note for $300, annual interest unpaid, and the amount of the same sum placed at compound interest for the same time at the same rate.

6. Find the amount due March 1, 1899, on a note for $500, dated March 1, 1893, with interest at 6%, annual interest unpaid after the third year.

U. S. GOVERNMENT LANDS.

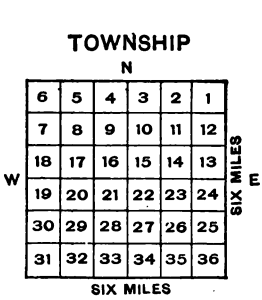
1173. In surveying government lands, a line is run east and west, called the *base line*, and one perpendicular to it, called the *principal meridian*.

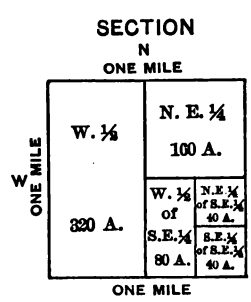
Parallel lines are run north and south, and east and west, 6 miles apart, forming squares, called *townships*. The row of townships adjoining the principal meridian is called *Range* 1 East or West, according to its location. The row of townships north of the base line is called *Township* 1, *North;* the row above, *Township* 2, *North*, etc.



The township in the diagram marked by a star (*) is designated 3 T. S., R. 2 E. (third township south of base line, in the second range east of the principal meridian).

1174. A township, which contains 36 square miles, is divided into sections one mile square, numbered as in the diagram, No. 1 being found at the northeast corner. Each section contains 640 acres.





1175. Sections are divided into half-sections (320 A.) and quarter-sections (160 A.), and the latter are subdivided into half quarter-sections (80 A.) and quarter quarter-sections (40 A.).

1176. Slate Problems.

1. Find the cost of the S.W. $\frac{1}{4}$ of the N. $\frac{1}{2}$ of sec. 13, T. 7 N., R. 4 E., at $\$1.87\frac{1}{2}$ per acre.

2. What will be the cost of fencing, at 75¢ per rod, the W. $\frac{1}{2}$ of the N.W. $\frac{1}{4}$ of sec. 36?

3. Mr. Thompson owns sec. 1, and his brother owns sec. 30 of the same township. What is the length of the shortest line between the boundaries of the two farms?

4. A road runs east and west between townships 4 and 5, south. Another road runs north and south between R. 7 and 8 east. How far is it by road from the north-east corner of T. 5 S., R. 10 W., to the north-west corner of T. 7 N., R. 8 E.?

5. How many feet of boards, 6 inches wide, would be needed to build an open fence, 4 boards high, around the N. $\frac{1}{2}$ of the S.W. $\frac{1}{4}$ of sec. 16?

6. The owner of secs. 19 and 20 has sold the W. $\frac{1}{2}$ of N.W. $\frac{1}{4}$ of sec. 19; also the N. $\frac{1}{2}$, the N. $\frac{1}{2}$ of S.E. $\frac{1}{4}$, and the S.E. $\frac{1}{4}$ of the S.E. $\frac{1}{4}$ of sec. 20. Draw a map of the land he still owns, and calculate its area.

METRIC SYSTEM.

1177. The metric system, which is used in nearly all the countries of continental Europe, is based upon the meter. The length of the meter is one ten-millionth part of the length of the meridian from the equator to the poles — about 39.37 inches.

1178. The subdivisions of the meter are denoted by the Latin prefixes milli ($\frac{1}{1000}$), centi ($\frac{1}{100}$), deci ($\frac{1}{10}$). For the multiples, the Greek prefixes deka (10), hecto (100), kilo (1,000), and myria (10,000) are used.

1179. It will be noticed, in the table below, that small letters are used for the abbreviations of the Latin prefixes of the

subdivisions, and capital letters for the Greek prefixes of the multiples.

The following is the table of

1180. Measures of Length.

10 millimeters (mm.)	1 centimeter (cm.)
10 centimeters	1 decimeter (dm.)
10 decimeters	1 *meter* (m.)
10 *meters*	1 dekameter (Dm.)
10 dekameters	1 hectometer (Hm.)
10 hectometers	1 kilometer (Km.)
10 kilometers	1 myriameter (Mm.)

1181. The units of this table in common use are the centimeter, the meter, and the kilometer.

1182. A person who wishes to buy 124 meters of cloth, would not ask for 1 hectometer 2 dekameters 4 meters, any more than a New York merchant would tell a person who owes him $38.75 that his bill is 3 eagles 8 dollars 7 dimes 5 cents.

1183. Long distances are expressed in kilometers. The thickness of wire is given in millimeters.

1184. Problems.

1. What will be the cost in francs of 380 m. 75 of dress goods at 2 f. 60 per meter? (380.75 meters @ 2.60 francs.)

2. How many square meters in a piece of carpet 26 m. 50 long, 85 cm. wide?

3. How many square meters in a circle whose diameter is 15 meters?

4. An *are* is a surface 10 meters long, 10 meters wide. How many ares in a field 135 meters long, 69 meters wide?

5. Find the area in ares of a right-angled triangle whose base is 245 meters, hypotenuse 875 meters.

6. A *stere* is a cubic meter. What will be the cost, at 8 f. 50 per stere, of a pile of wood 10 meters long, 1 meter wide, 3 m. 25 high?

7. A cube one decimeter each way contains a liter (l.), which is the principal unit of dry and liquid measure.

How many liters' capacity has a tank 10 m. 50 long, 8 m. wide, 6 m. 50 high?

8. How many bottles, each containing 0 l. 75, can be filled from a hogshead containing 222 l.?

9. How much will be received for 36 bags of beans, each containing 68 liters, at 1 mark 25 per dekaliter?

10. A liter of water weighs a kilogram (1,000 grams). How many kilos of oil would a tank contain, its dimensions being 5 meters $\times$ 4 meters $\times$ 3 meters, the weight of the oil being 92% of the weight of water?

11. Assuming the length of the meter as 39.37 inches, what is the length of the kilometer?

1185. Greater accuracy is assured in operations requiring multiplication and division by indicating the operations beforehand, and performing the division last.

Length of meter in yards $\frac{39.37}{36}$. 1 mile = 1,760 yd. 1 km. = 1,000 m.

$$Ans. = \frac{39.37 \times 1{,}000}{36 \times 1{,}760} = \frac{3{,}937}{36 \times 176} \text{ mile.}$$

12. Mt. Blanc is 4800 m. high. How many feet high is it?

1186. In the following ten problems call the meter 40 inches. Give answer in two decimal places.

13. How many cubic inches in a liter? (See problem 7.) How many quarts?

14. How many bushels in a hectoliter? How many gallons?

15. How many pounds in a kilo, when a cubic foot of water weighs 1,000 oz.? (See problem 10.)

16. What would be the circumference of the earth in miles if the meter measured 40 inches?

(The meter is $\frac{1}{10000000}$ of what part of circumference?)

17. How many square yards in a square meter?

18. How many acres in a hectare? (See problem 4.)

19. How many rods in a hectometer?

20. How many cubic feet in a stere? (See problem 6.)

21. How many troy grains (7,000 to av. lb.) in a gram? (See problem 15.)

22. How many kilometers in a mile?

1187. Measures of Surface.

100 sq. mm. = 1 sq. cm.
100 sq. cm. = 1 sq. dm.
100 sq. dm. = 1 sq. m. = 1.196 sq. yd.

1188. The square meter is the principal unit of surfaces, such as walls, ceilings, floors, etc.

100 centiares (ca.) = 1 are (a.) = 119.6 sq. yd.
100 ares = 1 hectare (Ha.) = 2.47 acres.

1189. The are is the principal unit of surface of small plots of land. The area of a farm is expressed in hectares, of a country in square kilometers.

1190. Measures of Volume.

1,000 cu. mm. = 1 cu. cm.
1,000 cu. cm. = 1 cu. dm.
1,000 cu. dm. = 1 cu. m. = 35.316 cu. ft.

1191. The principal unit is the cubic meter.

1192. The *stere* (cubic meter) is used for measuring wood.

10 decisteres (dst.) = 1 *stere* (st.) = 35.316 cu. ft.

10 *steres* = 1 dekastere (Dst.)

The stere is the only unit used.

1193. Dry and Liquid Measures.

		Dry.	Liquid.
10 milliliters	= 1 centiliter		
10 centiliters	= 1 deciliter		
10 deciliters	= 1 *liter* (l.)	= .908 qt.	= 1.057 qt.
10 *liters*	= 1 dekaliter	1.135 pk.	= 2.642 gal.
10 dekaliters	= 1 hectoliter	2.837 bu.	= 26.417 gal.
10 hectoliters	= 1 kiloliter		
10 kiloliters	= 1 myrialiter		

1194. The liter and the hectoliter are the principal units.

1195. Table of Weight.

10 milligrams (mg.)	1 centigram
10 centigrams	1 decigram
10 decigrams	1 *gram* (gr.)
10 *grams*	1 dekagram
10 dekagrams	1 hectogram
10 hectograms	1 kilogram (kilo) 2.2046 lb.
10 kilograms (Kg.)	1 myriagram
10 myriagrams	1 quintal
10 quintals	1 tonneau (ton)

1196. The kilo is the ordinary unit. Heavy articles are sold by the tonneau.

CHAPTER XV.

ALGEBRAIC EQUATIONS.—TWO UNKNOWN QUANTITIES.—THREE UNKNOWN QUANTITIES.—PURE QUADRATICS.—AFFECTED QUADRATICS.

ADDITION OF ALGEBRAIC QUANTITIES.

1199. Sight Exercises.

Add:

1.	2.	3.	4.	5.
2 fours	6 hundredths	\$4	3¢	$7x$
3 fours	8 hundredths	\$5	5¢	$4x$
4 fours	10 hundredths	\$7	8¢	$2x$
5 fours	12 hundredths	\$8	9¢	$5x$
? fours	? hundredths	\$?	?¢	$?x$

6.	7.	8.	9.	10.
$-2a$	$+3x$	$-5xy$	$9abc$	$-24xyz$
$-4a$	$+4x$	$-4xy$	$15abc$	$-5xyz$
$-6a$	$+5x$	$-xy$	$6abc$	$-xyz$
$-7a$	$+10x$	$-2xy$	abc	$-15xyz$
$-19a$	$+?x$	$-?xy$	$?abc$	$-?xyz$

1200. In the quantities $2a$, $3x$, $5xy$, $15abc$, the numbers 2, 3, 5, 15, are called coefficients. When no coefficient is expressed, 1 is understood. Thus, $abc = 1abc$.

Where no sign is expressed, + is understood.

1201. What a person has may be represented with a plus sign (+) placed before the amount; debts may be shown by a minus sign (−) placed before the amount.

A has \$500; B owes \$300. If they unite their fortunes, what will they be worth together?

$$\begin{array}{r} +\$500 \\ -\$300 \\ \hline +\$200 \end{array}$$

Both together are worth \$200.

The sum of $+500$ and -300 is $+200$.

1202. If A had \$300 and B owed \$500, the firm would be \$200 in debt.

$$(+\$300)+(-\$500)=-\$200.$$

1203. Add:

1.	2.	3.	4.	5.
$-2a$	$7x$	$-5xy$	$-9abc$	$-24xyz$
$-4a$	$-4x$	$-4xy$	$15abc$	$5xyz$
$-6a$	$-2x$	xy	$6abc$	xyz
$7a$	$5x$	$2xy$	$-abc$	$15xyz$
$-5a$	$6x$	$-?xy$	?	?

1204. Can you give the rule for addition where the quantities have different signs? Which sign does the sum take?

1205. Add:

6. $3x+14$, $-7x+9$, -23, $4x-5$, $-2x$, and $3x+11$.

$$\begin{array}{r} 3x+14 \\ -7x+9 \\ -23 \\ 4x-5 \\ -2x \\ \hline 3x+11 \end{array}$$

7. $4a+3x$, $-2a$, $-7x-3a$, $-5x$, $-9a+x$.

8. $-3b+c$, $4a+6b$, $5b-9c$, $-3a$, $-2a-3b+4c$.

9. $\frac{1}{2}x-8$, $-x+4$, $-\frac{1}{4}x-3$, $7x+16$, $-5x-10$.

10. $4x+23$, $-8x+2\frac{1}{2}$, $-\frac{2}{3}x+11$, $-x+5$, $9x-3$.

SUBTRACTION OF ALGEBRAIC QUANTITIES.

1206. Oral Problems.

1. The thermometer in the morning was 33 degrees, at noon it was 52 degrees. What was the difference in temperature?

2. In December the thermometer was 10 degrees below zero. In July it was 90 degrees above. What was the difference in temperature?

3. Two cities are in the same latitude. One is in 34° east longitude, and the other in 17° west longitude. What is their difference in longitude?

4. What is the difference in longitude between two cities on the equator, one being in 56° west longitude, and the other in 47° west longitude?

5. A boy makes 40¢ one day and 50¢ the next. How does he stand at the end of the two days?

6. How would he stand if he made 40¢ one day and lost 50¢ the next day?

7. A man traveled from the town M, 60 miles due north, and then traveled 50 miles due north. How far is he from his starting-point?

8. One day a man goes 50 miles due north; the next day he travels 70 miles due south. How far is he then from his starting-point?

9. On Monday A is worth $250; on Tuesday he is worth $150. What has he lost in a day?

10. A man has $150 Jan. 1. Feb. 1 he owes $250. What has he lost in a month?

1207. The degrees above zero on a thermometer may be indicated by a plus sign (+); those below, by a minus sign (−).

What is the difference between $+52°$ and $+33°$? Between $+90°$ and $-10°$?

Show by a diagram.

1208. A has \$600, B owes \$400. What are they worth together?

$$(+\$600)+(-\$400)=?$$

How much better off is A than B?

$$(+\$600)-(-\$400)=?$$

1209. In subtracting algebraic quantities, change the signs of the subtrahend, and proceed as in addition.

1. From $8a$ take $2a$.

$$\begin{array}{r} 8a \\ -2a \\ \hline \textit{Ans.}\quad 6a \end{array}$$

2. From $2a$ take $8a$.

$$\begin{array}{r} 2a \\ -8a \\ \hline \textit{Ans.}\quad -6a \end{array}$$

3. From $-8a$ take $2a$.

$$\begin{array}{r} -8a \\ -2a \\ \hline \textit{Ans.}\quad -10a \end{array}$$

4. From $8a$ take $-2a$.

$$\begin{array}{r} 8a \\ +2a \\ \hline \textit{Ans.}\quad 10a \end{array}$$

5. From $-8a$ take $-2a$.

6. From $-2a$ take $8a$.

7. From $-2a$ take $-8a$.

8. From $2a$ take $-8a$.

9. From $3x+14$ take $x+10$.

$$\begin{array}{r} 3x+14 \\ -x-10 \\ \hline \end{array}$$

10. From $5x-8$ take $-3x-9$.

11. From $x-28$ take $5x-37$.

12. From $7x+16$ take $9x-4$.

13. From $6x$ take $2x-5$.

14. From $8x$ take $9x+3$.

15. From $3x+2a-5$ take $x-a-9$.

16. From $7y-2z+b$ take $-8y+6b-z$.

17. From $c-d+e$ take $c+d-f$.

REMOVING PARENTHESES.

1210. From 84 take the difference between 49 and 25.

$$84 - (49 - 25) = \text{what?}$$

Would the result be the same if we should write the above

$$84 - 49 - 25?$$

What sign must be changed?

1211. Write the following without parentheses:

1. $57 + (33 - 16) = 74$
2. $92 - (63 + 25) = 4$
3. $(43 - 10) + (24 - 5) = 52$
4. $(17 - 8) - (16 - 14) = 7$
5. $75 + 4 \times (15 - 10) = 95$
6. $75 - 4 \times (15 - 10) = 55$

1212. Is there any change made in the signs of the first? In the signs of the second? Of the third? Of the fourth? Of the fifth? Of the sixth?

1213. Solve the following equations. Prove the correctness of your answers.

1. $6(2x - 5) = 5x + 12$

NOTE. $6(2x - 5)$ means 6 times $(2x - 5)$, or $12x - 30$.

2. $7(x + 2) = 3x + 50$
3. $5(3 + x) + 16 = 61$
4. $3(16 - x) = 4(13 - x)$
5. $15(x - 3) = 2(189 - 16x)$
6. $38 - (11 - 9x) = 10x$

Removing the parenthesis, we have

$$38 - 11 + 9x = 10x$$

Transposing, $$9x - 10x = -38 + 11$$

or, $$-x = -27$$

Bringing $-x$ to the right side of the equation, and -27 to the left side, we have

$$(+)\,27 = (+)\,x$$

In practice, however, when the result is such as the above, $-x = -27$, the signs of both members are changed, and the result becomes

$$x = 27$$

7. $2(x-1)-2(2x-19)=3(x-3)$

8. $6(2x-5)-5x=12$

9. $5x-6(2x-5)=-12$

10. $\frac{11-3x}{2}+5x=19.$

1214. $\frac{18-6}{2}-\frac{24-4}{5}=2$

Clear of fractions by multiplying both members of the equation by 10, and observe which sign must be changed to preserve the equality.

When $x=6$, the above may be written

$$\frac{3x-6}{2}-\frac{4x-4}{5}=2$$

Clearing of fractions,

$$15x-30-(8x-8)=20$$

Removing the parenthesis,

$$15x-30-8x+8=20$$

Transposing, $$15x-8x=20+30-8$$

or, $$7x=42$$

$$x=6$$

NOTE. — The horizontal line between the numerator and the denominator of the foregoing fractions has the effect of a parenthesis, the *entire* quantity above the line being divided by the number below.

$\frac{18-6}{2}=(18-6)\div 2$ $\qquad$ $\frac{24-4}{5}=\frac{1}{5}$ of $(24-4)$

$\frac{3x-6}{2}=\frac{1}{2}$ of $(3x-6)$ $\qquad$ $\frac{4x-4}{5}=(4x-4)\div 5$

1215. Solve:

11. $\frac{x-1}{2}+\frac{x-2}{3}=8$

12. $\frac{x-1}{2}-\frac{x-2}{3}=2$

13. $\frac{x-1}{2}-\frac{x-2}{3}-\frac{x-3}{4}+2=0$

14. $\frac{2x-5}{2}+\frac{x-7}{4}=\frac{5x-3}{6}$

15. $\frac{7x-8}{9}-(x+2)=\frac{4x+5}{6}-\frac{x+2}{3}$

16. $\frac{40-5x}{3}=\frac{52+9x}{7}$

17. $9\frac{3}{4}-\left(\frac{5}{4}x-\frac{x}{2}\right)=\frac{3}{8}x+3\frac{3}{4}x$

18. $2x=3+2\frac{1}{4}x-(5+\frac{2}{5}x)+2\frac{2}{5}$

19. $\frac{3}{4}x+9=2x+(\frac{3}{5}x-\frac{1}{2}x)$

20. $\frac{x}{4}+\frac{x}{5}+\frac{x}{6}+\frac{x}{8}+31=x$

21. $\frac{5}{4}x-120=\frac{x}{6}+10$

22. $x-20=\left(\frac{x}{7}+15\right)4$

23. $x+\frac{x}{3}+\frac{x}{4}=19$

24. $9(8x+1)-4=4(9x+5)+3$

25. $2x+3=\frac{5x-6}{2}$

1216. Slate Problems.

1. A certain number is multiplied by $3\frac{3}{8}$; 7 is subtracted from the product; the remainder is divided by 16, giving a quotient of 3. What is the number?

2. Three-eighths of what number is 60 less than the number itself?

3. Four persons are of the same age. If the first were $\frac{1}{8}$ of his age older, the second $\frac{1}{4}$ of his age older, the third $\frac{1}{8}$ of his age older, and the fourth $\frac{1}{8}$ of his age older, the sum of their ages would be 99 years. What is the age of each?

4. A man spends $\frac{1}{2}$ of his earnings on board and lodging, $\frac{1}{8}$ on clothing and repairs, and $\frac{1}{5}$ on sundries. At the end of the year he has \$280 left. What are his yearly earnings?

$$\left(x = \frac{x}{2} + \frac{x}{8} + \frac{x}{5} + 280.\right)$$

5. A boy gave $\frac{1}{8}$ of his marbles to one companion, and $\frac{1}{8}$ of them to another. He then bought $\frac{1}{8}$ as many as he originally had, and had 4 marbles more than he had at first. How many did he have at first?

6. A father's age and a son's age added together amount to 138 years. Twelve years ago the father was twice as old as the son. How old is each now?

Let x = son's age 12 years ago. $2x$ = father's age then.

7. John has 80 cents, and William has 60 cents. How many cents will William have to give John so that the latter shall have $2\frac{1}{2}$ times as much money as the former?

After William gives John x cents, the former has $(60 - x)$ cents, and the latter has $(80 + x)$ cents.

8. In how many years will a man, now 25, be double the age of his 11-year-old brother?

Let x = number of years. $25 + x$ and $11 + x$ = ages after x years.

9. A man has a cask of 60 gallons' capacity. He draws off one-fourth of its contents, and then fills it. If it takes 24 gallons to fill it, how many gallons did the cask originally contain?

10. A number is divided by 3, and 40 is subtracted from the quotient, leaving a remainder of 104. What is the number?

11. The difference between two numbers is 430. When the greater is divided by the less, the quotient is 4, and the remainder is 76. What are the numbers?

$$\text{Let } x = \text{less.} \quad \frac{\text{greater}}{\text{less}} = 4 + \frac{76}{\text{less}}$$

12. A person pays \$103 with 29 \$2 and \$5 bills. How many are there of each denomination?

13. A father is 30 years older than his daughter. In 4 years, his age will be four times her age. What are their present ages?

x and $x + 30$ = present ages. $x + 4$ and $x + 34$ = ages 4 years later.

14. The product of two numbers is 180. If the smaller number be increased by 3, the product of the two numbers will be 225. What are the numbers?

$$\therefore \quad \text{smaller} = x; \quad \frac{180}{x} = \text{greater.}$$

15. A man's wages are \$1 per day more than his son's. For 33 days' work, the father receives \$12 more than the son earns in 40 days. Find the wages of each.

16. The sum of two numbers is 47; their difference is 17. What are the numbers?

17. A mother is 41 years old. Her son's age is 5. In how many years will the son's age be $\frac{1}{3}$ of his mother's?

TWO UNKNOWN QUANTITIES.

1217. Preliminary Problems.

1. I paid a dollar for two 25¢ balls and five bats. How much did I pay apiece for the latter?

2. When three times one number is added to five times another, the sum is 84. If the second number is 12, what is the first number?

3. A girl paid 75¢ for ½ pound of tea and 2½ pounds of coffee. The coffee cost 20¢ per pound. What was the price of the tea per pound?

4. A man sold pigs at $5 each and lambs at $8 each, receiving $42. He sold 4 lambs. How many pigs did he sell?

5. Four times a father's age added to twice his daughter's age amounts to 180 years. The girl is 10 years old. What is the father's age?

6. Eight peaches and seven pears cost 44¢. The peaches cost 2¢ each. What is the cost of a pear?

7. Two pieces of cloth and eleven pieces of silk contain 152 yards. There are 10 yards in each piece of cloth. How many yards in each piece of silk?

8. Two-thirds of a yard of linen and three-fourths of a yard of lace cost 40¢. The price of the lace is 32¢ a yard. Find the price of the linen.

9. Three and one-half times one number added to four and one-third times a second number equals 60. The second number is 9. What is the first number?

1218. Slate Exercises.

Find the value of the unknown quantity:

1. $8x+7y=44$. When $x=2$, find the value of y.
2. $3y+5z=34$. Find the value of z; $y=3$.
3. $2x+11z=152$. $x=10$; $z=?$
4. $14x+7y=98$. $x=3\frac{1}{2}$; $y=?$
5. $\frac{2}{3}x+\frac{3}{4}z=40$. $z=32$.

6. $9x - 25y = 8.$ $x = 12.$

7. $3\frac{1}{2}y + 4\frac{1}{3}z = 60.$ $z = 9.$

8. $16x - 19z = 49.$ $z = 5.$

9. $7y - 3z = 18.$ $y = 6\frac{1}{7}.$

10. $32x + 50y = 2{,}600.$ $y = 20.$

1219. A boy gave 17¢ for 3 lemons and 4 oranges, another boy paid 25¢ for 3 lemons and 8 oranges. How much did the lemons cost apiece?

x = cost of lemons	$3x + 4y = 17$	(1)
y = cost of oranges	$3x + 8y = 25$	(2)
Subtracting (1) from (2)	$4y = 8$	
The oranges cost 2¢ each	$y = 2$	

How much apiece was paid for the lemons?

11. If 3 coats and 14 vests cost $78, and 2 coats and 14 vests, at the same rate, cost $66, how much does 1 coat cost? What is the price of a vest?

12. Given $4x + 7y = 53$ (1)

$2x + 3y = 25$ (2)

to find the value of y.

First multiply (2) by 2, making it $4x + 6y = 50$. Why?

13. What is the value of x in equation (1), when the value found for y is substituted therein? Substitute the same value for y in equation (2) and find the value of x.

1220. Find the values of x and y in the following equations:

14. $x + y = 15.$ $2x + 3y = 38.$

15. $2x + 2y = 30.$ $x + 3y = 27.$

16. $2x + 3y = 18.$ $4x + 3y = 24.$

17. $2x + 3y = 40.$ $3x + 2y = 35.$

18. $7x+5y=82.$ $2x+2y=28.$

19. $5x+9y=14.$ $9x+5y=14.$

20. $3x+5y=17.$ $8x+2y=17.$

1221. Given $\left.\begin{matrix}(1)\ x+3y=46\\(2)\ 7x-4y=22\end{matrix}\right\}$ To find values of x and y.

Multiply (1) by 7, $7x+21y=322$

(2) $7x-4y=22$ Subtract.

$25y=300$

$y=12$

Substituting this value of y in (1), we have

$x+36=46$

$x=46-36=10$

Answers. $x=10,\ y=12.$

21. $x+y=18$ Add or subtract.
$x-y=4$

22. $4x+3y=17$ (1) Multiply (2) by 2 and subtract.
$2x-y=1$ (2)

23. $3x+4y=48$ Add.
$x-4y=0$

24. $3x+5y=13$ (1) Multiply (1) by 7 and (2) by 3.
$7x+3y=13$ (2) Subtract.

25. $4x+5y=32$ Add.
$6x-5y=-2$

26. $3x+4y=3$ (1) Multiply (2) by 2. Add.
$12x-2y=3$ (2)

27. $5x=6y+5$ Transpose.
$3x=5y-4$

28. $3x+5y+8=0$
$2x-y-12=0$

29. $y-2x=8x-1$
$2y-4x=y+x+9$

30. $\frac{x}{4}+\frac{2y}{3}=17$ Clear of fractions.
$\frac{5x}{4}+\frac{5y}{8}=20$

31. $\frac{1}{2}x+\frac{1}{3}y=42$
$\frac{1}{5}x+\frac{1}{7}y=17\frac{1}{5}$

32. $23x-7y=8x+51$
$11y=15x+2$

33. $x+y=100{,}000$
$\frac{5x}{100}+\frac{4y}{100}=4{,}640$

34. $\frac{3x+7}{3y-4}=5$
$\frac{7x-6}{5y+3}=2$

35. $4\frac{1}{4}x+3\frac{3}{5}y=67$
$7\frac{1}{2}x-5\frac{1}{3}y=12$

36. $3(x+7)=9(y-9)$
$4(3x-8)=17y-155$

37. $2(x-11)-2(y-9)=6$
$\frac{x+9}{y-3}=\frac{32}{15}$

38. $\frac{x-4}{3}+\frac{y-1}{4}=5$
$\frac{x-4}{3}-\frac{y-1}{4}=1$

39. $\frac{2x+5y+3}{3x-4y-2}=6$
$\frac{4x-7y+5}{x-2y+2}=5$

1222. Slate Problems.

1. The sum of two numbers is 37. Twice the first added to three times the second is 96. What are the numbers?

(Let x = first number; y = second number.)

2. The difference between two numbers is 28. Five times the first less twice the second is 197. What are the numbers?

($x-y=28$; $5x-2y=197$.)

3. The product of the first of two numbers by 5, added to the product of the second by 3, gives 37. The product of the first by 6, diminished by 5 times the second, equals 10. Find the numbers.

4. Divide 65 into two parts whose difference shall be 19.

(Let x and y = parts. Solve also by one unknown quantity.)

5. A person pays \$103 with 32 bills, some of them \$2 bills, the others \$5 bills. How many of each does he use?

6. For 25 head of pigs and sheep, a farmer received \$145. How many of each did he sell, if he sold the former at \$7 each, the latter at \$5 each?

7. 10 oranges and 4 peaches cost 38¢; 6 oranges and 7 peaches cost 32¢. Find the cost of an orange. Of a peach.

8. 5 pounds of tea and 3 pounds of coffee cost \$3.75; 8 pounds of tea and 1 pound of coffee cost \$5.05. What is each worth per pound?

9. A farmer buys a certain number of horses at \$125 each, four times as many cows at \$45 each, eight times as many sheep at \$10 each, and half as many pigs at \$5 each, spending \$1,550 for all. How many of each does he buy?

10. A man paid 75¢ for 2 pounds of raisins and 3 pounds of cheese. 5 pounds of raisins and 2 pounds of cheese at the same price would have cost 94¢. What did each cost per pound?

11. The sum of two numbers is 19. The sum of the second number and ten times the first, minus the sum of the first and ten times the second, equals 45. What are the numbers?

12. Reduce $\frac{5}{13}$ to an equivalent fraction, the sum of whose numerator and denominator shall be 126.

x = numerator; y = denominator.

$$\frac{x}{y} = \frac{5}{13};\ x + y = 126.$$

13. What fraction equivalent to $\frac{8}{15}$ has 147 for the difference between its numerator and denominator?

$(x - y = -147.$ Why?)

14. 10 pounds of coffee at 30¢ per pound are mixed with x pounds of coffee at 25¢ per pound. What is x equal to, when the mixture is worth 26¢ per pound?

$$25x + (10 \times 30) = 26(10 + x).$$

15. A grocer mixes green tea costing 60¢ per pound with black tea costing 40¢ per pound. He uses 100 pounds in all, and the mixed tea costs him 48¢ per pound. How many pounds of each does he use?

Let x = number of pounds of black tea; y = number of green. Then $x+y$ = number of pounds of mixed tea.

$$x+y=100;\ 40x+60y=48(x+y).$$

THREE UNKNOWN QUANTITIES.

1223. 1. Given the following:

$$8x+2y-z=12\ (a)$$
$$5x-4y+3z=16\ (b)$$
$$2x+3y+2z=35\ (c)$$

to find the values of x, y, and z.

(a) multiplied by 5,		$15x+10y-5z=60$
(b) " " 3,		$15x-12y+9z=48$
	Subtract,	$22y-14z=12\ (d)$

an equation containing only two unknown quantities.

(b) multiplied by 2,		$10x-8y+6z=32$
(c) " " 5,		$10x+15y+10z=175$
	Subtract,	$-23y-4z=-143\ (e)$

an equation containing only two unknown quantities.

Compare the two equations (d) and (e), which contain the same two unknown quantities.

(d) multiplied by 2,		$44y-28z=24$
(e) " " 7,		$-161y-28z=-1{,}001$
	Subtract,	$205y=1{,}025$
		$y=5$

Substituting this value of y in (d), we have

$$110 - 14z = 12, \quad -14z = -98, \quad z = 7.$$

Substituting values of y and z in (a), we have

$$3x + 10 - 7 = 12, \quad 3x = 9, \quad x = 3.$$

$$\textit{Ans.} \quad \left.\begin{aligned} x &= 3, \\ y &= 5, \\ z &= 7. \end{aligned}\right\}$$

2. Find the values of the unknown quantities in the following equations:

$$x - 3y + 2z = 3 \quad (a)$$
$$2x + y + 3z = 22 \quad (b)$$
$$5x + 2y + 7z = 51 \quad (c)$$

Multiply (a) by 2, and subtract from (b). Multiply (a) by 5, and subtract from (c). This gives two equations, each of which contains two unknown quantities.

Compare these two resulting equations, and eliminate y.

3.
$$5x - 2y + z = 10 \quad (a)$$
$$3x + 8y - 5z = 120 \quad (b)$$
$$7x - 3y - 2z = 8 \quad (c)$$

Eliminate z by comparing (a) and (b), multiplying the former by 5. Compare (a) and (c), multiplying the former by 2.

4.
$$13x - 4y + 15z = 317$$
$$7x + 2y - 3z = 89$$
$$21x - 17y + 9z = -104$$

5.
$$-8x + y - 12z = -259$$
$$7x - 4y + 25z = 418$$
$$13x + 2y - 41z = -500$$

6. $\frac{x}{3}+\frac{x+y}{3}=14$

$\frac{x+y}{2}-\frac{x-y}{6}=16$

7. $\frac{3x-5y}{2}+3=\frac{2x+y}{5}$

$8-\frac{x-2y}{4}=\frac{x}{2}+\frac{y}{3}$

8. $2+\frac{5x-6y}{13}=4y-3x$

$12+\frac{5x-6y}{6}=2y+\frac{3x-2y}{4}$

9. $\frac{5x-3}{4}-\frac{3x-19}{4}=2-\frac{3y-x}{6}$

$\frac{2x+y}{2}-\frac{9x-7}{8}=\frac{3y+9}{4}-\frac{4x+5y}{16}$

1224. Slate Problems.

1. A man placed $\frac{2}{3}$ of his capital at 5% and the other third at 6%. At the end of a year, capital and interest amounted to $31,600. What was his capital?

$\frac{2x}{3}\times\frac{5}{100}$ and $\frac{x}{3}\times\frac{6}{100}$ = interest.

2. A has 18 chestnuts more than B. If each finds 4 more, A will have four times as many as B. How many chestnuts has each?

3. Two mechanics earn together $8 per day. One works 23 days and the other 17 days, for which they receive together $166. What does each earn per day?

4. The sum of the first and the second of three numbers is 55, of the first and the third 62, of the second and the third 83. What are the numbers?

5. The sum of two numbers is 53. Four times the first is 20 more than twice the second. Find the numbers.

6. A certain sum of money is divided among four persons. The first takes $\frac{1}{5}$ of it, the second takes $\frac{4}{5}$ of the remainder, the third takes $\frac{2}{3}$ of what then remains, the fourth receives the balance, $24. What is the share of each of the other three?

7. A merchant sold a lot of goods for $510, thereby losing $\frac{3}{20}$ of their cost. What did the goods cost?

8. A man collected a bill for a physician and deducted $\frac{1}{50}$ of the amount for his services. If he gave the physician $147, what was the amount collected?

9. Divide $130\frac{1}{4}$ acres of land among three persons, giving the first $27\frac{1}{2}$ acres more than the second, and the second $13\frac{3}{4}$ acres more than the third.

10. A merchant has sold $\frac{4}{7}$ of a piece of cloth, and has remaining 16 yards more than $\frac{1}{5}$ of the piece. How many yards did the piece contain originally?

11. A servant is engaged for a year for $280 and a suit of clothes; he leaves at the end of six months, and receives $130 and the suit. What is the value of the clothes?

MULTIPLICATION OF ALGEBRAIC QUANTITIES.

1225. Multiply $x+3$ by $x+4$.

The product is equal to x times $(x+3)+4$ times $(x+3)$.

$$x(x+3) = x^2+3x$$
$$4(x+3) = \quad 4x+12$$
$$(x+3)(x+4) = x^2+7x+12 \quad \textit{Ans.}$$

NOTE. — x^2 is read x square. The 2 is called an *exponent*.

Multiply $(x+7)$ by $(x+8)$.

$$\begin{array}{ll} & x+7 \\ & x+8 \\ \hline \text{Product by } x, & x^2+7x \\ \text{Product by } 8, & \quad 8x+56 \\ \hline & x^2+15x+56 \quad \textit{Ans.} \end{array}$$

1226. Multiply:

1. $(x+5)$ by $(x+2)$
2. $(x+8)$ by $(x+9)$
3. $(2x+5)$ by $(x+2)$
4. $(2x+8)$ by $(x+9)$
5. $(3x+1)$ by $(x+7)$
6. $(2x+1)$ by $(2x+1)$

1227. $(x-5)\times(x+4)=?$

$$\begin{array}{ll} & x-5 \\ & x+4 \\ \hline x(x-5) & x^2-5x \\ 4(x-5) & \quad 4x-20 \\ \hline & x^2-x-20 \quad \textit{Ans.} \end{array}$$

1228. Find products:

NOTE. — $(x-3)(x+9)$ means $x-3$ multiplied by $x+9$.

7. $(x-3)(x+9)$
8. $(x-6)(x+7)$
9. $(x-5)(x+5)$
10. $(x+5)(x-5)$
11. $(2x-6)(x+1)$
12. $(x-6)(2x+1)$
13. $(2x-6)(3x+3)$
14. $(3x+6)(2x-3)$
15. $(2x+3)(2x-3)$

1229. $(x-5)(x-4)=?$

The product is equal to $x(x-5)-4(x-5)$; that is, that $4(x-5)$ is to be subtracted from $x(x-5)$.

$$x(x-5)=x^2-5x;\ 4(x-5)=4x-20.$$

Placing the subtrahend under the minuend, and changing the signs of the former (Art. 1209), we have

$$\begin{array}{l} x^2-5x \\ \quad -4x+20 \\ \hline (x-5)(x-4)=x^2-9x+20 \quad \textit{Ans.} \end{array}$$

1230. $(x-7)(x-9)=?$

Using either as a multiplier, place one under the other. Commencing with x, say $x \times x = x^2$, $-9 \times x = -9x$. Taking -7 as a multiplier, say $x \times (-7) = -7x$, $(-9) \times (-7) = 63$. Combining, we get the product.

$$\begin{array}{r} x - 9 \\ x - 7 \\ \hline x^2 - 9x \\ \quad - 7x + 63 \\ \hline \textit{Ans.}\ x^2 - 16x + 63 \end{array}$$

1231. Note that the multiplication of a + (positive) quantity by a + (positive) quantity gives a + (positive) product; that $(+) \times (-)$ or $(-) \times (+)$ gives a − (negative) product; and that $(-) \times (-)$ gives a + (positive) product. This is usually stated as follows:

1232. *Like signs produce +, and unlike signs produce −.*

1233. Give results:

16. $(x-7)(x-7)$ 20. $(x+7)(x-6)$ 24. $(2x+7)(3x+3)$
17. $(x-5)(x-9)$ 21. $(x-4)(x-7)$ 25. $(2x-3)(3x-2)$
18. $(x+5)(x+5)$ 22. $(2x-4)(3x-6)$ 26. $(2x-3)(2x+3)$
19. $(x-3)(x+8)$ 23. $(2x+6)(3x-7)$ 27. $(2x+9)(4x-6)$

PURE QUADRATICS.

1234. Given $\frac{x^2+6}{5} = \frac{3x^2-66}{9}$, to find the value of x.

Clearing of fractions, $9x^2 + 54 = 15x^2 - 330$
Transposing and combining, $-6x^2 = -384$
Dividing by 6, and changing signs, $x^2 = 64$
Extracting square root, $x = \pm 8$.

1235. Since $(-8) \times (-8) = 64$, the square root of 64 may be either $+8$ or -8. It is written ± 8, and is read "*positive or negative* 8." (It is sometimes less correctly called *plus or minus* 8.)

1236. Slate Exercises.

Find value of x, y, z, etc.:

1. $x^2 - 13 = 36$
2. $3y^2 + 25 = 100$
3. $5z^2 - 13 = 3z^2 + 37$
4. $5(x^2+17) - 3x^2 + 63 = 198$
5. $5(x^2+17) - 3(x^2-21) = 198$
6. $y^2 + 2y + 1 - y^2 = 49$
7. $(x+1)^2 - x^2 = 49$
8. $\frac{y^2+5}{3} - \frac{2y^2-18}{4} = 2$
9. $\frac{z+7}{z-3} = \frac{z-5}{z-9}$
10. $\frac{20x}{x-1} = \frac{30x}{x+1}$
11. $(x-3)(x+3) = 40$
12. $(x+5)(x+5) = 10x + 26$
13. $(x+4)^2 = 8x + 80$
14. $z^2 + 64 = 5z^2$
15. $3x^2 + 18 = 2\frac{1}{2}x^2 + 36$
16. $(x-3)^2 - (x-5)^2 = 12$
17. $(x+7)(x-9) = (x-3)(x-5)$
18. $\frac{x}{4} + \frac{4}{x} = \frac{x}{9} + \frac{9}{x}$
19. $\frac{x+7}{x-5} = \frac{x-3}{x-9}$
20. $\frac{y-9}{y-5} = \frac{y-3}{y+7}$

1237. Slate Problems.

1. Find the dimensions of a field, the length of which is twice its breadth, its area being 1,800 square rods.

2. The surface of the six equal faces of a cube contains 96 square inches. Find the length of one edge.

3. One number is fourth-fifths of another, and their product is 80. What are the numbers?

4. One-third of a number multiplied by two-fifths of the same number gives a product of 270. Find the number.

5. Thirty per cent of a number multiplied by forty per cent of the same number gives a product of 300. What is the number?

6. Thirty per cent of forty per cent of a number is 300. What is the number?

7. The base of a right-angled triangle is $\frac{3}{4}$ as long as the perpendicular, and the area of the triangle is 96 square rods. Find the length of the base. What is the length of the hypotenuse?

8. The base of a right-angled triangle measures x yd., the perpendicular measures $\frac{3x}{4}$ yd. What is the length of the hypotenuse? If the hypotenuse measures 15 yd., find the length of the base.

9. The base of a right-angled triangle measures x ft., the hypotenuse measures $(x+9)$ ft., the perpendicular measures 15 ft. What is the length of the base?

10. The difference between the squares of two consecutive numbers is 49. What are the numbers?

AFFECTED QUADRATICS.

1238. Preliminary Exercises.

$$(x+1)(x+1) = x^2 + 2x + 1$$

The square of the sum of two quantities is equal to the square of the first + twice the product of the first and the second + the square of the second.

$$(x-1)(x-1) = x^2 - 2x + 1$$

The square of the difference of two quantities is equal to the square of the first − twice the product of the first and the second + the square of the second.

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(m-n)^2 = m^2 - 2mn + n^2$$
$$(10+5)^2 = 10^2 + 2 \times 10 \times 5 + 5^2$$
$$(10-3)^2 = 10^2 - 2 \times 10 \times 3 + 3^2$$

1239. Oral Exercises.

Square:

1. $x+3$	4. $x+10$	7. $30-1$	10. $x-y$
2. $x-7$	5. $a-b$	8. $40-1$	11. $80+5$
3. $x-9$	6. $x+y$	9. $m+n$	12. $60-5$

1240. Sight Exercises.

Extract the square root of

1. x^2+6x+9	6. $x^2+2xy+y^2$
2. $x^2-14x+49$	7. $x^2-2xy+y^2$
3. $x^2-18x+81$	8. $a^2-2ab+b^2$
4. $x^2+20x+100$	9. $x^2-24x+144$
5. $a^2+2ab+b^2$	10. $x^2+22x+121$

1241. The square of $(x+3)$ consists of how many terms? Of how many terms does $(x+4)^2$ consist? $(x+5)^2$?

1242. Supply term necessary to make a complete square:

1. $x^2+6x+?$	6. $x^2+2x+?$
2. $x^2-12x+?$	7. $x^2-4x+?$
3. $x^2-8x+?$	8. $x^2-10x+?$
4. $x^2-16x+?$	9. $x^2+14x+?$
5. $x^2+18x+?$	10. $x^2-22x+?$

1243. Slate Exercises.

Given $x^2+6x=27$

What number must be added to the first member of the equation to make it a "complete" square?

If a number is added to one member of an equation, what must be done to the other member to preserve the equality?

1244. Extract the square root of both members of the following equations, adding to both, where necessary, such a number as will make the first member a complete square.

1. $x^2+6x+9=40+9$ 2. $x^2-12x+36=28+36$

Remember that $(+7)\times(+7)=49$, and that $(-7)\times(-7)=49$.
$\therefore \sqrt{49}=+7$ or -7, written ± 7.

3. $x^2-8x+16=20+16$
4. $x^2-16x+64=-39+64$
5. $x^2+18x+?=19+?$
6. $x^2+2x+?=24+?$
7. $x^2-14x=15$
8. $x^2-22x=23$
9. $x^2+14x=51$
10. $x^2-22x=48$

1245. Given $x^2-10x=24$.

Completing the square, we have $x^2-10x+25=24+25=49$.
Extracting the square root of both sides, we have

$$x-5=\pm 7,$$
$$x=7+5=12, \text{ or } -7+5=-2.$$

Ans. 5 or -2.

1246. Find values of x:

1. $x^2-6x=7$
2. $x^2-12x=108$
3. $x^2+2x=48$
4. $x^2+18x=115$
5. $x^2-14x=-13$
6. $x^2-10x=0$
7. $x^2+20x=125$
8. $x^2+26x=56$
9. $x^2-24x=0$
10. $x^2-8x=384$
11. $x^2-4x=-3$
12. $x^2+30x=175$
13. $x^2+28x=29$
14. $x^2+22x=104$
15. $x^2-16x=-64$
16. $x^2+36x=76$

1247. To make the first member a complete square, you added the square of what part of the coefficient of x?

1248. Find values of x:

1. $x^2 + x = 12$
   $x^2 + x + (\frac{1}{2})^2 = 12 + (\frac{1}{2})^2$
2. $x^2 - 3x = 10$
   $x^2 - 3x + (\frac{3}{2})^2 = 10 + (\frac{3}{2})^2$
3. $x^2 + 5x = -4$
4. $x^2 - 7x = 8$
5. $x^2 + 9x = -20$
6. $x^2 - 11x = -28$
7. $x^2 + 13x = -42$
8. $x^2 - 15x = 76$
9. $x^2 - 17x = 18$
10. $x^2 + 19x = -18$

1249. When x^2 has a coefficient, divide both members by the coefficient.

$$3x^2 + 9x = 84.$$

Dividing by 3, $x^2 + 3x = 28.$

Completing the square,

$$x^2 + 3x + (\tfrac{3}{2})^2 = 28 + \tfrac{9}{4} = \frac{112 + 9}{4} = \frac{121}{4}$$

Extracting square root, $x + \frac{3}{2} = \pm \frac{11}{2}$.

$$\therefore x = \tfrac{11}{2} - \tfrac{3}{2} = \tfrac{8}{2} = 4; \text{ or } -\tfrac{11}{2} - \tfrac{3}{2} = -\tfrac{14}{2} = -7.$$

Ans. 4 or -7.

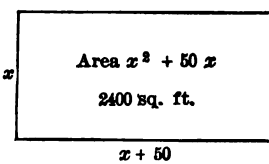
1. $6x^2 - 6x = 36$
2. $9x^2 + 9x = 180$
3. $7x^2 + 28x = 147$
4. $4x^2 - 40x = -64$
5. $8x^2 - 16x = 504$
6. $3x^2 + 9x = 54$
7. $8x^2 - 72x = -160$
8. $7x^2 + 49x = 56$
9. $3x^2 + 21x = 54$
10. $5x^2 - 25x = -20$

1250. Slate Problems.

1. The sum of two numbers is 12; their product is 32. What are the numbers?

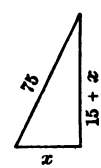
x and $12 - x$ = numbers. $(12 - x)x$ = product.

2. The base of a rectangle is 50 feet longer than its altitude. Its area is 2,400 square feet. How long is the base?



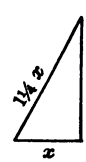
3. The perpendicular of a right-angled triangle measures 15 yards more than the base. The hypotenuse is 75 yards. Find the length of the perpendicular.

$[x^2 + (15 + x)^2 = 75^2.]$

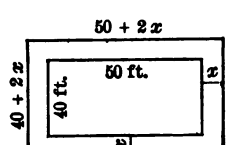


4. The hypotenuse of a right-angled triangle is $1\frac{1}{4}$ times as long as the base. The area of the triangle is 150 square yards. How long is the hypotenuse?

[Perpendicular = $\sqrt{(\frac{5}{4}x)^2 - x^2}$; area = $\frac{1}{2}$ base × perpendicular.]



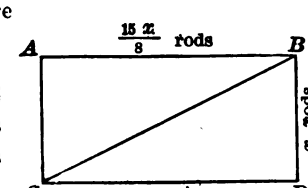
5. The entire surface of a square prism is 170 square feet. Its altitude is 6 feet, and one side of its base is x feet. Find the value of x.



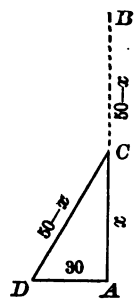
6. A garden 50 feet long, 40 feet wide, has a walk just outside it x feet wide. Find the area of the walk.

If the area of the walk is 784 square feet, what is its width?

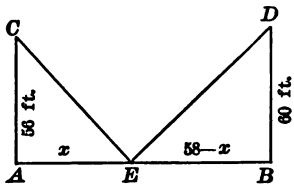
7. A field, $ABCD$, contains 12 acres. Its length is $1\frac{7}{8}$ times its breadth. How many rods long is the diagonal BC?



8. A flag-staff, AB, 50 feet high, was broken off at the point C. The broken part, resting on C, reached the ground D, 30 feet from the base of the staff. Find the length of the part broken off.

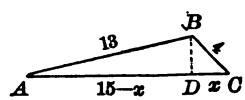


9. A ladder, CE or DE, placed at a point E, in a street 58 feet wide between the opposite houses, just touches the top of a house, DB, 60 feet high on one side of the street, or the top of a house, CA, 56 feet high on the other side. Find the length of the ladder.



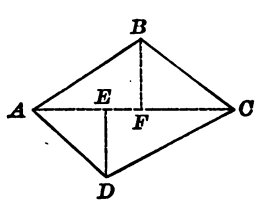
$$\overline{DE}^2 = 60^2 + (58 - x)^2 = \overline{CE}^2 = 56^2 + x^2.$$

10. ABC is a triangle. The side AB measures 13 feet; the side BC, 4 feet; AC, 15 feet. Find the altitude BD.



$$\overline{BD}^2 = \overline{AB}^2 - \overline{AD}^2 = \overline{BC}^2 - \overline{CD}^2.$$

11. $ABCD$ is a trapezium. $AB = 34$ ft.; $BC = 20$ ft.; $CD = 40$ ft.; $DA = 26$ ft. The perpendicular BF measures 16 ft. Find the length of the diagonal AC and of the perpendicular ED.



CHAPTER XVI.

ELEMENTARY GEOMETRY. — PROBLEMS IN CONSTRUCTION. — PRACTICAL APPLICATIONS. — CALCULATION OF HEIGHTS AND DISTANCES. — MENSURATION.

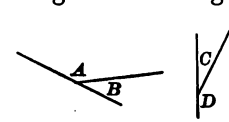
ELEMENTARY GEOMETRY.

1251. Angles.

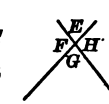
When two straight lines meet at a point, they are said to form an *angle*.

The point at which the lines meet is called the vertex of the angle.

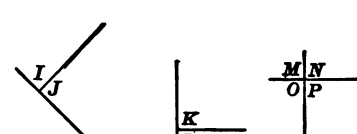
When two angles are formed by the meeting of two straight lines, they are called *adjacent* angles. A and B are adjacent angles. C and D are adjacent angles.



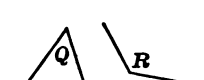
The angles E and G, formed by the intersection of two straight lines, are called *vertical*, or *opposite* angles. F and H are vertical angles. E and F, F and G, G and H, H and E, are adjacent angles.



When two adjacent angles are equal to each other, each is said to be a *right* angle. The angles I, J, K, L, M, N, O, P are right angles.



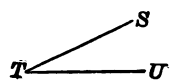
An angle that is smaller than a right angle is called an *acute* angle; one larger than a right angle is called an *obtuse* angle. Q is an acute angle; R is an obtuse angle.



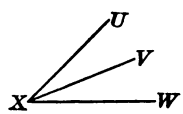
Angles that are not right angles are called, without regard to their size, *oblique* angles.

1252. Designation of Angles.

The angle formed by the lines ST and TU may be called the angle T. It is frequently better to call it the angle STU or UTS, the letter at the vertex being placed between the two others.

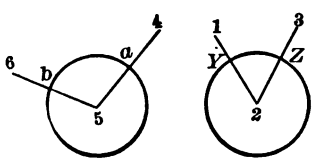


The use of the three letters is necessary where there is more than one angle having its vertex at the same point, as in the accompanying figure, where UX, VX, and WX meet at the point X.



1253. Measurement of Angles.

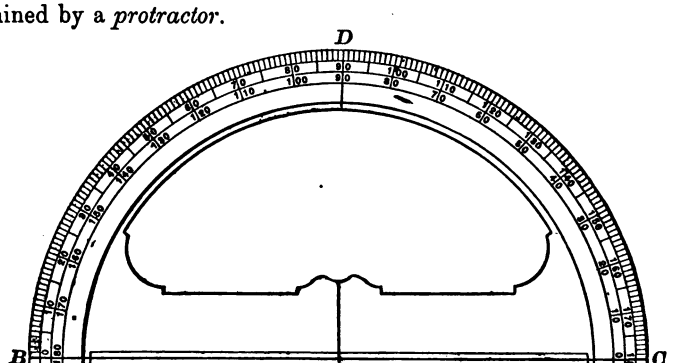
An angle is measured by the arc of a circle, the center of the circle being at the vertex of the angle. The angle 1 2 3 is measured by the arc YZ; the angle 4 5 6 by the arc ab.



1254. Circular Measure.

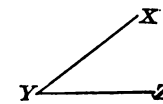
60 seconds	(″)	1 minute,
60 minutes	(′)	1 degree,
360 degrees	(°)	1 circle.

1255. The number of degrees in an arc or an angle is determined by a *protractor*.

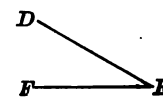


SEMI-CIRCULAR PROTRACTOR

To measure an angle, XYZ, for instance, produce the lines YX and YZ. Place the point A of the protractor on the vertex (Y) of the angle, and the edge AC on the line YZ produced. Using the lower line of figures, read off from the protractor the number of degrees at the point where the line YX produced cuts the semi-circle.



In measuring the angle DEF, the line AB is placed on EF, the point A on the vertex E. The number of degrees in this case is read from the upper row of figures.



EXERCISES IN CONSTRUCTION.

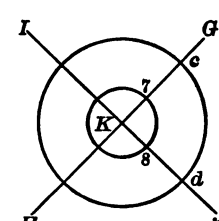
1256. NOTE. — In the following 100 exercises, the ruler, the compasses, and the protractor may be used.

The drawing should be carefully done with a sharp, hard pencil.

1. Draw an obtuse angle formed by two lines, each one inch long. Draw an acute angle formed by two lines, each six inches long. Which is the larger?

2. Fold a piece of paper twice, so that the lines made by the creases will form four right angles.

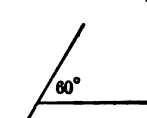
Fold a piece of paper so as to make four angles that are not right angles.



3. The lines GH and IJ intersect at K, making four right angles. Which arc is longer, 7 8, or cd? Which contains the greater number of degrees?

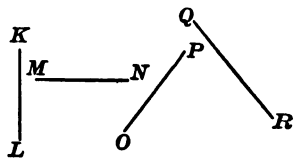
4. Draw two lines meeting at an angle of 45°. Two lines meeting at an angle of 90°. Two meeting at an angle of 135°.

5. Draw two lines making two angles, one of which measures 60°. How many degrees does the other angle contain?



1257. NOTE. — A line parallel to the right or the left side of the paper is called a *vertical* line; one parallel to the top or the bottom of the paper is called a *horizontal* line; others are called *oblique* lines.

KL is a vertical line, *MN* is a horizontal line, *OP* and *QR* are oblique lines.



6. To a horizontal line draw a line making two equal adjacent angles. How many degrees does each angle contain?

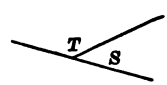
To a vertical line draw a line making two equal adjacent angles. How many degrees does each angle contain?

To an oblique line draw a line making two equal adjacent angles. How many degrees does each angle contain?

7. How many degrees are there in a right angle?

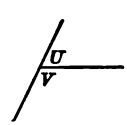
8. To an oblique line draw a line making two unequal adjacent angles. How many degrees are there in the sum of the two angles?

9. How many degrees in the angle *T*, if *S* contains 75°?



V measures 110°. How many degrees does *U* measure?

If one of two adjacent angles measures $63\frac{1}{2}°$, how many degrees are there in the other angle?



How many degrees are there in an angle adjacent to one of 47° 45′?

10. Construct angle 5, 60°; angle 4, 50°. Measure angle 3.

How many degrees and minutes will there be in angle 5, when 3 contains $49\frac{1}{2}°$ and 4 contains $83\frac{3}{4}°$?

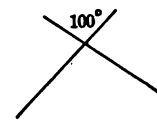
When angle 3 contains 36° 30′ and angle 5 contains 79° 45′, how many degrees and minutes will angle 4 contain?

11. Erect a perpendicular at each extremity of a horizontal line. At each extremity of a vertical line. At each extremity of an oblique line.

1258. NOTE. — A line making a right angle with another line is said to be *perpendicular* to it.

12. Construct a square upon a horizontal line. Upon an oblique line.

13. Draw two lines intersecting at an angle of 100°. Mark in each of the other three angles the number of degrees it contains.



14. If one of the four angles formed by two intersecting lines measures 90°, what does each of the other three measure?

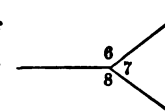
If one measures 60°, what does each of the others measure?

15. At each extremity of a horizontal line draw a line making an angle of 40° with the first line.

16. At each extremity of a vertical line draw a line making an angle of 100° with the first line.

17. At one extremity of an oblique line draw a line making with the first line an acute angle; at the other extremity draw a line making an obtuse angle with the first line.

18. Draw two lines making an angle (6) of 150°. Construct an adjoining angle (7) containing 80°. How many degrees will angle 8 contain?



19. How many degrees will there be in the sum of five angles having the same vertex?

20. Draw five equal angles having a common vertex.

Is any one of these five angles adjacent to any other?

21. Draw six equal angles having a common vertex. Is any angle adjacent to the angle next it? Why?

Are any of the angles vertical? Why?

22. Draw two angles, one of 65° and the other of 25°. Draw a third angle equal to the sum of both.

Draw an angle equal to their difference.

23. Draw an angle equal to the sum of three angles measuring, respectively, 40°, 50°, and 60°.

24. How many degrees are contained in the angle made by the hour and the minute hand of a watch at 1 o'clock? How many degrees in the angle made by the hands of a church-tower clock at the same hour?

25. How many degrees are passed over by the minute hand in a quarter of an hour? How many are passed over by the hour hand in an hour? In half an hour? In 15 minutes?

What angle is made by the hands at 12:15? At 6:30? At 8:20?

1259. NOTE. — The pupils should, of course, understand that the angles in questions 24 and 25 are those formed by imaginary lines passing through the centers of the hands. They should know, too, that a geometrical line has neither breadth nor thickness

1260. Parallels.

Lines lying in the same plane that will not meet, no matter how far they are produced, are said to be *parallel.*

26. Draw two or more lines that shall be perpendicular to a horizontal line. Where will they meet?

Draw two or more that shall be perpendicular to a vertical line. Where will they meet?

Draw two or more that shall be perpendicular to an oblique line. Where will they meet?

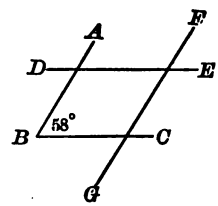
27. To a horizontal line draw two or more lines running in the same direction, and each making an angle of 35° with the first line. Will the oblique lines meet?

Draw two or more lines running in the same direction, and each making an angle of 125° with a vertical line. Will the oblique lines meet if produced very far?

Draw two or more lines running in the same direction, and each making an angle of 74° with an oblique line? Will the former lines meet?

28. Draw two lines making angles of 30° and 60°, respectively, with a third line. Will the two former lines meet if produced in either direction?

29. Draw a line, AB, meeting a horizontal line BC at an angle of 58°. Draw a third line, DE, parallel to the horizontal line, and cutting the oblique line. What angles does it make with the oblique line?

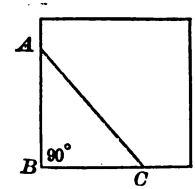


Draw a fourth line, FG, parallel to the oblique line, and cutting both horizontal lines.

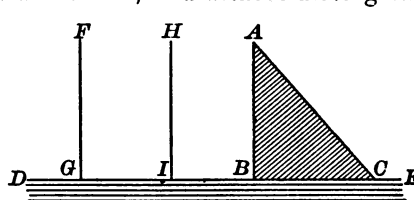
Mark in each of the twelve angles the number of degrees it contains.

Take a stiff card, having a square corner, and cut off the corner, making the side AB equal to the side BC, and each about 4 or 5 inches long.

Measure the angle at A and the angle at C.



30. With a ruler, draw a line DE; and without moving the ruler, draw lines FG, HI, AB, by placing the side BC of the triangle firmly against the edge of the ruler.

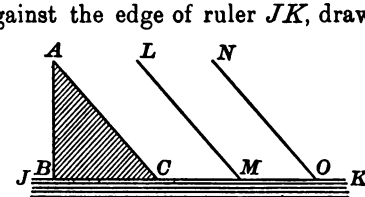


FG, HI, and AB are parallel. Why?

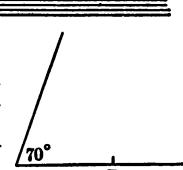
31. Holding the triangle against the edge of ruler JK, draw oblique lines AC, LM, NO.

If the angle ACB contains 45°, how many degrees will LMJ contain? How many will NOJ contain?

Are AC, LM, and NO parallel? Why?

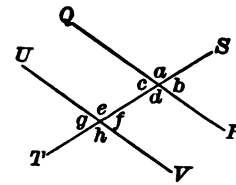


32. Draw a line making an angle of 70° with a horizontal line. At the point P, using only the ruler and the triangle, draw a line parallel to the oblique line.



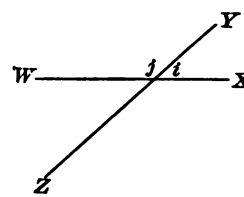
33. QR and UV are parallel lines, cut by a line ST. If the angle b measures 50°, how many degrees does a measure?

Find the number of degrees in each of the other six angles.



34. Draw WX and YZ intersecting at any angle. With the ruler and the triangle draw a line parallel to WX.

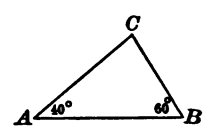
Mark all the angles that are equal to i. Mark all those that are equal to j.



1261. Triangles.

35. From the extremities of the line AB, draw lines that shall make angles of 60° and 40°, respectively, with AB. Prolong the lines until they meet at C, forming a triangle.

Measure the angle at C. How many degrees does it contain? How many degrees are there in the sum of the three angles of the triangle?

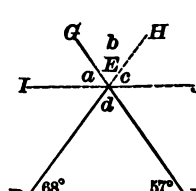


36. Construct a triangle having one angle of 90° and one of 30°. Measure the third angle.

How many degrees are there in the sum of the three angles?

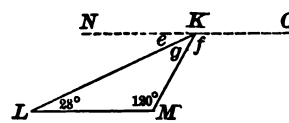
37. Draw a triangle having angles at the base measuring, respectively, 57° and 68°. With the ruler and the triangle draw IJ parallel to DF. Produce DE to H and FE to G.

How many degrees will the angle a contain? How many degrees in the angle c? Calculate the number of degrees in the angle b when you know



the number of degrees in a and c, respectively. What angle is vertical to b? How many degrees in d?

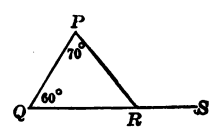
38. Construct a triangle, KLM, making the angles at the base 28° and 120°, respectively. Draw, as before, NO, parallel to LM.



Is the angle e equal to any angle of the triangle? How many degrees does it contain? Is the angle f equal to any angle of the triangle? How many degrees does it contain?

How many degrees are there in the sum of the angles e, g, and f? How many degrees are there in the angle g?

39. The angle at P is 70°, the angle at Q is 60°. Can you tell how many degrees there are in the angle PRS, formed by producing the base RS?



The angle PRS is called an *exterior* angle of the triangle PQR.

40. How many degrees are there in the three angles of any triangle?

41. Two angles of a triangle measure 36° and 65°, respectively. How many degrees does the third angle contain?

1262. A triangle containing a right angle is called a *right-angled* triangle.

A triangle containing an obtuse angle is called an *obtuse-angled* triangle.

A triangle all of whose angles are acute, is called an *acute-angled* triangle.

Obtuse-angled triangles and acute-angled triangles are also called *oblique-angled* triangles.

42. Draw a triangle that shall contain an acute angle. Mark the acute angle.

Draw one that shall contain two acute angles. One that shall contain three acute angles.

43. See if you can draw a triangle containing three right angles.

A triangle containing two right angles.

A triangle containing one right angle.

44. Try to draw a triangle that shall contain three obtuse angles.

A triangle that shall contain two obtuse angles.

A triangle containing one obtuse angle.

45. Draw a triangle containing two angles of 50° and 70°, respectively. How many degrees are there in the third angle?

Measure each side, and mark on the side its length.

Opposite which angle is found the longest side? Opposite which, the shortest side?

46. Draw a triangle having two angles of 75° each. Are any two of its sides equal?

Draw a triangle having two angles of 50° each. Are any of its sides equal?

47. Draw a triangle having two angles of 60° each. How many degrees does the third angle contain?

Are any of its sides equal?

1263. A triangle having all its sides equal, is called an *equilateral* triangle.

A triangle having two equal sides, is called an *isosceles* triangle.

A triangle having all its sides unequal, is called a *scalene* triangle.

48. Fold evenly on the line AC (Fig. 1), a rectangular sheet of paper. Cut off, on the line AB, one of the folded corners. When this piece is opened out (Fig. 2), it makes what kind of a triangle? Which are the equal sides? How does the crease line, AC, divide the base, BB?

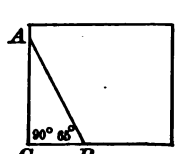


FIG. 1.

If the angle ABC, Fig. 1, is 65°, and the angle at C is a right angle, what is the angle BAC?

How many degrees in the angle BAB', Fig. 2? How does a perpendicular let fall upon the base of an isosceles triangle from the opposite angle divide the angle? How does it divide the base? How do the angles at the base of an isosceles triangle compare with each other as to size?

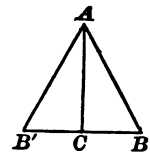


FIG. 2.

1264. The *unequal* side of an isosceles triangle is called the base.

49. Draw an isosceles triangle having the base a vertical line.

An isosceles triangle having the vertex below the base.

One having an oblique line for the base.

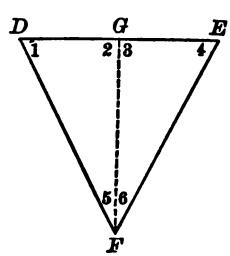
50. Draw a right-angled isosceles triangle. How many degrees will there be in each of the other angles?

Draw an obtuse-angled isosceles triangle.

51. How many degrees will there be in each angle of an equilateral triangle?

Draw an equilateral triangle having one side vertical.

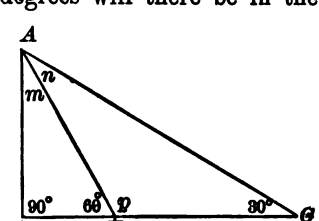
Draw an equilateral triangle having its vertex below the base.



52. DEF is an isosceles triangle, DF and EF being the equal sides. If the angle 1 measures 50°, how many degrees are there in each of the other five angles, when the line FG bisects the base?

53. If the angle DFE in the above triangle measures 45°, how many degrees will there be in the angles 1, 2, 3, 4, 5, and 6?

54. ABC is a right-angled triangle, the angle at B measuring 90°, and the angle at C measuring 30°. If the line AX is so drawn as to make the angle AXB equal to 60°, find the number of degrees in the angles m, n, and p, respectively?



1265. Quadrilaterals.

A plane figure of four sides is called a *quadrilateral.*

When the opposite sides are parallel, the quadrilateral is called a *parallelogram.* (Fig. 1 to 8.)

A *rectangle* is a parallelogram all of whose angles are right angles. (Fig. 1 to 4.)

When the four sides of a rectangle are equal to each other, it is called a *square.* (Fig. 1 and 2.)

The term *oblong* is frequently applied to rectangles whose adjacent sides are unequal. (Fig. 3 and 4.)

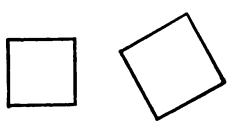


FIG. 1. FIG. 2.

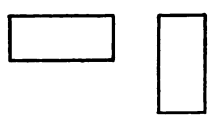


FIG. 3. FIG. 4.

A *rhombus* is a parallelogram all of whose sides are equal, but whose angles are oblique. (Fig. 5 and 6.)

When the adjacent sides of a parallelogram are unequal and the angles are oblique, it is called a *rhomboid.* (Fig. 7 and 8.)

FIG. 5. FIG. 6.

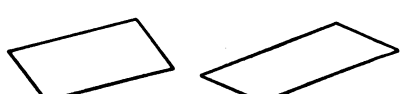


FIG. 7. FIG. 8.

A *trapezoid* is a quadrilateral having only two of its sides parallel. (Fig. 9 and 10.)

A *trapezium* is a quadrilateral having no two sides parallel. (Fig. 11 and 12.)

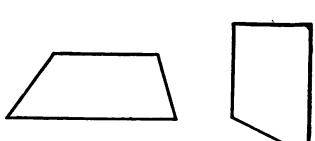
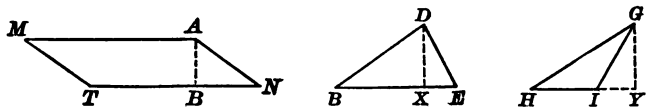


FIG. 9. FIG. 10

FIG. 11. FIG. 12.

1266. The *altitude* of a *parallelogram* is the perpendicular distance between its base and the side opposite.



The *altitude* of a *triangle* is the perpendicular distance between the vertex and the base, or between the vertex and the base produced.

AB is the altitude of $MANT$; DX is the altitude of DBE; GY of GHI.

55. Draw a parallelogram. How many angles does it contain? Into how few triangles can you divide a parallelogram? How many degrees are there in the sum of the angles of each triangle? How many degrees are there in the sum of the angles of a parallelogram?

56. Construct a parallelogram, the adjacent sides of which shall measure 2 inches and 3 inches, respectively, and the angle between them 60°. How long will each of the other two sides be? Measure each of the other angles. How many degrees are there in the sum of the four angles?

57. Construct a trapezoid having a base of 5 inches, altitude 3 inches, the angles at the base measuring 90° and 60°, respectively. Measure the remaining angles, and find the sum of the four angles. How long is each of the remaining sides?

58. Fold a piece of paper twice at right angles, and cut off the folded corner, making a rhombus when the part cut off is opened out.

Can you cut out a rhombus having two angles of 60° each? A rhombus having two angles of 80° each?

59. Can you so cut a piece of paper, folded twice at right angles, that the part cut off will be a square?

60. We have seen that an equilateral triangle is *equiangular;* that is, that all its angles are equal. Are the four angles of a square equal? Are the four angles of a rhombus equal?

61. A triangle that has three equal angles is *equilateral;* that is, all its sides are equal. Can you draw an equiangular parallelogram that will be equilateral? Can you draw an equiangular parallelogram that will not be equilateral?

62. Draw a rectangle, base $2\frac{1}{2}$ inches, altitude 2 inches.
A rhomboid, base $2\frac{1}{2}$ inches, altitude 2 inches.

63. Make, out of paper, a rectangle and a rhomboid, each having the above dimensions, and endeavor to ascertain, by cutting, whether or not they are equal to each other in area.

64. Show, by cutting, that any parallelogram can be divided into two equal triangles.

65. Make three rhomboids of different shapes, the base of each to measure 3 inches, and the altitude 2 inches. Are they equal to each other in area?

66. Make three rhomboids of different shapes, the base of each to measure 3 inches, and the adjacent side 2 inches. Are they equal to each other in area?

67. How do we calculate the area of a rectangle 3 inches by 2 inches? Of a rhombus whose base is 3 inches, altitude 2 inches? Of a rhomboid, base 3 inches, altitude 2 inches? Of a right-angled triangle, base 3 inches, altitude 2 inches? Of an obtuse-angled triangle, base 3 inches, altitude 2 inches? Of an acute-angled triangle, base 3 inches, altitude 2 inches? Draw each of the foregoing.

68. Draw a rectangle having two adjacent sides measuring 3 inches and 2 inches, respectively. A rhomboid having two adjacent sides measuring 3 inches and 2 inches, respectively. A trapezium having two adjacent sides measuring 3 inches and 2 inches, respectively. A trapezoid having two adjacent sides

measuring 3 inches and 2 inches, respectively. A right-angled triangle, an obtuse-angled triangle, and an acute-angled triangle, each having two adjacent sides measuring 3 inches and 2 inches, respectively.

Calculate the area of each, making such measurements as may be necessary.

69. Draw three trapezoids of different shapes, whose parallel sides shall measure 3 inches and $2\frac{1}{2}$ inches, respectively, and whose altitude shall be 2 inches.

Draw a rectangle 2 inches high that shall be equal in area to each of the foregoing. How long will be its base? What will be the relation of the length of its base to the lengths of the parallel sides of the trapezoids?

70. Draw three trapeziums of different shapes, each having one diagonal of 3 inches, the perpendiculars let fall on this diagonal from the opposite corners measuring $2\frac{1}{2}$ and 2 inches, respectively.

What is the altitude of a rectangle that is equal in area to each of the foregoing, when the base of the rectangle measures 3 inches?

1267. The Circle.

71. With the compass points one inch apart, draw a circle.

Draw a radius. How long is it?

The line made by the compass pencil is called the *circumference*. A line drawn from the center to the circumference is called a *radius*.

72. Draw a circle, radius $1\frac{1}{2}$ inches. From a point on the circumference, draw a line through the center to an opposite point on the circumference.

This line is called a *diameter*. How long is the diameter?

73. Draw a circle, radius $1\frac{3}{4}$ inches. Draw a diameter and several radii.

Write *diameter*, *circumference*, *radius*, each in its proper place.

74. Draw a circle. Between two points on the circumference draw a line that does not pass through the center.

This line is called a *chord*.

75. Draw a circle. In it draw two diameters, a radius, and three chords. Write on each line its name.

76. Draw a part of the circumference of a circle greater than one-half of it. Draw the chord.

A part of the circumference is called an *arc*.

77. Draw an arc less than a semi-circumference. Draw a chord. Write the name on each.

Can you make a chord that will be longer than the diameter?

78. Draw two equal circles. In the first draw the chord of an arc of 120°. In the second, the chord of an arc of 240°. What is the ratio between the two chords you have drawn?

79. In a circle draw a chord equal in length to the radius. How many degrees are there in the arc whose chord has been drawn?

80. Draw an arc of 72°. To its extremities draw two radii.

The part of the surface of a circle enclosed by two radii and the intercepted arc is called a *sector*.

81. Draw a sector of 60° (a *sextant*). A sector of 90° (a *quadrant*).

82. Draw an arc of 120 degrees. Draw the chord.

The part of the surface of a circle bounded by an arc and its chord is called a *segment*.

83. Draw several circles having the same center, but of unequal radii (*concentric* circles).

84. Draw two equal circles just touching each other (*tangent*). Draw two unequal circles tangent to each other.

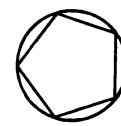
Within a large circle draw a smaller one tangent to it.

85. Draw circles of equal radii cutting each other. Draw intersecting circles of unequal radii.

1268. Pentagons, Hexagons, Octagons.

86. Divide the circumference of a circle into four equal arcs. Draw the chords, forming an inscribed square.

87. If you wish to inscribe in a circle a figure of five equal sides, into how many equal arcs must the circumference be divided? How many degrees will each arc contain?



1269. A plane figure bounded by straight lines is called a *polygon.*

A five-sided polygon is called a *pentagon;* one of six sides, a *hexagon;* of seven, a *heptagon;* of eight, an *octagon;* of nine, a *nonagon;* of ten, a *decagon;* etc.

A *regular* polygon is one that is both equilateral and equiangular.

88. Inscribe a regular pentagon in a circle. Use the protractor.

89. Inscribe in a circle a regular hexagon. A regular octagon. An equilateral triangle.

90. Inscribe in a circle a regular hexagon. Connect the opposite corners by lines passing through the center of the circle, forming six triangles.

How many degrees are there in each of the six angles about the center of the circle? In each of the twelve angles at the circumference? How many degrees are there in the sum of angles 1 and 2?

Is each of the six triangles scalene, equilateral, or isosceles?

91. Divide a regular inscribed pentagon into five equal triangles by lines drawn from the center of the circle.

What kind of triangles are formed; isosceles, scalene, or equilateral?

How many degrees are there in each angle at the center? In each angle at the circumference? How many degrees are there in the sum of two adjoining angles at the circumference?

92. Circumscribe a square about a circle. An equilateral triangle. A regular pentagon. A regular hexagon. A regular octagon.

93. Draw a pentagon. What is the smallest number of triangles into which a pentagon can be divided? How many degrees are there in all of these triangles? How many degrees are there in each angle of a regular pentagon?

94. Draw a hexagon. What is the smallest number of triangles into which it can be divided? How many degrees do the six angles of a hexagon contain? How many degrees are there in each angle of a regular hexagon?

95. A quadrilateral is divisible into how few triangles? A pentagon into how few? A hexagon? A heptagon? An octagon?

The smallest number of triangles into which a polygon is divisible is how many less than the number of its sides?

96. How many degrees are there in each angle of a regular octagon?

97. Using a protractor, construct a regular pentagon on a line two inches long.

98. On a three-inch line construct an equilateral triangle. On the same line construct a square, a regular pentagon, a regular hexagon, heptagon, octagon, nonagon, etc.

99. Inscribe a square in a circle. Circumscribe a square about the same circle by drawing lines touching the four corners of the inscribed square.

What is the ratio between the areas of the two squares?

100. Inscribe an equilateral triangle in a circle. Circumscribe an equilateral triangle about the same circle by drawing lines touching the vertex of each of the three angles of the inscribed triangle.

What is the ratio between a side of the inscribed triangle and a side of the circumscribed triangle? What is the ratio between the areas of the two triangles?

PROBLEMS IN CONSTRUCTION.

1270. In drawing the following one hundred exercises, only the ruler and the compasses are to be used. Use neither the protractor nor the triangle.

1. Draw a circle, radius an inch and a half. Outside of it, and tangent to it, draw a second circle of an inch radius.

How far apart are the centers?

2. Draw two tangent circles having radii of an inch and a half and an inch, respectively, one within the other.

How long is the line joining the centers?

3. With centers 3 inches apart draw two equal circles tangent to each other. How long is the radius of each?

4. With centers three inches apart draw two equal circles of 2 inches radius. Connect the centers.

Draw a line joining the two points in which the circles intersect. How does this line divide the line connecting the centers?

Draw radii from each center to each point of intersection.

5. Construct an isosceles triangle, base 3 inches, equal sides 2 inches.

NOTE. — Use circles or arcs where necessary.

6. Construct an isosceles triangle, base $3\frac{1}{2}$ inches, equal sides 4 inches.

Divide it into two equal parts. Do not locate the center of the base by measurements.

7. On a vertical line construct an isosceles triangle. Without measuring the length of the base draw a perpendicular to the center of the base.

8. Bisect a vertical line. An oblique line.

Do not measure the length of the line.

9. Construct an equilateral triangle on a two-inch line.

10. Construct an equilateral triangle on a vertical line. On an oblique line.

11. Construct a scalene triangle.

A triangle having sides measuring 1, $1\frac{1}{4}$, 2 inches, respectively.

One whose sides measure 2, $2\frac{1}{2}$, and 3 inches, respectively.

12. Can you construct an isosceles triangle whose base measures 4 inches, equal sides 2 inches?

Try to construct a scalene triangle with sides measuring 1, 2, and 3 inches, respectively.

13. Draw a circle. In it draw a chord.

Bisect the chord, using as few lines and as short ones as you can.

NOTE. — Do not use the ruler to ascertain the length of the chord before bisecting it.

14. Divide a sector into two equal parts.

15. Draw a circle. Draw a chord. Draw a radius through the center of the chord.

Is the radius perpendicular to the chord? Why?

16. Bisect the arc of a circle and its chord.

Bisect the arc of a circle without drawing the chord.

17. Draw a perpendicular to the middle point of a horizontal line. To the middle point of a vertical line. To the middle point of an oblique line.

18. Draw in a circle two diameters perpendicular to each other.

19. Divide the circumference of a circle into four equal parts. Into eight equal parts.

Inscribe a square in a circle.

20. Inscribe a regular octagon in a circle.

21. Connect the opposite vertices of a regular octagon inscribed in a circle by lines passing through the center of the circle.

Lines connecting the opposite vertices of a polygon are called *diagonals*.

22. Inscribe a square in a circle. Circumscribe a square whose sides shall be perpendicular to the diagonals of the inscribed square.

23. Construct an equilateral triangle on a horizontal line 1 inch long. On the right side as a base, construct a second equilateral triangle. On the left side of the first triangle, construct a third. Construct three more, completing the hexagon.

24. Can you circumscribe a circle about the above hexagon? What is the radius of the circle?

25. Inscribe a regular hexagon in a circle whose radius is $1\frac{1}{2}$ inch. What is the length of each side of the hexagon?

26. Inscribe in a circle an equilateral triangle. On each of its three sides construct an equilateral triangle.

27. Construct an arc of 60°. Draw two lines meeting at an angle of 60°.

28. Bisect an arc of 60°. Draw two lines meeting at an angle of 30°.

29. Construct an angle of 60° and an angle of 30°. Draw two lines making an angle equal to the sum of the two angles first constructed.

30. Erect a perpendicular at the end of a horizontal line. At the end of a vertical line. At the end of an oblique line.

31. Construct an angle of 45°. An angle of $22\frac{1}{2}$°. An angle of 135°. An angle of 15°. An angle of 75°.

32. Draw a circle, radius 1 inch. Draw a diameter, and produce it an inch beyond the circumference. At the center of the circle erect a perpendicular to the diameter.

33. An inch from one end of a 3-inch line, erect a perpendicular, using as few and as short lines as possible.

34. Erect a perpendicular at the center of an oblique line. Erect a perpendicular at one end of an oblique line. Erect a

perpendicular at a point in a line between one end and the center.

35. Construct a right-angled triangle, base 2 inches, altitude 2 inches.

A right-angled triangle, base 3 inches, altitude $2\frac{1}{2}$ inches.

36. On a line 2 inches long, construct a square. Construct a rectangle, base 3 inches, altitude 2 inches.

37. On an oblique line 3 inches long, construct a square. On an oblique line 3 inches long, construct a rectangle, altitude $2\frac{1}{2}$ inches.

38. Construct a rhombus whose side is 3 inches, altitude $2\frac{1}{2}$ inches.

A rhombus whose side is 3 inches, and which contains an angle of 60°.

39. Construct a rhomboid whose adjacent sides measure 4 inches and 3 inches, respectively, altitude $2\frac{1}{2}$ inches.

40. Construct a right-angled isosceles triangle. An isosceles triangle containing an angle of 120°. One containing an angle of 135°.

41. Construct an isosceles triangle, base 3 inches, altitude 3 inches. Can you construct an equilateral triangle whose altitude shall be 3 inches?

42. Construct a scalene triangle, base 3 inches, altitude 3 inches. An obtuse-angled triangle having a 3-inch base, and the altitude 3 inches.

43. Construct a triangle whose base measures 5 inches, the other sides being 3 inches and 4 inches, respectively. Draw its altitude geometrically.

44. Construct a triangle whose base measures 4 inches, the angles at the base measuring 120° and 30°, respectively. Draw its altitude geometrically.

45. Draw a line. From a point above the line, let fall a perpendicular to the line.

46. Inscribe a regular hexagon in a circle. Divide it into six equilateral triangles by diagonals.

How many degrees are there in each of the six angles at the center of the circle? How many degrees are there in the arc on which each angle at the center stands?

47. Inscribe an equilateral triangle in a circle. Mark in each angle the number of degrees it contains. Mark on each arc the number of degrees it contains.

An angle at the circumference of a circle is measured by what part of the arc on which it stands?

48. Do you know how many degrees there are in each angle of a regular hexagon?

Construct a regular hexagon without drawing a circle and without using triangles.

49. Construct a regular octagon, each side one inch. Do not draw a circle.

50. If an angle at the circumference is measured by one-half of the arc on which it stands, how many degrees are contained between two lines forming a right angle whose vertex is at the circumference?

51. Inscribe several right angles in a circle, their sides intercepting the same arc.

52. Inscribe in a circle a right-angled isosceles triangle.

53. Construct a square whose diagonal measures 4 inches.

A rhombus whose side is 3 inches and which has one diagonal 3 inches long.

54. Inscribe in a circle a scalene triangle. How many degrees are there in the sum of the arcs intercepted by the sides forming the three angles?

How many degrees are there in the sum of the three angles?

55. Draw two equilateral triangles having one side common to both triangles. What figure have you drawn?

56. Construct two triangles, each having two sides measuring 2 inches and $2\frac{1}{2}$ inches, respectively, and the angle made by these sides (the included angle) measuring 60°.

Cut out the triangles. Place one upon the other, and ascertain if the third side of one triangle is equal to the third side of the other. Are the two remaining angles of the first triangle equal to the corresponding angles of the second?

57. Construct a triangle having a base of 3 inches, and angles at the base containing 60° and 45°, respectively. Construct a second triangle having angles of 60° and 45°, respectively, and the included side 3 inches.

Cut both out of paper, and test the equality of the corresponding parts of each.

58. Construct a triangle whose angles measure 30°, 60°, and 90°, respectively. Can you construct another triangle having angles equal to those in the first triangle, but with sides not equal to those of the first triangle?

59. Construct a triangle having sides of 2, $2\frac{1}{2}$, and 3 inches, respectively.

Try to construct a second triangle having sides equal to those of the foregoing triangle, but having the corresponding angles unequal.

60. Can you construct a triangle containing three angles of 75° each?

61. In a circle, inscribe a right-angled triangle whose hypotenuse is 3 inches.

One whose hypotenuse is 3 inches, base 2 inches.

Hypotenuse 3 inches, perpendicular $1\frac{1}{2}$ inches.

What is the diameter of the circle in each case?

62. Draw two equal circles. Mark off an arc in one. Mark off an equal arc in the other.

63. Draw two lines meeting at any angle. Construct a second angle equal to the first.

64. Draw a horizontal line. From each end, and from two points between, draw four oblique lines that shall be parallel to each other.

From one end of a three-inch horizontal line, draw an oblique line running upwards; at the other end, draw a line running downwards that shall be parallel to the first oblique line.

65. On a three-inch line as a diagonal, construct a rhomboid having two opposite sides each $2\frac{1}{2}$ inches long. Mark off on each of these two sides half-inch divisions. Draw lines through the diagonal, dividing the rhomboid into 5 equal parts. How do these lines divide the diagonal?

66. Divide a 3-inch line into 5 equal parts.

Draw a line exactly $\frac{3}{5}$ inch in length. One exactly $\frac{4}{7}$ inch.

67. Draw a right-angled triangle whose base is 4 inches, perpendicular 3 inches. How long is the hypotenuse?

One whose base is 3 inches, perpendicular 2 inches. How long is the hypotenuse?

One whose hypotenuse is 4 inches, base 3 inches. How long is the perpendicular?

68. Can you draw a line measuring exactly $\sqrt{13}$ inches? One measuring exactly $\sqrt{7}$ inches? One measuring exactly $\sqrt{5}$ inches?

69. Divide a 2-inch equilateral triangle into two right-angled triangles. Mark the number of degrees in each of the six angles.

In each right-angled triangle, find the ratio between the length of a side opposite an angle of 30° and the length of a side opposite an angle of 90°.

70. Construct a right-angled triangle, hypotenuse 4 inches, angle at base 30°. Measure the length of the perpendicular.

Construct a right-angled triangle whose perpendicular is $1\frac{1}{2}$

inches, the angle formed by the perpendicular and the hypotenuse being 60°. How long is the hypotenuse?

71. Draw in a circle whose radius is two inches, the chord of an arc of 60°. Draw the chord of an arc of 180°. What is the ratio between their respective lengths?

What is the ratio between the chord of an arc of 60° and the chord of an arc of 300°?

72. In a circle of 2 inches radius, draw a chord. At one extremity erect a perpendicular. From the other extremity draw a diameter. Where will this diameter meet the perpendicular? Why?

73. Make a triangle of splints, fastening them by one tack at each angle. Will the triangle retain its shape under pressure?

Make a rectangle of splints. Fasten each corner by a single tack. Will the rectangle retain its shape under pressure?

74. In a circle draw three parallel chords the same distance apart.

Draw three parallel chords not equidistant.

75. At the circumference of a circle, draw a line perpendicular to a radius. This perpendicular is tangent to the circumference.

A line that touches the circumference in a single point is called a *tangent*. A line that cuts the circumference is called a *secant*.

76. Inscribe a circle in a square.

77. In a circle draw two chords not parallel. Bisect each by a perpendicular. Where will these perpendiculars intersect?

78. Inscribe a circle in an equilateral triangle. In an isosceles triangle. In a scalene triangle.

79. Circumscribe a circle about an equilateral triangle. About an isosceles triangle. About a scalene triangle.

80. Draw a circle, using a coin or something similar. Can you find the center?

81. Draw an obtuse-angled triangle. Circumscribe a circle about it.

82. Without using the compasses, draw an arc. Can you find the center of the circle of which the arc forms a part?

83. Construct a rhombus whose sides are 4 inches, the acute angle being 30°. What is its altitude?

84. Construct a square on a line 3 inches long, using only arcs of 3 inches radius.

85. Can you construct a triangle with sides 2, 4, and 6 inches?

86. Draw a circle, radius 3 inches. Draw an equilateral triangle around it.

87. Construct an equilateral triangle, altitude $1\frac{1}{2}$ inches. Construct one having an altitude twice as great.

88. Construct a triangle whose sides measure 1, $1\frac{1}{2}$, and 2 inches, respectively. Construct a second having its corresponding sides twice as large as those of the first triangle.

Construct a triangle whose angles measure 30°, 60°, and 90°, respectively. Try to construct a second triangle having its corresponding angles twice as large as those of the first triangle.

89. Construct a square which shall be equal in area to two squares, one having a side of 2 inches, the other having a side of 3 inches.

90. Can you construct a square whose area will be 13 square inches?

91. Construct a square whose area will be equal to the difference of area between two squares, one having a side of 3 inches, the other having a side of 2 inches.

92. Construct a square whose side is 3 inches. Construct another having double the area.

93. Construct an equilateral triangle, side 2 inches. How many 1-inch equilateral triangles can be made from it?

94. How many 1-inch equilateral triangles can be made from an equilateral triangle whose sides are 3 inches?

From one whose sides are 4 inches? 5 inches?

95. Construct a triangle, sides 2, 3, 4 inches. Divide it into four equal triangles. Give the dimensions of each of the latter.

96. Construct a triangle 1, $1\frac{1}{2}$, 2 inches. Make a triangle nine times as large as the first by producing two of the sides and drawing a fourth line. What are the dimensions of the second triangle?

97. Draw a circle of 1 inch radius. Tangent to it, and enclosing it, draw one having four times the area of the first.

98. Can you draw a circle having half the area of a circle whose radius is 2 inches?

99. Draw an equilateral triangle, side 2 inches. Construct another of half the area of the first.

100. Can you tell the ratio between the area of an inscribed and that of a circumscribed hexagon?

EQUAL TRIANGLES. EQUIVALENT TRIANGLES.

1271. NOTE. — The protractor and the triangle may be used in the following exercises.

1. Draw a rectangle, base $2\frac{1}{2}$ inches, altitude 2 inches. Draw a rhomboid, base $2\frac{1}{2}$ inches, altitude 2 inches. Find the area of each.

2. With a base $2\frac{1}{2}$ inches, altitude 2 inches, draw

(*a*) A right-angled triangle.

(*b*) An isosceles triangle.

(*c*) One or more acute-angled scalene triangles.

(*d*) One or more obtuse-angled triangles.

Calculate the area of each.

3. Can you show, by cutting from paper, that a right-angled triangle having its base and perpendicular 4 inches and 3 inches, respectively, has the same surface as an acute-angled triangle whose base and altitude are 4 inches and 3 inches, respectively, and an obtuse-angled triangle whose base and altitude are 4 inches and 3 inches, respectively?

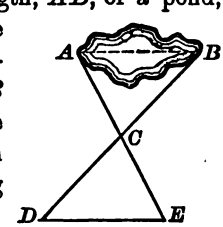
Two triangles that have the same area are called *equivalent* triangles; those having their corresponding sides and angles equal, each to each, are called *equal* triangles.

4. Construct a triangle whose sides measure $1\frac{1}{2}$, 2, and $2\frac{1}{2}$ inches, respectively. Construct another triangle having its sides of the same lengths. Are the angles of the second equal to the angles of the first? Are the triangles equal?

5. Draw two triangles each of which has two sides measuring $1\frac{1}{2}$ and 3 inches, respectively, and the included angle 60 degrees. Is the third side of one triangle equal to the third side of the other? Are the remaining angles of the first triangle equal to the remaining angles of the second?

6. With a base 2 inches long, and with angles at the base measuring 50° and 60°, respectively, construct a triangle. Construct a second triangle having a base measuring 2 inches, and angles at the base measuring 60° and 50°, respectively. Are the two triangles equal?

7. A person wishing to ascertain the length, AB, of a pond, places a pole at a convenient point, C, visible from A and B. The distance BC is measured, and a pole is set up, on a line with B and C, at D, the distance CD being made equal to BC. A pole is also placed at E, on a line with A and C, the distance CE being made equal to AC.

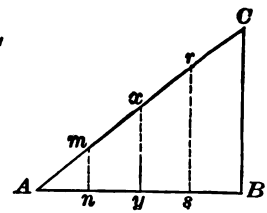


Can you show that the length, AB, of the pond can be ascertained by measuring the distance DE?

SIMILAR TRIANGLES.

1272. Construct a right-angled triangle ABC. Make AB 4 inches, AC 5 inches, BC 3 inches.

At the points n, y, and s, distant from A one, two, and three inches, respectively, erect perpendiculars. Measure the length of each.



8. An is one-fourth of AB; ascertain the ratio between mn and CB.

Compare xy and CB; see if $\frac{CB}{xy} = \frac{AB}{Ay} = 2$.

As is three-fourths of AB; is $rs = \frac{3}{4}BC$?

Measure Am, Ax, and Ar. How does each compare with AC?

Would the relations as found above, exist if mn were not perpendicular to AB?

How do the angles in the triangle Amn compare with the angles in the triangle ABC?

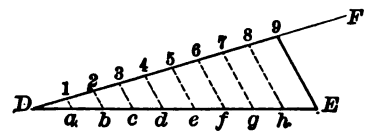
9. Draw a triangle whose sides measure 4, 5, and 6 inches, respectively. Cut out of paper a triangle whose sides measure 2, $2\frac{1}{2}$, and 3 inches, respectively. Place an angle of the small triangle on the corresponding angle of the large triangle, and compare their respective sizes.

How does the area of the small triangle compare with the area of the large triangle?

10. Two angles of a triangle measure about 37° and 53°, respectively. The sides opposite those angles measure 3 inches and 4 inches, respectively. How many degrees does the third angle contain? Calculate the length of the third side.

What will be the dimensions of a similar triangle whose area is one-fourth that of the given triangle? Give the approximate number of degrees in each angle of the small triangle.

11. Draw a line DE, $2\frac{7}{8}$ inches long. At any angle draw DF. Commencing at D, mark off on DF quarter-inch portions, 1, 2, 3, etc., to 9. Join 9 E, and with a triangle and a ruler draw 8 h, 7 g, 6 f, etc., parallel to 9 E. Measure Eh. Is it equal to hg, gf, fe, etc.? Why?



Into how many parts is the line ED divided? What fraction of an inch does each part contain?

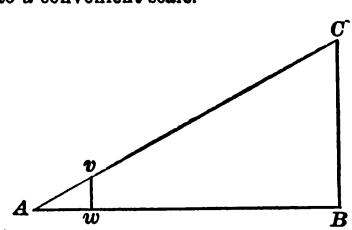
In locating the points 1, 2, 3, 4, etc., is it necessary to make the divisions $\frac{1}{4}$ inch? Would it be sufficient to use the compasses with the points any convenient distance apart?

12. Divide a line $2\frac{7}{8}$ inches long into 5 equal parts.

CALCULATING HEIGHTS AND DISTANCES.

1273. To verify the results obtained by calculation, the pupil should make diagrams, drawing the figures to a convenient scale.

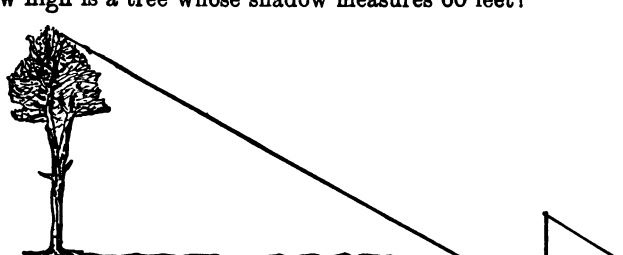
1. If AB in a right-angled triangle measures 120 feet, and a perpendicular, vw, erected 10 feet from A measures 5 feet, calculate the length of BC.



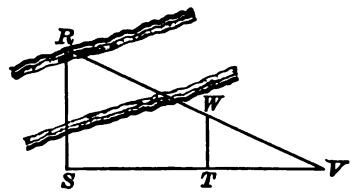
$Aw : AB :: wv : BC$;

i.e. $10 : 120 :: 5 : BC$.

2. A post 6 feet above ground throws a shadow of $7\frac{1}{2}$ feet. How high is a tree whose shadow measures 60 feet?

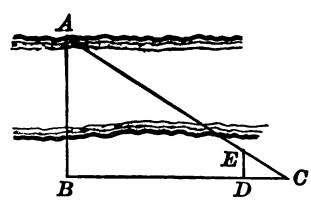


3. Wishing to ascertain the distance between two houses, R and S, on opposite sides of a stream, I measure a line SV, at right angles to SR, 200 feet. At T, 90 feet from S, the perpendicular TW measures 60 feet. Required the distance SR.



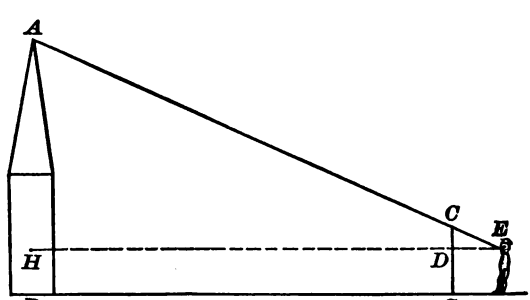
$VT : TW :: VS : SR$ $\quad VT = VS - ST.$

4. Beginning at B, 100 feet from the bank of a river, a line, BC, is measured 1,200 feet long. At D, distant from C 50 feet, the perpendicular DE is found to measure 90 feet. What is the distance from B to A, a tree on the opposite bank? How wide is the river?

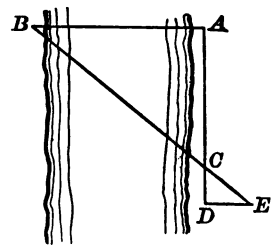


5. A boy, whose eye (E) is 4 feet from the ground, can just see the top (A) of a steeple when he stands back 3 feet from a fence (CG) 6 feet high. The distance from the foot of the fence to the center of the base of the steeple is 177 feet. Find the height of the steeple AB.

$CD = ?$ $\quad EH = ?$ $\quad ED : CD :: EH : AH$

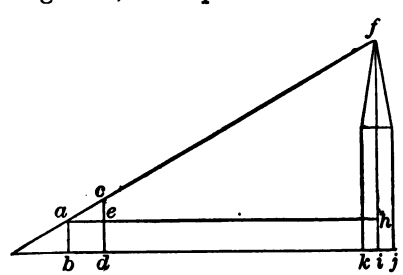


6. Wishing to ascertain the distance AB, I measure a line, AD, at right angles to AB, 12 chains; DE, at right angles to AD, 5 chains; and find that a line sighted from E to B intersects AD at C, distant from D 3.25 chains. What is the distance from A to B?



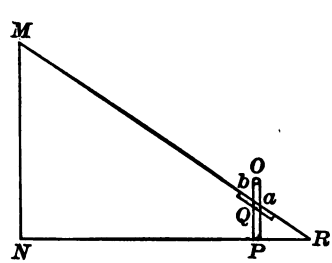
NOTE. — The triangles DCE and ACB are similar. Why?

7. Wishing to find the height of a tower fi, I set up a pole, cd, 12 feet long above the ground. Another pole ab, $4\frac{1}{2}$ feet above ground, is set up at such a distance that the tops of the two poles and of the tower are in a line. The distance between the poles (ae or db) is $10\frac{1}{2}$ feet. The distance from d to the foot of the tower is 195 feet. The width of the tower (kj) is 30 feet.

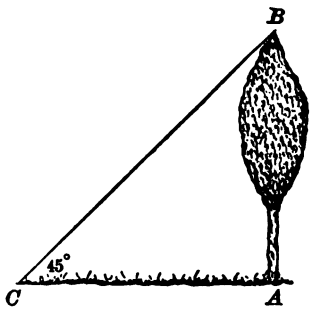


The similar triangles aec and ahf give us the proportion $ae : ah :: ec : hf$. What is the distance ec? $ah = bi = bd + dk + ki$. $ki = \frac{1}{2}\ kj$. When fh is found, what must be added to get the height of the tower?

8. To determine the height of a building, MN, a person attached a strip of wood, ab (a tin tube or a piece of narrow pipe would be better), to a post, OP, in such a manner that sighting from a, he could just see M, the top of the building. He then sighted down from b, and marked on the ground the point R, on a line with ab.



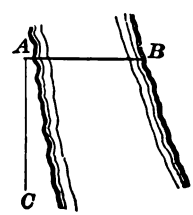
PQ was found by measurement to be 4 feet, RP 6 feet, PN 120 feet. Required MN.



9. Wood-choppers, desiring to know the height of a tree before cutting it, sometimes make an isosceles right-angled triangle of wood or paper, and "step off" the distance on level ground from the point at which they find they can just see the top of the tree looking along the hypotenuse of the triangle, the base being parallel to the ground.

How high is the tree AB, if AC is 36 paces of 3 feet each, and the angle ACB is 45°?

10. B is a point on the bank of a stream due east of A on the other bank. A boy walks due south of A until he reaches a point at which he finds, from his pocket compass, that he is directly south-west of B. If the distance AC measures 119 yards, how wide is the stream?

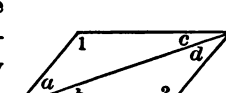


1274. Miscellaneous Exercises.

1. Calculate the length in inches of an arc of 60°, the radius of the circle being two inches. Calculate the length of an arc of 120°. Of 180°. Of 240°. Of 300°.

2. Calculate the length in inches of a chord of 60° in the above circle. Of a chord of 120°. Of a chord of 180°. Of a chord of 240°. Of a chord of 300°.

3. In the parallelogram shown in the accompanying diagram, the angle a measures 40°, and the angle b 35°. How many degrees does the angle c contain? The angle d? The angle 1? The angle 2?



4. Inscribe a regular nonagon in a circle of 2 inches radius, using the protractor.

5. How many degrees does each angle of a regular nonagon contain? Draw a regular nonagon, each side measuring two inches. (Use the protractor.)

6. The distance around a polygon is called its *perimeter.* What is the perimeter of a regular hexagon, inscribed in a circle whose radius is 1 inch?

What is the circumference of the circle?

7. The distance from the center of a regular polygon to the middle point of one side is called the *apothem.*

Draw the apothem of a regular hexagon inscribed in a circle of 1 inch radius. About how long is it?

8. Cut a regular hexagon, side one inch, into six triangles. Place three in a line, and fit in the other three so as to make a rectangle. (Divide one of the triangles into two equal parts.)

How long is the base of the rectangle? What part of the perimeter? About how long is the perpendicular of the rectangle?

9. In a circle, radius 1 inch, inscribe a regular octagon. Divide it into eight triangles, and make out of them a rectangle.

About what is the half perimeter of the octagon? Its apothem? Its area?

Which has the greater perimeter, apothem, area, the hexagon or the octagon?

10. Find the approximate area of a regular hexagon, side 1 inch, apothem about $\frac{7}{8}$ inch.

11. Find the approximate area of a regular octagon, side about $\frac{3}{4}$ inch, apothem about $\frac{15}{16}$ inch.

12. If we inscribe in a circle a regular polygon of 16 sides, will its perimeter be greater or less than that of the octagon? Which polygon will have the greater apothem?

13. If we inscribe polygons of 32, 64, 128, etc., sides, what will be the greatest perimeter we can have in a circle of 1 inch radius? What will be the greatest apothem?

14. Draw a rectangle that will be about equal to a polygon of a million sides inscribed in a circle whose radius is 1 inch.

Mark upon it the dimensions. Calculate the area.

15. What is the area of a circle whose radius is 2 inches?

16. Find the area of a circle whose diameter is 10 inches.

17. Find the area of a circle whose circumference is 6.2832 inches.

18. Calculate the area of a sector of a circle whose radius is 10 inches, the arc of the sector being 60°.

19. How many square inches are there between the circumferences of two concentric circles whose radii measure 3 and 6 inches, respectively?

SURFACES OF SOLIDS.

1275. Prisms, Cylinders, Pyramids, Cones.

NOTE. — The pupils should first examine a number of prisms and pyramids, right and oblique, regular and irregular, triangular, quadrangular, pentagonal, etc. Right and oblique cylinders and cones should also be at hand.

A **prism** is a body bounded by plane faces, two of which are equal and parallel polygons, the remaining faces being parallelograms.

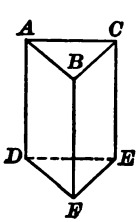


FIG. 1.

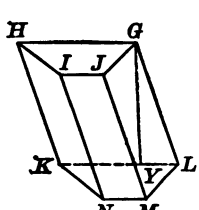


FIG. 2.

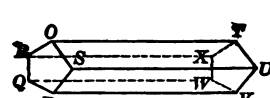


FIG. 3.

The two parallel faces of a prism are called its *bases*. The remaining faces taken together constitute its *convex surface*.

In Fig. 1, ABC and DEF are the bases; in Fig. 2, the bases are $GHIJ$ and $KLMN$; in Fig. 3, $OPQRS$ and $TUVWX$.

The sides AB, CE, etc., GH, IN, etc., QR, OT, etc., are called *edges*.

1276. Prisms may be either *right* or *oblique*. The convex surface of a right prism consists of rectangles.

Fig. 1 is a right prism; Fig. 2 is an oblique prism.

NOTE.—When a prism is spoken of, a right prism is meant unless the word *oblique* is used.

The *altitude* of a prism is the perpendicular distance between the bases.

AD, BF, or CE is the altitude in Fig. 1. GY is the altitude in Fig. 2.

1277. The number of sides in each base determines the name, as *triangular* (Fig. 1), *quadrangular* (Fig. 2), *pentagonal* (Fig 3), etc.

A quadrangular prism whose bases are parallelograms is called a *parallelopipedon*. Fig. 4 is an oblique parallelopipedon. Fig. 5 is a right parallelopipedon. Any two opposite faces of a parallelopipedon may be considered the bases.

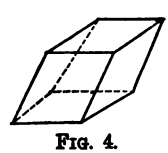


FIG. 4.

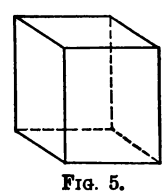


FIG. 5.

1278. When the bases are regular polygons, the prism is said to be *regular*.

Fig. 1 is a right regular triangular prism; Fig. 2 is an oblique irregular quadrangular prism.

1279. A cylinder is a body having two circular parallel plane faces, and one curved face. The plane faces are the bases. The curved face constitutes the convex surface.

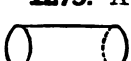


FIG. 6.

Cylinders, like prisms, are either *right* or *oblique*. The altitude of a cylinder is the perpendicular distance between the bases.

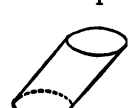


FIG. 7.

FIG. 8.

1280. A **pyramid** is a body whose convex surface is made up of triangles having a common vertex, the base of the pyramid being a polygon.

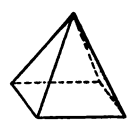


FIG. 9.

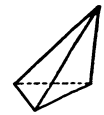


FIG. 10.

Pyramids are either *right* or *oblique; regular* or *irregular; triangular, quadrangular, pentagonal*, etc.

In a right pyramid, each of the triangles that make up the convex surface, is isosceles. When, in addition, the pyramid is a regular one, these triangles will be equal to each other.

The altitude of any of these equal triangles constitutes the *slant height* of a right regular pyramid. The *altitude* of a pyramid is measured by a line drawn from the apex to the center of the base.

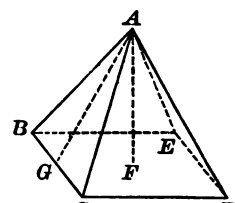


FIG. 11.

AG is the slant height of the square pyramid, Fig. 11. *AF* is its altitude.

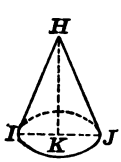


FIG. 12.

1281. The *cone* has a single circular base; its convex surface is curved, sloping to the apex.

In the right cone, Fig. 12, *HI* is the slant height, and *HK* is the altitude. *LO* is the altitude of the oblique cone, Fig. 13.

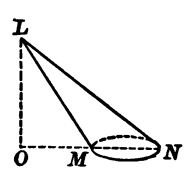


FIG. 13.

1282. Surfaces of Prism, Cylinder, Pyramid, Cone.

1. Draw the developed convex surface of a square prism, height 3 inches, one side of base 1 inch.

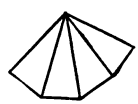
2. Draw the developed (entire) surface of a triangular prism, height 3 inches, each side of base 2 inches.

3. Draw the developed convex surface of a prism 3 inches high, each base being a triangle having sides of 1, 1½, and 2 inches, respectively.

4. Find the convex surface of each prism, and show that the convex surface is found by multiplying the perimeter of the base by the altitude.

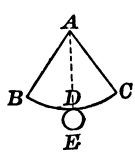
5. Show that the convex surface of a cylinder is found by multiplying the circumference of the base by the altitude.

6. Draw the developed convex surface of a square pyramid, slant height 3 inches, each side of base 2 inches. Cut it out of paper, leaving on one edge a small strip for gumming. Fold into a hollow pyramid, and measure its altitude.



7. Show that the convex surface of a regular pyramid is found by multiplying the perimeter of the base by one-half the slant height.

8. Make out of paper a hollow square pyramid whose altitude shall be 3 inches, each side of base 2 inches.



1283. The developed convex surface of a cone is a sector, *ABDC*.

How many inches does the arc *BDC* measure when the diameter of the base of the cone is 2 inches?

The slant height of the cone is the radius of the circle of which the sector is a part. *AB*, *AD*, or *AC* is the slant height.

9. Cut out of paper a semicircle, radius 3 inches, adding a narrow strip for gumming, and fold into a cone.

What is the slant height of the cone? The diameter of the base? The radius of the base? The circumference of the base?

What is the ratio between the radius of the base and the slant height? Between the diameter of the base and the slant height?

10. Calculate the convex surface of the above cone, and show that it is equal to the circumference of the base multiplied by one-half the slant height.

11. Find the diameter of the base of a cone made by folding a paper sector of 90 degrees (quadrant), the radius of the sector being 3 inches. What is its slant height?

When a sector of 60 degrees is used, what will be the diameter and the slant height, the radius of the sector being 3 inches?

(Make no allowance for overlapping.)

12. If you wish to make a hollow paper cone whose slant height shall be 5 inches, and the diameter of whose base shall be 3 inches, how many degrees should the arc of the sector contain?

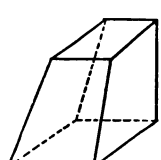
13. Draw the development of a right pyramid 4 inches in altitude, whose base is a rectangle 3 inches by 2 inches. Is the altitude (slant height) of each of the four triangles the same?

14. Calculate the slant height of each convex face of a rectangular pyramid whose altitude is 12 inches, and whose base measures 10 inches by 18 inches.

Find the convex surface.

1284. Surface of Frustum of Pyramid and Cone.

When the upper part of a pyramid or of a cone is cut off by a plane parallel to the base, the remaining part is called a frustum.



15. Draw one face of the frustum of a square pyramid. Of the frustum of a triangular pyramid. What figure have you drawn?

Calculate its surface when the length of the upper side is 4 inches, that of the lower side is 8 inches, and the slant height of the frustum is 10 inches.

16. Draw the developed convex surface of the frustum of a regular triangular pyramid, each side of the upper base measuring 1 inch, of the lower base 2 inches, the slant height being 2 inches.

SUGGESTION. — Locate the apex of the whole pyramid of which the given frustum forms a part.

17. Find the convex surface of the frustum of a square pyramid, one side of the upper base measuring 2 feet, of the lower base 3 feet, and having a slant height of 4 feet.

Find the entire surface.

18. Show that the convex surface of the frustum of a pyramid is equal to one-half the sum of the perimeters of the upper and the lower bases multiplied by the slant height.

19. Draw the pattern of a small shade for a candle. Make the upper opening $1\frac{1}{2}$ inches in diameter, the lower one $2\frac{1}{2}$ inches in diameter, and the slant height 2 inches.

20. How many square inches of tin will be required to make a pan, its upper base being 9 inches in diameter, the lower base 6 inches in diameter, and the slant height 4 inches?

(Do not forget the bottom of the pan.)

1285. The frustum of a cone may be considered the frustum of a pyramid whose bases contain a very great number of sides. The convex surface of the frustum of a cone may, therefore, be found by multiplying the half sum of the circumference of the two bases by the slant height.

21. Find the convex surface of a frustum of a cone, the circumferences of the bases being 15 inches and 20 inches, respectively, and the slant height 10 inches.

22. How many square yards are there in the entire surface of a frustum of a cone, the radius of the upper base (r) being 3 yards, of the lower base (R) 5 yards, and the slant height 6 yards?

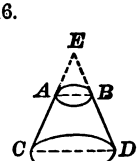
Circumference of upper base $= 2\pi r$; of lower base $= 2\pi R$.

Convex surface $= (2\pi r + 2\pi R) \times \frac{\text{slant height}}{2} = \pi(r + R) \times$ slant height.

Surface of upper base $= \pi r^2$; of lower base $= \pi R^2$.

Entire surface $= \pi \times$ what? Multiply only once by 3.1416.

23. The diameter, AB, of the upper base of the frustum of a cone measures 6 feet, CD measures 8 feet, the slant height AC measures 9 feet. Find the slant height EA of the part cut from the cone in making the frustum.



Let $EA = x$; $EC = x + 9$; $AB = 6$; $CD = 8$.

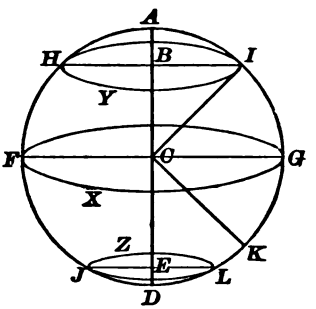
The triangles EAB and ECD are similar.

24. Find the convex surface of the whole cone, ECD, and the convex surface of the part cut off, EAB.

1286. The Sphere.

A *sphere* is a body all points on whose surface are equally distant from the center.

The distance from the center to the surface is called the *radius* of the sphere. The diameter is a line running between two points on the surface and passing through the center.



CG, CK, CD, CF, CA, and CI are radii; AD and FG are diameters.

1287. If a sphere be cut through at any part, the cut surface will be a circle. When the cutting plane passes through the center of the sphere, the circle is called a *great circle;* other circles are called *small circles.*

$FXGC$ is a great circle; $HYIB$ and $JLEZ$ are small circles.

25. Find the length of an arc of 60° of a great circle of a sphere whose circumference is 25,000 miles.

26. Calling the arc AI in the preceding figure, 30°, the angle BCI will measure 30°. Calculate the radius BI of the small circle when the radius CI of the large circle is 4,000 miles. (IAH = arc of 60°; IH = chord of 60°.)

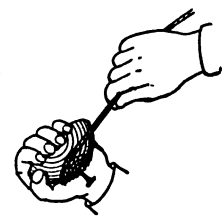
27. If I is 60° from G, a point on the equator, find the length of the circumference of the small circle HYI, assuming the circumference of a great circle to be 25,000 miles.

28. What is the ratio between the length of a degree on the small circle HYI, and the length of a degree of a great circle?

29. Calculate the radius of a small circle formed by passing a plane parallel to $GCXF$ through a point on GA 45 degrees from G.

1288. Surface of Sphere.

We have seen (Art. 1151) that it may be experimentally shown that the surface of a sphere is equal to the surface of four of its great circles.



Calling the radius of the sphere R, its surface is $4\pi R^2$.

30. Find the surface of a sphere whose diameter is 6 inches.

31. How many square inches are there in the convex surface of a hemisphere whose radius is 3 inches?

What is the area of the great circle that forms the base of the hemisphere?

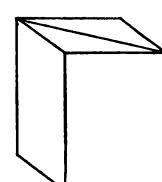
Find the entire surface of the hemisphere.

32. Is there any difference between the convex surface of a sphere and its entire surface? Why?

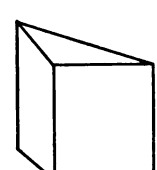
VOLUMES.

1289. Prisms and Cylinders.

1. How many one-inch cubes will cover the base of a box 4 inches by 3 inches? If the box is 2 inches deep, how many one-inch cubes will it contain? How many cubic inches are there in the volume of a right prism whose base is a rectangle measuring 4 inches by 3 inches, and whose altitude is 6 inches?



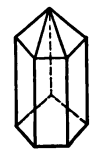
2. If the above hollow prism were divided into two equal parts by a thin partition extending from a vertical edge to one diagonally opposite, how many cubic inches of sand would each part contain?



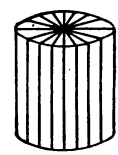
3. How many cubic inches are there in the volume of a prism whose base is a right-angled triangle 3 by 4 by 5 inches, and whose altitude is 6 inches?

4. Find the volume of a triangular prism, the area of the base being 6 square inches, and the altitude 6 inches.

Find the volume of a triangular prism, each side of whose base measures 6 inches, its altitude being 8 inches.



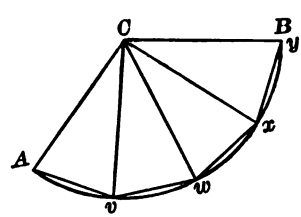
5. What are the solid contents of a pentagonal prism formed by fastening together three triangular prisms whose bases contain, respectively, 12, 16, and 18 square inches, the altitude of each being 15 inches?



6. If a very great number of triangular prisms of the same height are united so as to form a cylinder whose base contains 12.5664 square inches, and whose altitude measures 5 inches, what are the solid contents?

1290. Pyramids and Cones.

With a center at C, and a convenient radius, describe an arc AB. Mark off four equal portions v, w, x, and y; and draw the equal chords. Cutting out $CAvwxy$, with an additional narrow strip along Cy for gumming, and creasing along the lines Cv, Cw, Cx, and Cy, we can fold the paper into a square pyramid.



Measure its altitude, and make a square prism of equal altitude and with an equal base.

Filling the pyramid with sand, and pouring the sand into the prism, it will be found that the latter will contain the contents of the former three times; that is, the volume of a square pyramid is one-third that of a square prism having an equal base and an equal altitude.

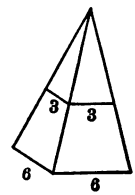
The same ratio will be found true in the case of a triangular, or any other pyramid, as compared with the corresponding prism, and of the cone as compared with a cylinder.

1291. The volume of a pyramid or of a cone is equal, therefore, to the area of the base multiplied by one-third of the altitude.

1292. Frustums of Pyramids and Cones.

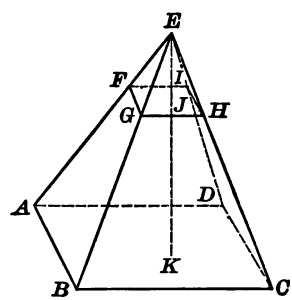
7. Find the volume of a square pyramid whose altitude is 12 inches, one side of the base measuring 6 inches.

Find the volume of a square pyramid whose altitude is 6 inches, one side of the base measuring 3 inches.



8. Find the volume of the frustum of a square pyramid whose altitude is 6 inches, one side of the upper base 3 inches, and one side of the lower base 6 inches.

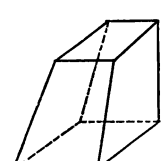
9. A square pyramid whose altitude measures 18 inches, and each side of whose base measures 15 inches, is divided into two parts by a plane, $FGHI$, parallel to the base, the distance, EJ, of the plane from the vertex, E, being 6 inches.



The ratio between the edge, EB, of the whole pyramid and the edge, EG, of the part cut off will be equal to that between EJ and EK; that is, $6 : 18 = 1 : 3$. The same will be the ratio between BC and GH, and the latter will be one-third of 15 inches long, or 5 inches.

Find the volume of the large pyramid and that of the small pyramid.

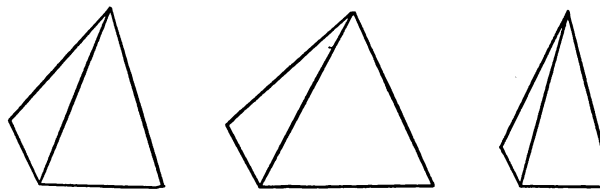
10. Each side of the upper base of the frustum of a square pyramid measures 5 inches; each side of the lower base measures 15 inches; the perpendicular distance between them measures 12 inches. Find the solid contents.



Find the convex surface of the above frustum. Find its entire surface.

NOTE. — What is the slant height?

11. Find the total volume of three square pyramids, the altitude of each being 12 inches, and the areas of their bases being 25 sq. in., 225 sq. in., and 75 sq. in., respectively.



12. Find the number of cubic feet in a block of stone whose shape is that of a frustum of a square pyramid 4 feet high, each side of the upper base measuring 3 feet, and each side of the lower base 5 feet.

1293. The volume of the frustum of a pyramid is equal to the sum of the volumes of three pyramids, each having an altitude equal to that of the frustum; the base of one of them being equal in area to that of the lower base of the frustum, the base of a second being equal in area to that of the upper base of the frustum, and the base of a third being a mean proportional between the area of the other two.

Base of first $= 3 \times 3$ sq. ft.; of second, 5×5 sq. ft.; of third, $\sqrt{9 \times 25}$ sq. ft. $= 15$ sq. ft.

NOTE. — The mean proportional between two numbers is equal to the square root of their product.

13. Find the volume of the frustum of a square pyramid, its upper base containing 64 square inches, and its lower base 196 square inches, its altitude being 18 inches.

1294. Note that the mean proportional between 64 and 196 is 8×14, or 112. Since each is multiplied by one-third of the altitude, the operation is shortened by adding together the three areas, 64, 196, and 112, and multiplying their sum by one-third of 18.

Calling the altitude A, the side of the large square S, of small square s, the volume V, we have

$$V = \frac{A(S^2 + s^2 + Ss)}{3}.$$

1295. The volume of the frustum of a cone is found in the same way as that of the frustum of a pyramid.

$\frac{1}{3}$ altitude (area upper base + area lower base + area mean proportional).

Calling the radius of the upper base r, and that of the lower base R, the area of the upper base will be πr^2, of the lower base πR^2, of mean proportional πrR.

$$V = \tfrac{1}{3} A(\pi r^2 + \pi R^2 + \pi rR)$$

Since π (or 3.1416) is a common factor, we can save time by first adding r^2, R^2, and rR, and then multiplying by π.

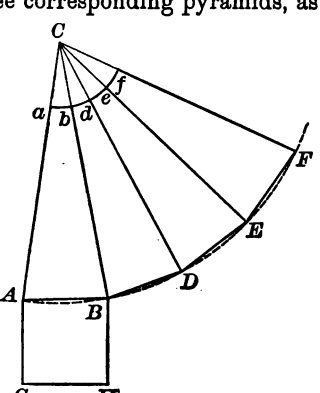
$$V = \frac{A\pi(r^2 + R^2 + rR)}{3}.$$

14. The diameters of the bases of the frustum of a cone measure 8 and 15 inches, respectively; the altitude is 18 inches. Find the volume.

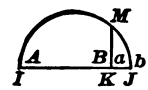
15. How many cubic inches of water will a pan hold, whose lower base is 12 inches in diameter, whose upper base is 16 inches in diameter, and whose depth is 6 inches? How many gallons?

1296. The pupils should make a frustum of a square pyramid of convenient size, and the three corresponding pyramids, as given in the rule. Fill the latter with sand, and pour the contents of all three into the frustum.

To make the frustum, draw two concentric circles. Lay off equal arcs, *AB*, *BD*, *DE*, *EF*. Draw the chords and radii from the extremities of each chord. Draw the chords *ab*, *bd*, *de*, and *ef*. Cut out, after constructing a square for either the upper or the lower base, and taking care to provide flaps for pasting.

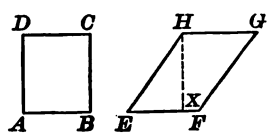


To get a mean proportional between ab and AB for one side of the base of the third pyramid, lay off a line IJ equal in length to $ab + AB$. On this line construct a semicircle. Make IK equal to AB, and at K erect a perpendicular KM. KM is a mean proportional between ab and AB.



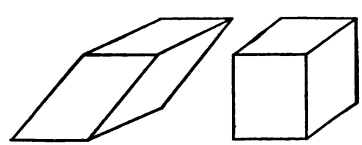
1297. Oblique Prisms.

We have seen that a rectangle, $ABCD$, and a parallelogram, $EFGH$, are equal in area when the bases, AB and EF, and the altitudes, AD and HX, are equal, each to each.



This can be shown by cutting both out of paper, and by shifting the triangle HEX to the right side of the parallelogram.

1298. In a somewhat similar way, we can show that an oblique prism is equal to a right prism that has an equal base and an equal altitude.



Make from a potato or a turnip an oblique prism having rectangular bases, and change it to a right prism of the same height by cutting and shifting a portion.

1299. The volume of *any* prism (or cylinder) is found by multiplying the area of the base by the altitude.

1300. In the same way it can be shown that the volume of any pyramid or cone is equal to the product of the area of the base by one-third the altitude.

1301. The Sphere.

A sphere may be considered as made up of a great number of pyramids whose bases together make the surface of the sphere,

and whose vertices all meet at the center of the sphere, making their altitudes each equal to the radius of the sphere.

The volume of a sphere is equal, therefore, to its surface $\times \frac{1}{3}$ radius.

$$\text{Surface} = 4 \text{ great circles} = 4\pi R^2;$$

$$\text{Volume} = 4\pi R^2 \times \frac{R}{3} \quad = \frac{4\pi R^3}{3}.$$

16. Find the volume of a sphere whose radius is 9 inches.

17. What is the volume of a sphere whose diameter is 9 inches?

Find the volume of a cone whose altitude is 9 inches, diameter of base 9 inches.

How does the volume of the cone compare with the volume of the sphere?

How does the volume of the sphere compare with the volume of a cylinder 9 inches in diameter and 9 inches high?

1302. Take a clay sphere of a convenient size. Make a paper cylinder that will exactly contain it, the height of the cylinder being equal to the diameter of the sphere. Make a hollow cone of the same diameter and altitude.

Place the sphere in the cylinder, carefully fill the cone with water, and pour it into the cylinder, which should then be filled to the top, showing that the volume of the cylinder is equal to that of the sphere and the cone together.

APPENDIX.

TABLES.

1303. Measures of Extension.

Long Measure.

12 inches (in.) . . . 1 foot (ft.)
3 feet . . . . . 1 yard (yd.)
5½ yards . . . . . 1 rod (rd.)
320 rods . . . . . 1 mile (mi.)

A furlong (fur.) = ⅛ mile.

Surveyors' Measure.

7.92 inches . . . . . 1 link (li.)
100 links . . . . . 1 chain (ch.)
80 chains . . . . . 1 mile

1 chain = 4 rods = 66 feet.

Square Measure.

144 sq. in. . . . . 1 square foot
9 sq. ft. . . . . 1 square yard
30¼ sq. yd. . . . . 1 square rod
160 sq. rd. . . . . 1 acre
640 acres . . . . 1 square mile

10 square chains = 1 acre.

Cubic Measure.

1728 cu. in. . . . 1 cubic foot
27 cu. ft. . . . . 1 cubic yard

1 cord = 128 cu. ft.
1 bushel = 2150.4 cu. in.
1 gallon = 231 cu. in.

Measures of Capacity.

Dry Measure.

2 pints (pt.) . . . 1 quart (qt.)
8 quarts . . . . . 1 peck (pk.)
4 pecks . . . . . 1 bushel (bu.)

Liquid Measure.

2 pints (pt.) . . . 1 quart (qt.)
4 quarts . . . . 1 gallon (gal.)

A gill is ¼ pint.

The capacity of tanks, vats, etc., is frequently expressed in barrels of 31½ gallons.

1 qt. dry measure = 67⅕ cu. in.
1 qt. liquid measure = 57¾ cu. in.

Measures of Weight.

Troy Weight.

24 grains (gr.) 1 pennyweight (pwt.)
20 pennyweights, 1 ounce (oz.)
12 ounces . . . 1 pound (lb.)

Troy weight is used in weighing gold, silver, precious stones, etc.

Apothecaries' Weight.

20 grains (gr) . 1 scruple (℈)
3 scruples . . . . . 1 dram (ʒ)
8 drams . . . . . 1 ounce (℥)
12 ounces . . . . 1 pound (℔)

Apothecaries' weight is used in prescriptions. Drugs are bought and sold by avoirdupois weight. The grain, the ounce, and the pound apothecaries' weight are the same as the corresponding denominations of troy weight.

AVOIRDUPOIS WEIGHT.

16 ounces (oz.) . 1 pound (lb.)
2000 pounds . . . . 1 ton (T.)

1 lb. avoirdupois = 7000 grains.
1 lb. troy = 5760 "
1 oz. avoirdupois = 437½ "
1 oz. troy = 480 "

In weighing ores and coal at the mines and in calculating duties at the U. S. custom houses, the following table is used:

28 pounds . . . . . 1 quarter (qr.)
4 quarters, 1 hundredweight (cwt.)
20 hundredweight . . . . 1 ton (T.)
1 cwt. = 112 lb. 1 T. = 2240 lb.

Measures of Value.

U. S. MONEY.

10 mills . . . . . 1 cent
100 cents . . . . . 1 dollar
1 dime = 10c.
1 eagle = $10.

ENGLISH MONEY.

12 pence (d.) . . 1 shilling (s)
20 shillings . . . 1 pound (£)
1 farthing = ¼ penny.
£1 = $4.8665.

The Canadian dollar is equal in value to that of the United States, and is also divided into 100 cents.

The French franc (fr.) = 19.3¢, is divided into 100 centimes (c.).

The German mark (reichsmark) (M., m.) = 23.8¢, is divided into 100 pfennigs (pf.).

Circular Measure.

60 seconds (″) . . 1 minute (′)
60 minutes . . . . 1 degree (°)
360 degrees . 1 circumference

Time Measure.

60 seconds (sec.) 1 minute (min.)
60 minutes . . . . .1 hour (hr.)
24 hours . . . . . 1 day (da.)
7 days . . . . . 1 week (wk.)
365 days 1 common year (yr.)
366 days . . . . . 1 leap year

Years divisible by 4 and not by 100 are leap years.

1892, 1896 are leap years.

Years divisible by 100 but not by 400 are common years.

1700, 1800, 1900 are common years; 1600 and 2000 are leap years.

1304. Time between Dates.

1. In the common method, by compound subtraction, each month is considered one of 30 days, regardless of its length.

EXAMPLE: Find the time between May 13, 1895 and March 2, 1899.

1899	3	2
1895	5	13
3	9	19

Taking 30 days to the month, the difference in time is found to be 3 years, 9 months, 19 days.

2. A more exact method is to take first the number of entire years, then the number of entire months, and lastly the number of days.

May 13, 1895 to May 13, 1898 = 3 years.
May 13, 1898 to Feb. 13, 1899 = 9 months.
Feb. 13, 1899 to Mar. 2, 1899 = 17 days.
Ans. 3 years, 9 months, 17 days.

3. Another method is to find the difference in years and days.

May 13, 1895 to May 13, 1898 = 3 years.
May 13, 1898 to Mar. 2, 1899 = 293 days.
Ans. 3 years, 293 days.

1305. Days of Grace.

Grace is not allowed on notes, time drafts, etc., in California, Colorado, Connecticut, District of Columbia, Florida, Idaho, Illinois, Maine, Maryland, Massachusetts, Montana, New Hampshire, New Jersey, New York, North Dakota, Ohio, Oregon, Pennsylvania, Utah, Vermont, and Wisconsin.

Answers in which days of grace are not included, are inclosed in a parenthesis.

When a note falls due on a Sunday or a legal holiday, it is generally payable on the next business day following. In this case, banks include the extra days in calculating discount. Some states, however, require payment on the next preceding business day.

In a majority of the states, days of grace are not allowed on sight drafts.

1306. Rates of Interest.

The following table shows the rates of interest allowed by law in the several states and territories, the first column giving the rate allowed where none is specified in the note or other document. Any rate up to the one given in the second column is permitted when an agreement is made in writing.

States and Territories.	Legal Rate.	Contract Rate.	States and Territories.	Legal Rate.	Contract Rate.
Alabama	8	8	Montana	10	Any
Arizona	7	Any	Nebraska	7	10
Arkansas	6	10	Nevada	7	Any
California	7	Any	New Hampshire	6	6
Colorado	8	Any	New Jersey	6	6
Connecticut	6	6	New Mexico	6	12
Delaware	6	6	New York	6	6
District of Columbia	6	10	North Carolina	6	6
Florida	8	10	North Dakota	7	12
Georgia	7	8	Ohio	6	8
Idaho	7	12	Oklahoma	7	12
Illinois	5	7	Oregon	8	10
Indiana	6	8	Pennsylvania	6	6
Indian Territory	6	10	Rhode Island	6	Any
Iowa	6	8	South Carolina	7	8
Kansas	6	10	South Dakota	7	12
Kentucky	6	6	Tennessee	6	6
Louisiana	5	8	Texas	6	10
Maine	6	Any	Utah	8	Any
Maryland	6	6	Vermont	6	6
Massachusetts	6	Any	Virginia	6	6
Michigan	6	8	Washington	7	12
Minnesota	7	10	West Virginia	6	6
Mississippi	6	10	Wisconsin	6	10
Missouri	6	8	Wyoming	8	12

PARTIAL PAYMENTS.

1307. Connecticut Rule.

Find the amount of the principal to the time when the payment, or the sum of two or more payments equals or exceeds the interest, if such time is one year or more from the time the interest commenced. From this amount deduct the payment or sum of payments. Use the balance then due as a new principal, and proceed as before.

If, however, any payments exceeding the interest then due, are made within a year from the time from which the interest is reckoned, the amount of the principal is calculated for one year, and from this amount are deducted the amounts of such payments from the time made until the end of the year. When the settlement is made in less than one year from the time from which interest is reckoned, the amount of the principal and those of the payments are calculated to the date of settlement.

1. NORWICH, CONN., Jan. 5, 1896.

Two years after date, we, jointly and severally, promise to pay to Emerson W. Keyes, or order, Three Hundred Dollars, value received, with interest at 6 per cent. JOSIAH H. PITTS,
300\frac{00}{100}$. CHARLES W. FIELD.

Endorsements: May 20, 1896, $100; Nov. 2, 1896, $100; March 17, 1897, $50.

How much is due Jan. 5, 1898?

Amount of $300 to Jan. 5, 1897, 1 year		$318.00
Amount of $100 from May 20, 1896 to Jan. 5, 1897, 7½ mo.	$103.75	
Amount of $100 from Nov. 2, 1896 to Jan. 5, 1897, 2 mo. 3 da.	101.05	204.80
Balance Jan. 5, 1897		$113.20
Interest on $113.20 to Jan. 5, 1898		6.79
Amount		$119.99
Amount of $50 from March 17, 1897 to Jan. 5, 1898, 9 mo. 18 da.		52.40
Due Jan. 5, 1898		$67.59

2. DANIELSONVILLE, CONN., April 15, 1896.

On demand, I promise to pay to the order of Robert P. Webb, Two Thousand Dollars, value received, with interest.

2000\frac{00}{100}$. JOHN J. BARNICLE.

On this note are endorsed the following payments: $50, Sept. 20, 1896; $100, May 26, 1897; $1,000, June 20, 1898. How much is due Dec. 27, 1899?

Face of note	$2000.00
As the first payment, $50, is not large enough to meet the interest then due, the interest on $2,000 is calculated to date of second payment, May 26, 1897, 1 yr. 1 mo. 11 da.	133.67
Amount due May 26, 1897	$2133.67
Less payments, $50 and $100	150.00
Balance May 26, 1897	$1983.67
Interest on $1,983.67 to June 20, 1898, 1 yr. 24 da.	126.95
Amount due June 20, 1898	$2110.62
Less third payment	1000.00
Balance June 20, 1898	$1110.62
Interest on $1110.62 to Dec. 27, 1899, 1 yr. 6 mo. 7 da.	101.25
Due Dec. 27, 1899	$1211.87

3. Find the amount due April 16, 1898, on an interest-bearing note for $1,000 drawn June 1, 1896; on which were paid $150, Sept. 16, 1896; and $50, Sept. 16, 1897.

4. NEW HAVEN, CONN., July 25, 1896.

On demand, I promise to pay to the order of Frank Regan, Jr., Five Hundred Dollars, with interest at 6%.

500\frac{00}{100}$. JAMES W. NAUGHTON.

Endorsements: Sept. 18, 1896, $100; Feb. 5, 1897, $200. How much is due April 1, 1897?

5. HARTFORD, CONN., Jan. 2, 1894.

Three years after date, we promise to pay to the order of George G. Brown, Eight Hundred Seventy and $\frac{50}{100}$ Dollars, value received, with interest.

870\frac{50}{100}$. WHITLOCK, TULLY, & Co.

Find the sum due at maturity on the above note, the following payments having been made: Sept. 12, 1895, $35; March 18, 1896, $200; Aug. 24, 1896, $250.

ANNUAL INTEREST.

1308. Partial Payments.

When payments have been made on a note subject to "annual interest," the amount due at any time is calculated as follows:

Find the amount of the principal by the method given for annual interest (Art. 1172) *until the end of the first year in which the amount of the payment or payments equals the interest then due. From the amount of the principal, subtract the amounts of the payments. Use the balance then due as a new principal, and proceed as before.*

6. LANSING, MICH., June 1, 1895.

For value received, I promise to pay to the order of Jere. Smith, Six Hundred Dollars, with interest annually at six per cent.

600\frac{00}{100}$. FRANK H. COOPER.

Endorsed: Feb 1, 1896, $30; July 1, 1896, $100; Oct. 1, 1897, $200.

How much is due Oct. 1, 1898?

Principal		$600.00
Interest to June 1, 1897, 2 years		72.00
Interest on $36 unpaid interest, 1 year		2.16
Amount June 1, 1897		$674.16
Amount of $30 Feb. 1, 1896 to June 1, 1897	$32.40	
Amount of $100 July 1, 1896 to June 1, 1897	105.50	137.90
New principal, June 1, 1897		$536.26
Interest to June 1, 1898		32.18
Amount June 1, 1898		$568.44
Amount of $200 Oct. 1, 1897 to June 1, 1898		208.00
New principal June 1, 1898		$360.44
Interest to Oct. 1, 1898		7.21
Amount due Oct. 1, 1898		$367.65

7. Hiram Stevens, of Hallowell, Me., borrowed of John Karst, June 15, 1895, seven hundred dollars, agreeing to repay it on demand with six per cent per annum, and six per cent on the deferred payments of annual interest. The following payments were made: Nov. 15, 1895, $20; Feb. 15, 1897, $80; Sept. 15, 1898, $15. Find the amount due Oct. 15, 1899.

1309. New Hampshire Rule.

In New Hampshire, partial payments on a note "with interest annually" are subject to the following rule:

If the payments during a year equal or exceed the interest due at the end of the year, the amount of the payments to that date is deducted from the amount of the principal.

If the payments are less than the interest due, the amount of these payments is used (1) *to cancel the interest due on the annual interest, and* (2) *to reduce the annual interest then due. But if no interest is due except what is accruing during the year, only the principal of the payments is thus applied, no interest being added.*

NOTE. — When a payment is less than the interest due on the annual interest, the remaining simple interest is not subject to interest.

8. PORTSMOUTH, N.H., June 1, 1895.

On demand I promise to pay to the order of W. M. Davis & Co., Six Hundred and $\frac{00}{100}$ Dollars, value received, with interest annually, at six per cent.

$600\frac{00}{100}$. HENRY C. McLEAN.

Endorsements: Feb. 1, 1896, $30; July 1, 1896, $100; Oct. 1, 1897, $200. How much is due Oct. 1, 1898?

Principal		$600.00
Annual interest due June 1, 1896	$36.00	
Payment Feb. 1, 1896	30.00	
Balance of interest due June 1, 1896	$6.00	
Annual interest due June 1, 1897	36.00	
Interest on $6 for 1 year	.36	42.36
Amount due June 1, 1897		$642.36

Brought forward	$642.36
Amount of $100 July 1, 1896 to June 1, 1897	105.50
New principal June 1, 1897	$536.86
Interest on $536.86 to June 1, 1898	32.21
Amount due June 1, 1898	$569.07
Amount of $200 Oct. 1, 1897 to June 1, 1898	208.00
New principal June 1, 1898	$361.07
Interest on $361.07 to Oct. 1, 1898	7.22
Due Oct. 1, 1898	$368.29

9. CONCORD, N.H., May 1, 1896.

I promise to pay Albert S. Caswell, on demand, Two Thousand Five Hundred Dollars, with interest annually.

2500\frac{00}{100}$. WALTER B. GOODNOUGH.

Endorsements: Oct. 1, 1896, $100; June 1, 1897, $1,000; Nov. 1, 1898, $50. Find the amount due Oct. 1, 1899.

10. How much is due March 11, 1899, upon a note for $1,000, with annual interest, drawn Jan. 3, 1895, on which the following payments have been made: June 1, 1895, $10; March 14, 1896, $10; Sept. 30, 1898, $500?

NOTE. — Ordinary interest-bearing notes on which partial payments have been made, are subject to the U.S. Rule.

1310. Vermont Rule.

Partial payments on "annual interest" notes are used in Vermont (1) to cancel the interest due on the annual interest; (2) to reduce the annual interest then due; (3) to reduce the principal.

RULE.

If the payments during a year equal or exceed the interest due at the end of the year, the total of the amounts of the payments is deducted from the amount of the principal.

If the total payments are less than the interest due, the amounts of these payments are used (1) *to cancel the interest due on the annual interest, and* (2) *to reduce the annual interest then due.*

NOTE. — When the amount of a payment is not sufficient to cancel all the interest due on the annual interest, the balance of this simple interest is not subject to interest.

11. BURLINGTON, VT., June 1, 1895.

Due Halsey Clark, Six Hundred Dollars, payable on demand, with interest at six per cent annually.

600\frac{00}{100}$. CHARLES HERMAN.

Endorsements: Feb. 1, 1896, $30; July 1, 1896, $100; Oct. 1, 1897, $200. How much is due Oct. 1, 1898?

Principal		$600.00
Annual interest due June 1, 1896	$36.00	
Amount of $30 Feb. 1, 1896 to June 1, 1896	30.60	
Balance of interest due June 1, 1896	$5.40	
Annual interest due June 1, 1897	36.00	
Interest on $5.40 for one year	.32	41.72
Amount due June 1, 1897		$641.72
Amount of $100 July 1, 1896 to June 1, 1897		105.50
New principal June 1, 1897		$536.22
Interest on $536.22 to June 1, 1898		32.17
Amount due June 1, 1898		$568.39
Amount of $200 Oct. 1, 1897 to June 1, 1898		208.00
New principal June 1, 1898		$360.39
Interest on $360.39 to Oct. 1, 1898		7.21
Due Oct. 1, 1898		$367.60

12. RUTLAND, VT., June 1, 1895.

I promise to pay Ahern & Bro., on demand, Two Thousand Five Hundred Dollars, with interest annually.

2500\frac{00}{100}$. MAGNUS SCHULER.

Endorsements: Oct. 1, 1896, $100; June 1, 1897, $1,000; Nov. 1, 1898, $50. Find the amount due Oct. 1, 1899.

13. Find the amount due March 11, 1899, upon a note for $1,000, with annual interest, drawn Jan. 3, 1895, on which the following payments have been made: June 1, 1895, $10; March 14, 1896, $10; Sept. 30, 1898, $500.

14. BENNINGTON, VT., March 17, 1896.

On demand, I promise to pay Francis de Raismes, Three Thousand Dollars, with interest annually.

3000\frac{00}{100}$. EDWARD HUTCHINSON.

Endorsements: May 17, 1899, $20; Sept. 17, 1902, $1000. How much is due March 17, 1903?

15. A demand note for $1200, with annual interest, dated Feb. 25, 1893, bears the following endorsements: Feb. 26, 1896, $400; June 11, 1898, $10; Sept. 26, 1900, $400. What sum will pay the note in full Feb. 26, 1902?

TAXES.

1311. The real estate of a town in which a tax of $3,300 is to be raised, is assessed at $180,000, and the personal property at $20,000. There are 100 taxable polls at $2 each. What is F. D. Clarke's tax, his real and personal property being assessed at $10,000, and who pays two poll taxes?

$2 × 100	= $200, amount to be raised by poll taxes.
$3,300 − $200	= $3,100, amount to be raised on property.
$180,000 + $20,000	= $200,000, assessed value of property.
$3,100 ÷ 200,000	= 15½ mills, tax on $1, or the rate.
$10,000 × .0155	= $155, tax on Mr. Clarke's property.
$2 × 2	= $4, Mr. Clarke's poll taxes.
$155 + $4	= $159, Mr. Clarke's total taxes.

1. Find the taxes of W. E. Pulsifer, of the above town, whose real property is assessed at $8,500, whose personal property is assessed at $600, and who pays one poll tax.

2. What is the rate of taxation in a town that proposes to raise $2,500, the real and personal property being valued at $275,000 and the number of polls at $2 each being 150?

1312. Vermont Method of Levying Taxes.

In Vermont, all male inhabitants between 21 and 70, unless specially exempted, are "listed" for poll taxes in their respective towns, at $2 each.

Real and personal property is placed in the town "list" at 1% of its appraised value.

The "grand list" comprises the total of the poll list and 1% of the assessed (appraised) value of the real and the personal property, and is the amount on which all taxes are levied.

3. The real estate of a town in which a tax of $3,300 is to be raised, is appraised at $180,000, and the personal property at $20,000. There are 100 taxable polls. What is John Griffin's tax, who pays two polls, the appraised value of his real property being $7,500 and of his personal property, $2,500?

$180,000 + $20,000	= $200,000, appraised value of real and personal property.
$\frac{1}{100}$ of $200,000 = $2,000	= 1% of appraised value.
$2 × 100	= $200, the poll list.
$2,000 + $200	= $2,200, the grand list.
$3,300 ÷ 2,200	= $1.50, rate on $1 of the grand list.
[1% of ($7,500 + $2,500)] + ($2 × 2)	= $104, John Griffin's grand list.
$1.50 × 104	= $156, John Griffin's town tax.

4. Find the city tax of Parker P. Simmons, whose property, real and personal, is assessed at $9,500, and who pays one poll, the rate being $2.45.

5. How many taxable polls are there in a town in which the appraised value of the real and personal property is $150,000, the tax rate necessary to raise $3,600 being $2?

Let x = the number of polls; $2x$ = poll list.

6. Henry W. Hallock's taxes on real and personal property, appraised at $12,000, amount to $252, including 3 poll taxes. The total amount levied is $6,800. Find the appraised value of all the property, the number of taxable polls being 200.

1313. Values of Foreign Coins.

The following estimate by the Director of the Mint, of the values of foreign coins, has been proclaimed by the Secretary of the Treasury, January 1, 1897:

VALUES OF FOREIGN COINS.

Country.	Standard.	Monetary unit.		Value in terms of U. S. gold dollar.
Argentine Republic	Gold and silver	Peso		$0.96,5
Austria-Hungary	Gold	Crown		.20,3
Belgium	Gold and silver	Franc		.19,3
Bolivia	Silver	Boliviano		.47,4
Brazil	Gold	Milreis		.54,6
British Possessions N. A. (except Newfoundland)	Gold	Dollar		1.00,0
Central Amer. States	Silver	Peso		.47,4
Chile	Gold	Peso		.86,5
China	Silver	Tael	Amoy	.76,7
			Canton	.76,5
			Chefoo	.73,3
			Chin Kiang	.74,9
			Fuchau	.70,9
			Haikwan	.78,0
			Hankow	.71,7
			Niuchwang	.71,9
			Ningpo	.73,7
			Shanghai	.70,0
			Swatow	.70,8
			Takau	.77,2
			Tientsin	.74,3
Colombia	Silver	Peso		.47,4
Cuba	Gold and silver	Peso		.92,6
Denmark	Gold	Crown		.26,8
Ecuador	Silver	Sucre		.47,4
Egypt	Gold	Pound (100 piasters)		4.94,3
Finland	Gold	Mark		.19,3
France	Gold and silver	Franc		.19,3
German Empire	Gold	Mark		.23,8
Great Britain	Gold	Pound sterling		4.86,6½
Greece	Gold and silver	Drachma		.19,3
Haiti	Gold and silver	Gourde		.96,5
India	Silver	Rupee		.22,5
Italy	Gold and silver	Lira		.19,3
Japan	Gold and silver	Yen	Gold	.99,7
			Silver	.51,1
Liberia	Gold	Dollar		1.00,0
Mexico	Silver	Dollar		.51,5
Netherlands	Gold and silver	Florin		.40,2
Newfoundland	Gold	Dollar		1.01,4
Norway	Gold	Crown		.26,8
Persia	Silver	Kran		.08,7
Peru	Silver	Sol		.47,4
Portugal	Gold	Milreis		1.08,0
Russia	Silver	Ruble	Gold	.77,2
			Silver	.37,9
Spain	Gold and silver	Peseta		.19,3
Sweden	Gold	Crown		.26,8
Switzerland	Gold and silver	Franc		.19,3
Turkey	Gold	Piaster		.04,4
Uruguay	Gold	Peso		1.03,4
Venezuela	Gold and silver	Bolivar		.19,3

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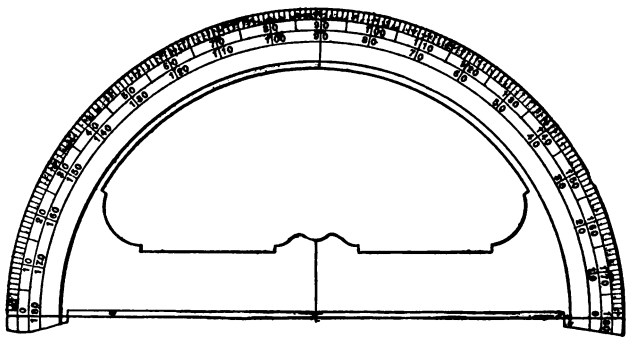

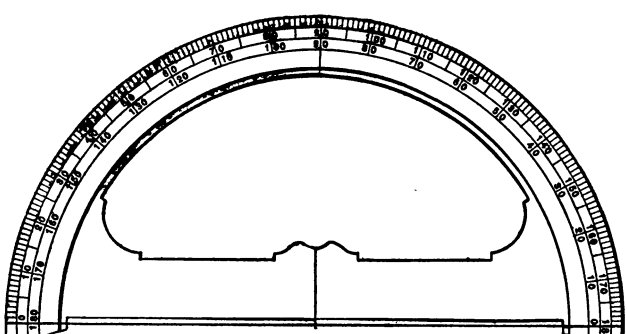

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